1-Span Cast-in-Place Post-Tensioned Concrete Box Girder [CIPPTCBGB] Bridge Example

Cast-in-place post-tensioned concrete box girder bridge. The bridge has a 160 feet span with a 15 degree skew. Standard ADOT 32-inch f-shape barriers will be used resulting in a bridge configuration of 1'-5" barrier, 12'-0" outside shoulder, two 12'-0" lanes, a 6'-0" inside shoulder and a 1'-5" barrier. The overall out-to-out width of the bridge is 44'-10". A plan view and typical section of the bridge are shown in Figures 1 and 2.

The following legend is used for the references shown in the left-hand column:

[2.2.2]	AASHTO LRFD Specification Article Number
[2.2.2-1]	AASHTOLRFD Specification Table or Equation Number
[C2.2.2]	AASHTO LRFD Specification Commentary
[A2.2.2]	AASHTO LRFD Specification Appendix
[BDG]	ADOT LRFD Bridge Design Guidelines

Bridge Geometry

Bridge length	160.00 ft
Bridge width	44.83 ft
Roadway width	42.00 ft
Superstructure depth	7.50 ft
Web spacing	9.25 ft
Web thickness	12.00 in
Top slab thickness	8.50 in
Bottom slab thickness	6.00 in
Deck overhang	3.33 ft

Minimum Requirements

The minimum span to depth ratio for a single span bridge should be taken as 0.045 resulting in a minimum depth of 7.20 feet. Use 7'-6"

[Table 2.5.2.6.3-1]

The minimum top slab thickness shall be as shown in the LRFD Bridge Design Guidelines. For a centerline spacing of 9.25 feet, the effective length is 8.25 feet resulting in a minimum thickness of 8.50 inches.

[9.7.1.1]
[BDG]

The minimum bottom slab thickness shall be the larger of:

1/30 the clear web spacing = (8.25)(12)/30 = 3.30 inches
6.0 inches

[5.14.1.5.1b]
[BDG]

The minimum thickness of the web shall be 12 inches.

[C5.14.1.5.1c]
[5.14.1.5.1c]
[BDG]

Concrete Deck Slab Minimum Requirements

Slab thickness	8.50 in
Top concrete cover	2.50 in
Bottom concrete cover	1.00 in
Wearing surface	0.50 in

This example illustrates the design of a single span cast-

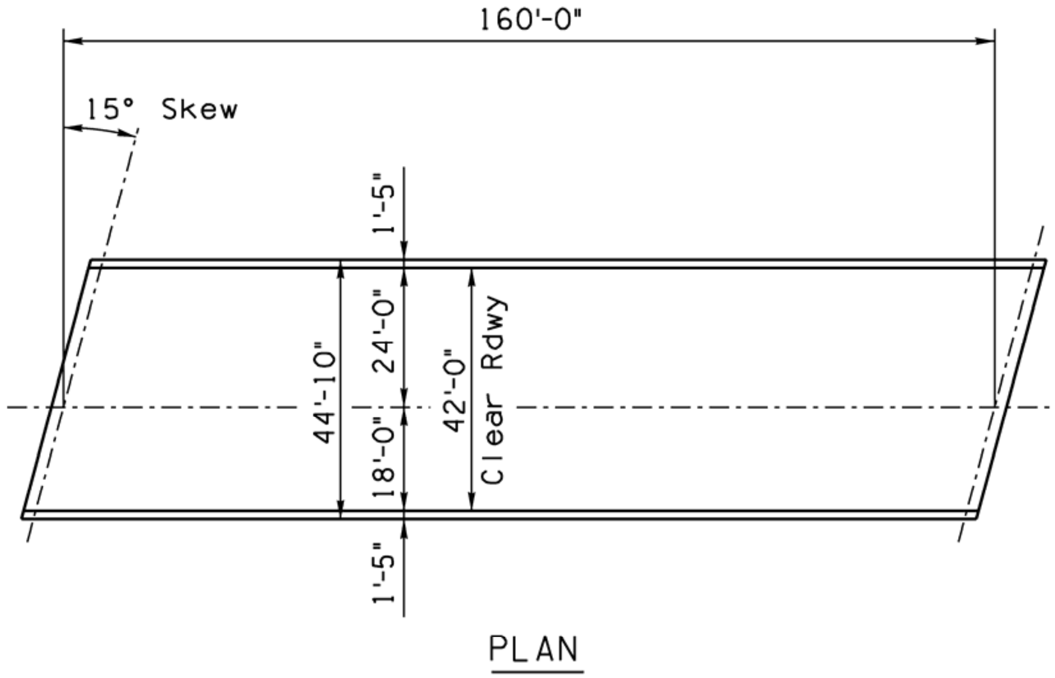


Figure 1

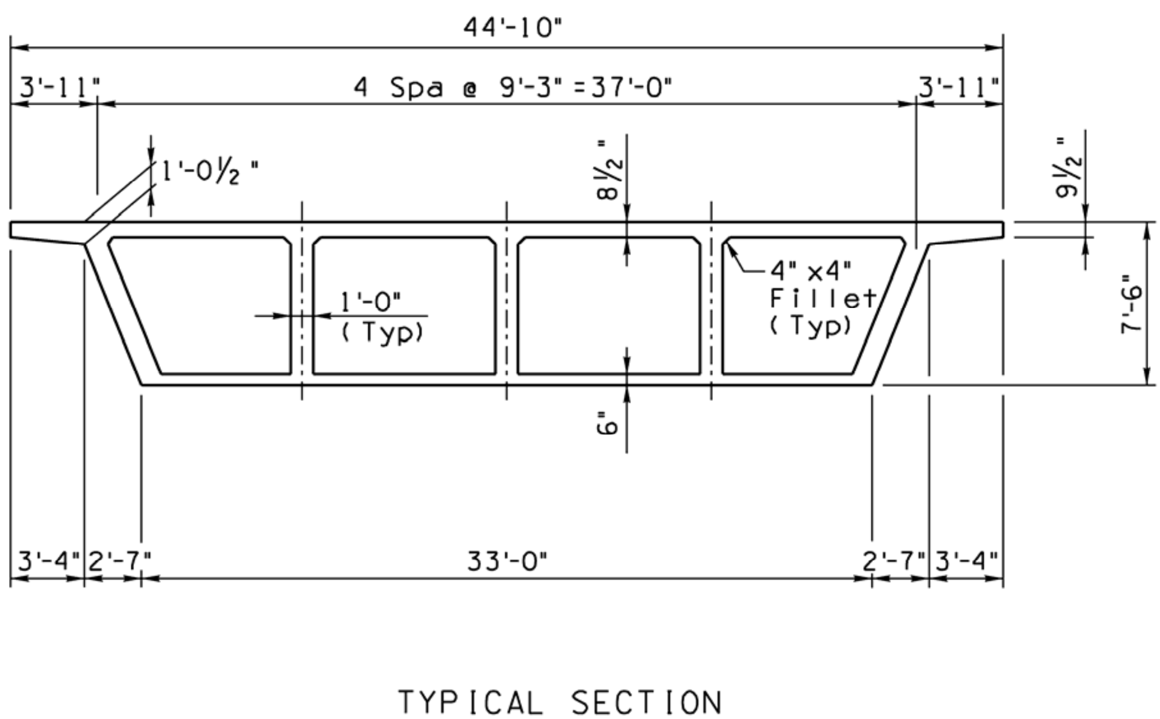


Figure 2

Material Properties

Reinforcing Steel

[5.4.3.1] Yield Strength f_y = 60 ksi

[5.4.3.2] Modulus of Elasticity E_s = 29,000 ksi

Prestressing Strand

Low relaxation prestressing strands

0.6" diameter strand A_{ps} = 0.217 in^2

[Table 5.4.4.1-1] Tensile Strength f_{pu} = 270 ksi

Yield Strength f_{py} = 243 ksi

[5.4.4.2] Modulus Elasticity E_p = 28500 ksi

[5.4.2.1] **Concrete**

[BDG] The final and release concrete strengths are specified below:

<u>Superstructure</u>	<u>Substructure</u>
f'_c = 4.5 ksi	f'_c = 3.5 ksi
f'_{ci} = 3.5 ksi	

[Table 3.5.1-1] Unit weight for normal weight concrete is listed below:

Unit weight for computing E_c = 0.145 kcf

[C3.5.1] Unit weight for DL calculation = 0.150 kcf

[C5.4.2.4] The modulus of elasticity for normal weight concrete where the unit weight is 0.145 kcf may be taken as shown below:

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{4.5} = 3861\,ksi$$

$$E_{ci} = 1820\sqrt{f'_{ci}} = 1820\sqrt{3.5} = 3405\,ksi$$

[5.7.1] The modular ratio of reinforcing to concrete should be rounded to the nearest whole number.

$$n = \frac{29000}{3861} = 7.51 \text{ Use n = 8}$$

[5.7.2.2] β_1 = The ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone stress block.

$$\beta_1 = 0.85 - 0.05 \cdot \left[\frac{f'_c - 4.0}{1.0}\right] = 0.85 - 0.05 \cdot \left[\frac{4.5 - 4.0}{1.0}\right] = 0.825$$

Modulus of Rupture [5.4.2.6]

The modulus of rupture for normal weight concrete has two values. When used to calculate service level cracking, as specified in Article 5.7.3.4 for side reinforcing or in Article 5.7.3.6.2 for determination of deflections, the following equation should be used:

Service Level Cracking

$$f_r = 0.24\sqrt{f'_c}$$

For superstructure calculations:

$$f_r = 0.24\sqrt{4.5} = 0.509\ ksi$$

For substructure calculations:

$$f_r = 0.24\sqrt{3.5} = 0.449\ ksi$$

When the modulus of rupture is used to calculate the cracking moment of a member for determination of the minimum reinforcing requirement as specified in Article 5.7.3.3.2, the following equation should be used:

Minimum Reinforcing

$$f_r = 0.37\sqrt{f'_c}$$

For superstructure calculations:

$$f_r = 0.37\sqrt{4.5} = 0.785\ ksi$$

For substructure calculations:

$$f_r = 0.37\sqrt{3.5} = 0.692\ ksi$$

Limit States
[1.3.2]

In the LRFD Specification, the general equation for design is shown below:

$$\sum \eta_i \gamma_i Q_i \leq \varphi R_n = R_r$$

For loads for which a maximum value of γ_i is appropriate:

$$\eta_i = \eta_D \eta_R \eta_I \geq 0.95$$

For loads for which a minimum value of γ_i is appropriate:

$$\eta_i = \frac{1}{\eta_D \eta_R \eta_I} \leq 1.0$$

[1.3.3]

Ductility
For strength limit state for conventional design and details complying with the LRFD Specifications and for all other limit states:

[3.4.1]
[BDG]

$$\eta_D = 1.00$$

Redundancy
For the strength limit state for conventional levels of redundancy and for all other limit states:

[1.3.4]

$$\eta_R = 1.0$$

[1.3.5]

Operational Importance
For the strength limit state for typical bridges and for all other limit states:

$$\eta_I = 1.0$$

[3.4.1]
[BDG]

For an ordinary structure with conventional design and details and conventional levels of ductility, redundancy, and operational importance, it can be seen that η_i = 1.0 for all cases. Since multiplying by 1.0 will not change any answers, the load modifier η_i has not been included in this example.

For actual designs, the importance factor may be a value other than one. The importance factor should be selected in accordance with the ADOT LRFD Bridge Practice Guidelines.

DECK DESIGN

[BDG]

As bridges age, decks are one of the first element to show signs of wear and tear. As such ADOT has modified some LRFD deck design criteria to reflect past performance of decks in Arizona. Section 9 of the Bridge Design Guidelines provides a thorough background and guidance on deck design.

Effective Length
[9.7.2.3]

[BDG]

ADOT Bridge Design Guidelines specify that deck design be based on the effective length rather than the centerline-to-centerline distance specified in the AASHTO LRFD Specification. The effective length for monolithic cast-in-place concrete is the clear distance between supports. For this example with center line-to-center line web spacing of 9.25 feet and web width of 12 inches, the effective length is 8.25 feet. The resulting minimum deck slab thickness per ADOT guidelines is 8.50 inches.

Method of Analysis

[9.6.1]
[BDG]

In-depth rigorous analysis for deck design is not warranted for ordinary bridges. The empirical design method specified in [9.7.2] is not allowed by ADOT Bridge Group. Therefore the approximate elastic methods specified in [4.6.2.1] will be used. Dead load analysis will be based on a strip analysis using the simplified moment equation of [$w\ S^2 / 10$] where “S” is the effective length.

Live Loads
[A4.1]

[9.6.1]
[BDG]

The unfactored live loads found in Appendix A4.1 will be used. Multiple presence and dynamic load allowance are included in the chart. Since ADOT bases deck design on the effective length, the chart should be entered under S equal to the effective length of 8.25 feet rather than the centerline-to-centerline distance of 9.25 feet. Since the effective length is used the correction for negative moment from centerline of the web to the design section should be zero. Entering the chart yields the following live load moments:

Pos M = 5.83 ft-k/ft
Neg M = -6.58 ft-k/ft (0 inches from centerline)

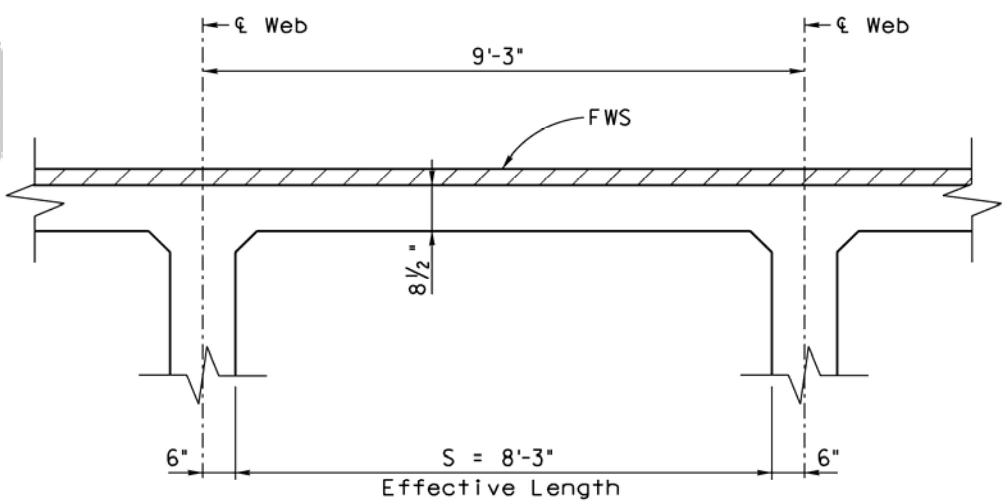


Figure 3

Positive Moment Design

A summary of positive moments follows:

DC Loads
Deck $M_{DC} =$ [0.150(8.50 / 12)(8.25)^2] ÷ 10 = 0.72 ft-k / ft

DW Loads
FWS $M_{DW} =$ [0.025(8.25)^2] ÷ 10 = 0.17 ft-k / ft

Vehicle
LL + IM = 5.83 ft-k / ft

Service I Limit State [9.5.2] [BDG]

Deck design is normally controlled by the service limit state. The working stress in the deck is calculated by the standard methods used in the past. For this check Service I moments should be used.

$$M_s = 1.0 \cdot (M_{DC} + M_{DW}) + 1.0 \cdot (M_{LL+IM})$$

M_s = 1.0(0.72 + 0.17) + 1.0(5.83) = 6.72 ft-k / ft

[9.4] [9.7.1.1] [BDG]

Try #5 reinforcing bars

d_s = 8.50 – 0.50 ws - 1 clr – 0.625 / 2 = 6.69 in

Determine approximate area reinforcing as follows:

[9.5.2] [BDG]

$$A_s \approx \frac{M_s}{f_s j d_s} = \frac{(6.72)\cdot(12)}{(24.0)\cdot(0.9)\cdot(6.69)} = 0.558\ \text{in}^2$$

Try #5 @ 6 ½ inches

A_s = (0.31)(12 / 6.50) = 0.572 in^2

The allowable stress for a deck under service loads is not limited by the LRFD Specifications. The 2006 Interim Revisions replaced the direct stress check with a maximum spacing requirement to control cracking. However, the maximum allowable stress in a deck is limited to 24 ksi per the ADOT LRFD Bridge Design Guidelines.

$$p = \frac{A_s}{bd_s} = \frac{0.572}{(12)\cdot(6.69)} = 0.007125$$

np = 8(0.007125) = 0.05700

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.05700) + (0.05700)^2} - 0.05700 = 0.285$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.285}{3} = 0.905$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(6.72)\cdot(12)}{(0.572)\cdot(0.905)\cdot(6.69)} = 23.29 \text{ ksi} < 24 \text{ ksi}$$

Since the applied stress is less than 24 ksi, the LRFD Bridge Design Guidelines service limit state requirement is satisfied.

Control of Cracking [5.7.3.4]

For all concrete components in which the tension in the cross-section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in equation 5.7.3.4-1 shall be satisfied.

$f_{sa} = 0.80f_r = 0.80(0.24\sqrt{f'_c}) = 0.80(0.509) = 0.407$ ksi

$S_{cr} = [(12.00)(8.00)^2] \div 6 = 128 \text{ in}^3$

$$f_{cr} = \frac{M_s}{S_{cr}} = \frac{(6.72)\cdot(12)}{128} = 0.630 \text{ ksi} > f_{sa} = 0.407 \text{ ksi}$$

Since the service limit state stress exceeds the allowable, the spacing, s, of mild steel reinforcing in the layer closest to the tension force shall satisfy the following:

[5.7.3.4-1]

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

where

Determine stress due to service moment:

γ_e = 0.75 for Class 2 exposure condition for decks

d_c = 1.0 clear + (0.625 ÷ 2) = 1.31 inches

h_{net} = 8.00 inches

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{1.31}{0.7 \cdot (8.00 - 1.31)} = 1.28$$

$$s \le \frac{(700)\cdot(0.75)}{(1.28)\cdot(23.29)} - (2)\cdot(1.31) = 14.99 \text{ in}$$

Since the spacing of 6.50 inches is less than 14.99 the cracking criteria is satisfied.

Strength I Limit State [Table 3.4.1-1]

Factored moment for Strength I is as follows:

$$M_n = \gamma_{DC}(M_{DC}) + \gamma_{DW}(M_{DW}) + 1.75(M_{LL+IM})$$

$$M_n = 1.25 \cdot (0.72) + 1.50 \cdot (0.17) + 1.75 \cdot (5.83) = 11.36 \text{ ft-k}$$

Flexural Resistance [5.7.3] [5.7.3.2.2-1]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \phi M_n = \phi A_s f_y \left(d_s - \frac{a}{2} \right)$$

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.572)\cdot(60)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(12)} = 0.906 \text{ in}$$

[5.7.3.2.3]

$a = \beta_1 c$ = (0.825)(0.906) = 0.75 in

The tensile strain must be calculated as follows:

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_s}{c} - 1 \right) = 0.003 \cdot \left(\frac{6.69}{0.906} - 1 \right) = 0.019$$

[5.5.4.2.1]

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 0.90$.

$$M_r = (0.90)\cdot(0.572)\cdot(60)\cdot\left(6.69 - \frac{0.75}{2}\right) \div 12 = 16.25 \text{ ft-k}$$

f_s = 23.29 ksi

Since the flexural resistance, M_r, is greater than the factored moment, M_n, the strength limit state is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated this limit. Below a net tensile strain in the extreme tension steel of 0.005, the factored resistance is reduced as the tension reinforcement quantity increases. This reduction compensates for the decreasing ductility with the increasing overstrength.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_n. A conservative simplification for positive moments is to ignore the 0.5 inch wearing surface for this calculation. If this check is satisfied there are no further calculations required. If the criterion is not satisfied one check should be made with the wearing surface subtracted and one with the full section to determine which of the two is more critical.

$$S_c = (12.0)(8.50)^2 / 6 = 144.5 \text{ in}^3$$
$$f_r = 0.37\sqrt{f'_c} = 0.785\ ksi$$

$$1.2M_{cr} = 1.2 f_r S_c = (1.2)\cdot(0.785)\cdot(144.5) \div 12 = 11.34 \text{ ft-k}$$
$$1.2M_{cr} = 11.34 \le M_r = 16.25 \text{ ft-k}$$

$\therefore$ The minimum reinforcement limit is satisfied.

Fatigue Limit State [9.5.3] & [5.5.3.1]

Fatigue need not be investigated for concrete deck slabs in multi-girder applications.

The interior deck is adequately reinforced for positive moment using #5 @ 6½ inches

Distribution Reinforcement [9.7.3.2]

Reinforcement shall be placed in the secondary direction in the bottom of slabs as a percentage of the primary reinforcement for positive moments as follows:

$$\frac{220}{\sqrt{S}} = \frac{220}{\sqrt{8.25}} = 77 \text{ percent} > 67 \text{ percent maximum}$$

Use 67% Maximum.

$A_s = 0.67(0.572) = 0.383 \text{ in}^2$

Use #5 @ 9” $\Rightarrow A_s = 0.413 \text{ in}^2$

The LRFD Specification does not allow for a reduction of this reinforcing in the outer quarter of the span as was allowed in the Standard Specifications.

Skewed Decks [9.7.1.3] [BDG]

For bridges with skews less than 20 degrees, the ADOT LRFD Bridge Design Guidelines specifies that the primary reinforcing shall be placed parallel to the skew. For the 15 degree skew in this example, the transverse deck reinforcing is placed parallel to the skew.

Negative Moment Design

A summary of negative moments follows:

DC Loads

Deck $M_{DC} = 0.150\ (8.50 / 12)(8.25)^2 \div 10$ = -0.72 ft-k

DW Loads

FWS $M_{WS} = 0.025\ (8.25)^2 \div 10$ = -0.17 ft-k

Vehicle

LL + IM $M_{(LL+IM)}$ = -6.58 ft-k

Service I Limit State [9.5.2] [BDG] [Table 3.4.1-1]

Deck design is normally controlled by the service limit state. The working stress in the deck is calculated by the standard methods. For this check Service I moments should be used.

$$M_s = 1.0(M_{DC} + M_{DW}) + 1.0(M_{LL+IM})$$

M_s = 1.0(0.72 + 0.17) + 1.0(6.58) = 7.47 ft-k

Try #5 reinforcing bars

d_s = 8.50 – 2.50 clear – 0.625 / 2 = 5.69 inches

Determine approximate area reinforcing as follows:

$$A_s \approx \frac{M_s}{f_s jd_s} = \frac{(7.47)\cdot(12)}{(24.0)\cdot(0.9)\cdot(5.69)} = 0.729\ \text{in}^2$$

Try #5 @ 5 inches

$A_s = (0.31)(12 / 5) = 0.744\ \text{in}^2$

Allowable Stress

Determine stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.744}{(12)\cdot(5.69)} = 0.01090$$

np = 8(0.01090) = 0.08720

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.08720) + (0.08720)^2} - 0.08720 = 0.339$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.339}{3} = 0.887$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(7.47)\cdot(12)}{(0.744)\cdot(0.887)\cdot(5.69)} = 23.87 \text{ ksi} \le 24.0 \text{ ksi}$$

[9.5.2]
[BDG]

Since the applied stress is less than the allowable specified in the ADOT LRFD Bridge Design Guidelines, the service limit state stress requirement is satisfied.

Control of Cracking
[5.7.3.4]

The deck must be checked for control of cracking. For all concrete components in which the tension in the cross section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in Equation 5.7.3.4-1 shall be satisfied.

$f_{sa} = 0.80f_r = 0.80(0.24\sqrt{f'_c}) = 0.80(0.509) = 0.407$ ksi

$S_{cr} = (12.00)(8.00)^2 \div 6 = 128$ in^3

$$f_{cr} = \frac{M_s}{S_{cr}} = \frac{(7.47)\cdot(12)}{128} = 0.700 \text{ ksi} > f_{sa} = 0.407 \text{ ksi}$$

Since the service limit state stress exceeds the allowable, the spacing, s, of mild steel reinforcing in the layer closest to the tension force shall satisfy the following:

[5.7.3.4-1]

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

$\gamma_e = 0.75$ for Class 2 exposure condition for decks

$d_c = 2.50$ clear $+ 0.625 \div 2 = 2.81$ inches

$f_s = 23.87$ ksi

$h = 8.50$ inches

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{2.81}{0.7\cdot(8.50 - 2.81)} = 1.71$$

$$s \le \frac{(700)\cdot(0.75)}{(1.71)\cdot(23.87)} - (2)\cdot(2.81) = 7.24 \text{ in}$$

Since the spacing of 5 inches is less than 7.24 the cracking criteria is satisfied.

Strength I Limit State [3.4.1]

Factored moment for Strength I is as follows:

$$Mn = \gamma_{DC}\left(M_{DC}\right) + \gamma_{DW}\left(M_{DW}\right) + 1.75\left(M_{LL+IM}\right)$$

$$Mn = 1.25 \cdot (0.72) + 1.50 \cdot (0.17) + 1.75 \cdot (6.58) = 12.67 \text{ ft-k}$$

Flexural Resistance [5.7.3]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \phi M_n = \phi A_s f_y \left(d_s - \frac{a}{2} \right)$$

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.744)\cdot(60)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(12)} = 1.179 \text{ in}$$

[5.7.3.2.3]

$a = \beta_1 c$ = (0.825)(1.179) = 0.97 inches

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_s}{c} - 1 \right) = 0.003 \cdot \left(\frac{5.69}{1.179} - 1 \right) = 0.011$$

[5.5.4.2.1]

Since $\varepsilon_T > 0.005$, the member is tension controlled and φ= 0.90.

$$M_r = (0.90)\cdot(0.744)\cdot(60)\cdot\left(5.69 - \frac{0.97}{2}\right) \div 12 = 17.43 \text{ ft-k}$$

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the strength limit state is satisfied.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u. The most critical cracking load for negative moment will be caused by ignoring the 0.5 inch wearing surface and considering the full depth of the section.

S_c = 144.5 in^3

$$1.2M_{cr} = 1.2 f_r S_c = (1.2)\cdot(0.785)\cdot(144.5) \div 12 = 11.34 \text{ ft-k}$$

$$1.2M_{cr} = 11.34 \le M_r = 17.43 \text{ ft-k}$$

$\therefore$ The minimum reinforcement limit is satisfied.

Fatigue Limit State [9.5.3] & [5.5.3.1]

Fatigue need not be investigated for concrete deck slabs in multi girder applications.

The interior deck is adequately reinforced for negative moment using #5 @ 5 inches.

Shear [C4.6.2.1.6]

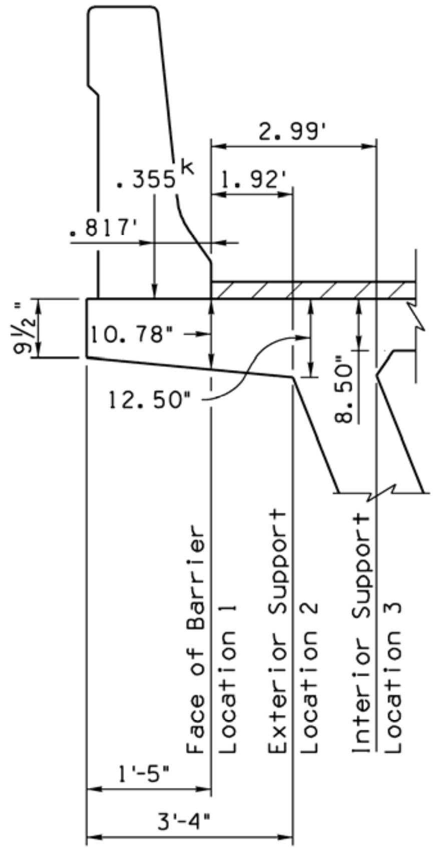
Past practice has been not to check shear in typical decks. For a standard concrete deck shear need not be investigated.

OVERHANG DESIGN [A13.4.1]

The overhang shall be analyzed for the three design cases described below:

Design Case 1

Design Case 1: Transverse forces specified in Article A13.2
Extreme Event Limit Combination II Limit State



DESIGN CASE 1

Figure 4

The deck overhang must be designed to resist the forces from a railing collision using the forces given in Section 13, Appendix A, Table A13.2-1. A TL-4 railing is generally acceptable for the majority of applications on major roadways and freeways. A TL-4 rail will be used. A summary of the design forces is shown below:

[Table A13.2-1]

Design Forces		Units
F_t, Transverse	54.0	kips
F_l, Longitudinal	18.0	kips
F_v, Vertical Down	18.0	kips
L_t and L_l	3.5	feet
L_v	18.0	feet
H_e Minimum	32.0	inch

Barrier Design

The philosophy behind the overhang analysis is that the deck should be stronger than the barrier. This ensures that any damage will be done to the barrier which is easier to repair and that the assumptions made in the barrier analysis are valid. The forces in the barrier must be known to analyze the deck. Required design values for the ADOT 32-inch F-shape barrier shown in SD-1.01 are published in the Bridge Design Guidelines and are repeated below:

[Section 9]
[BPG]

M_b = 0 ft-k
M_c = 6.17 ft-k
M_w = 28.66 ft-k

Concrete Railing

R_w = total transverse resistance of the railing.
L_c = critical length of yield line failure. See Figures 5 and 6.

<u>For impacts within a wall segment:</u>

[A13.3.1-1]

$$R_w = \left(\frac{2}{2L_c - L_t}\right)\left(8M_b + 8M_w + \frac{M_c L_c^{\,2}}{H}\right)$$

[A13.3.1-2]

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_w)}{M_c}}$$

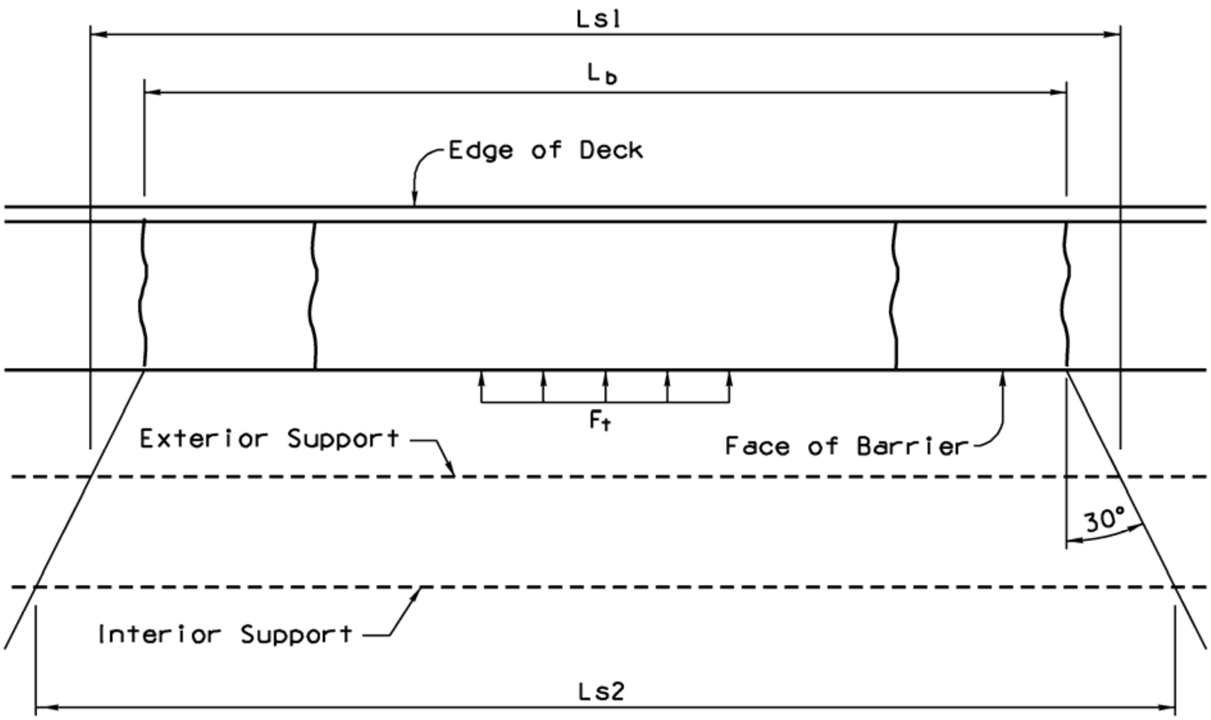
Substituting values for the 32-inch barrier yields:

$$L_c = \frac{3.50}{2} + \sqrt{\left(\frac{3.50}{2}\right)^2 + \frac{8\cdot(2.67)\cdot(0+28.66)}{6.17}} = 11.86\,\text{ft}$$

$$R_w = \left(\frac{2}{2\cdot(11.86)-3.50}\right)\cdot\left(8\cdot(0)+8\cdot(28.66)+\frac{(6.17)\cdot(11.86)^2}{2.67}\right) = 54.83\,\text{k}$$

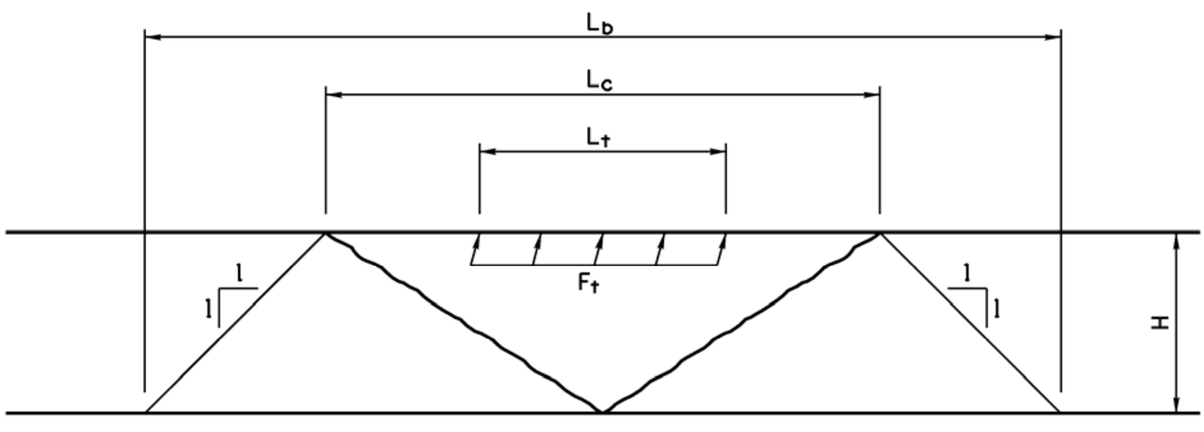
Since R_w = 54.83 kips is greater than F_t = 54.0 kips, the barrier is adequately designed for impacts within a wall segment.

At the expansion joint opening the barrier must also be investigated for impact. This is not demonstrated in this example.



PLAN

Figure 5



ELEVATION

Figure 6

Barrier Connection To Deck

The strength of the attachment of the barrier to the deck must also be checked. The deck will only see the lesser of the strength of the barrier or the strength of the connection. For the 32-inch barrier, #4 reinforcing at 16 inches connects the barrier to the deck.

$A_s = (0.20)(12) / (16) = 0.150 \text{ in}^2$

$d_s = 14.75 – 1 \tfrac{1}{2} \text{ clear} – 0.50 / 2 = 13.00 \text{ inches}$

Flexure

For a reinforcing bar not parallel to the compression face only the parallel component is considered. The #4 reinforcing is oriented at an angle of 26 degrees.

$$c = \frac{A_s f_y \cos\theta}{0.85 f'_c \beta_1 b} = \frac{(0.150)\cdot(60)\cos(26)}{(0.85)\cdot(4.0)\cdot(0.85)\cdot(12)} = 0.233 \text{ in}$$

$a = \beta_1 c = (0.85)(0.233) = 0.20 \text{ inches}$

$$M_n = A_s f_y \cos(\theta)\left(d_s - \frac{a}{2}\right)$$

$$M_n = (0.150)\cdot(60)\cdot\cos(26)\cdot\left(13.00 - \frac{0.20}{2}\right) \div 12 = 8.70 \text{ ft-k / ft}$$

$\varphi = 1.0$ for extreme event

[1.3.2.1]

$\varphi M_n = (1.00)(8.70) = 8.70 \text{ ft-k / ft}$

$\varphi P_u = (8.70)(12) \div (32) = \underline{3.261 \text{ k/ft}}$

Shear

The barrier to deck interface must also resist the factored collision load. The normal method of determining the strength is to use a shear friction analysis. However, in this case with the sloping reinforcing, the horizontal component of reinforcing force will also directly resist the horizontal force.

$R_n = A_s f_y \sin\theta = (0.150)(60) \sin (26) = 3.945 \text{ k/ft}$

The strength of the connection is limited by the lesser of the shear or flexural strength. In this case, the resistance of the connection is controlled by flexure with a value equal to 3.261 k/ft.

The design of the deck overhang is complicated because both a bending moment and a tension force are applied. The problem can be solved using equilibrium and strain compatibility as described in Appendix A of Example 1. A simplified design method is presented here.

Face of Barrier
Location 1
Figure 4

The design horizontal force in the barrier is distributed over the length L_b equal to L_c plus twice the height of the barrier. See Figures 5 and 6.

L_b = 11.86 + 2(2.67) = 17.20 ft
P_u = 54.83 / 17.20 = 3.188 k/ft < 3.261 k/ft per connection

Dimensions

h = 9.50 + (3.00) (1.42) / (3.33) = 10.78 in
d_1 = 10.78 – 2.50 clr – 0.625 / 2 = 7.97 in

Moment at Face of Barrier

Deck	$= 0.150(9.50 / 12)(1.42)^2 \div 2$	= 0.12 ft-k
	$0.150(1.28 / 12)(1.42)^2 \div 6$	= <u>0.01 ft-k</u>
		= 0.13 ft-k
Barrier	= 0.355(0.817)	= 0.29 ft-k

[Article 3.5.1]
[BDG]

Collision = 3.188[2.67 + ((10.78/12) / 2)] = 9.94 ft-k

The load factor for dead load shall be taken as 1.0.

[A13.4.1]

M_u = 1.00(0.13 + 0.29) + 1.00(9.94) = 10.36 ft-k

Extreme Event II
[3.4.1]

e = M_u / P_u = (10.36)(12) / (3.188) = 39.00 in

Assume the top layer of the deck reinforcing yields and f_s = 60 ksi. The resulting force in the top layer of reinforcing (#5 @ 5”) is:

T_1 = (0.744)(60) = 44.64 k

Simplified Method

A simplified method of analysis is available. If only the top layer of reinforcing is considered in determining strength, the assumption can be made that the reinforcing will yield. By assuming the safety factor for axial tension is 1.0 the strength equation can be solved directly. This method will determine whether the section has adequate strength. However the method does not consider the bottom layer of reinforcing does not maintain the required constant eccentricity and does not determine the maximum strain. For an in-depth review of the development of this equation refer to Appendix A of this Example 1.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c b} = \dfrac{44.64 - 3.188}{(0.85)\cdot(4.5)\cdot(12)} = 0.90$ in

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(7.97 - \frac{0.90}{2}\right) - (3.188)\cdot\left(\frac{10.78}{2} - \frac{0.90}{2}\right)\right] \div 12$$

φM_n = 26.66 ft-k

Since $\varphi\ M_n$ =26.66 ft-k > M_u = 10.36 ft-k, the overhang has adequate strength at Location 1. Note that the resulting eccentricity equals (26.66)(12) ÷ 3.188 = 100.35 inches compared to the actual eccentricity of 39.00 inches that is fixed by the geometry of the deck thickness and barrier height.

Development Length

The reinforcing must be properly developed from the barrier face towards the edge of deck. The available embedment length equals 17 inches minus 2 inches clear or 15 inches. For the #5 transverse reinforcing in the deck the required development length is as follows:

[**5.11.2.1.1]**

For No. 11 bar and smaller: $\dfrac{1.25 A_b f_y}{\sqrt{f'_c}} = \dfrac{(1.25)\cdot(0.31)\cdot(60)}{\sqrt{4.5}} = 10.96 \text{ in}$

But not less than 0.4 d_b f_y = (0.4)(0.625)(60) = 15.00 in

Since the available length is equal to the required length, the reinforcing is adequately developed using straight bars.

If #6 transverse reinforcing was used as the top deck reinforcing, the development length would have been inadequate and the bars would require hooks. The reduction for excess reinforcing cannot be directly used since the analysis is based on a strain that ensures that the reinforcing yields. To use a reduced development length based on excess reinforcing, a stress-strain analysis must be performed that limits the strain in the reinforcing to a limit that produces less than the yield stress in the reinforcing. Consideration must also be given to the magnitude of the strain in the compressive zone in the concrete. For low levels of stress the analysis will default to a working stress limit in the concrete with the standard triangular stress block. Use of this method is complicated, of questionable validity and is not recommended.

The deck slab must also be evaluated at the exterior overhang support. At this location the design horizontal force is distributed over a length L_{s1} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the exterior support. See Figures 4, 5 and 6. Using a distribution of 30 degrees from the face of barrier to the exterior support results in the following:

Exterior Support Location 2
Figure 4

L_{S1} = 11.86 + 2(2.67) + (2)tan(30)(1.92) = 19.42 ft
P_u = 54.83 / 19.42 = 2.823 k/ft

Dimensions

h = 12.50 in
d_1 = 12.50 – 2.50 clr – 0.625 / 2 = 9.69 in

Moment at Exterior Support

.

DC Loads			
Deck	= 0.150(9.50 / 12)(3.33)^2 / 2		= 0.66 ft-k
	= 0.150(3.00 / 12)(3.33)^2 / 6		= 0.07 ft-k
Barrier	= 0.355(0.817 + 1.917)		= <u>0.97</u> ft-k
		DC	= 1.70 ft-k
DW Loads			
FWS	= 0.025(1.917)^2 / 2		= 0.05 ft-k
Collision	= 2.823[2.67 + (12.50 / 12) / 2]		= 9.01 ft-k

The load factor for dead load shall be taken as 1.0.

[A13.4.1]

M_u = 1.00(1.70) + 1.00(0.05) + 1.00(9.01) = 10.76 ft-k

Extreme Event II
[3.4.1]

e = M_u / P_u = (10.76)(12) / (2.823) = 45.74 in

The top layer of reinforcing yields and f_s = 60 ksi. The resulting force in the reinforcing is:

T_1 = (0.744)(60) = 44.64 k

Simplified Method

A simplified method of analysis is available based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \frac{T_1 - P_u}{0.85 f'_c b} = \frac{44.64 - 2.823}{(0.85)\cdot(4.5)\cdot(12)} = 0.91$ in

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(9.69 - \frac{0.91}{2}\right) - (2.823)\cdot\left(\frac{12.50}{2} - \frac{0.91}{2}\right)\right] \div 12$$

φM_n = 32.99 ft-k

Since φM_n = 32.99 ft-k > M_u = 10.76 ft-k, the overhang has adequate strength at Location 2.

Interior Support
Location 3
Figure 4

The deck slab must also be evaluated at the interior point of support. At this location the design horizontal force is distributed over a length L_{s2} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the interior support. See Figures 4, 5 and 6. Using a distribution of 30 degree from the face of the barrier to the interior support results in the following:

L_{S2} = 11.86 + 2(2.67) + (2)tan(30)(2.99) = 20.65 ft
P_u = 54.83 / 20.65 = 2.655 k/ft

Dimensions
h = 8.50 in
d_1 = 8.50 – 2.50 clr – 0.625 / 2 = 5.69 in

Moment at Interior Support
For dead loads use the maximum negative moments for the interior cells used in the interior deck analysis
.

DC = 0.72 ft-k
DW = 0.17 ft-k
Collision = 2.655[2.67 + (8.50 / 12) / 2] = 8.03 ft-k

[A13.4.1]
Extreme Event II
[3.4.1]

The load factor for dead load shall be taken as 1.0.

M_u = 1.00(0.72) + 1.00(0.17) + 1.00(8.03) = 8.92 ft-k
e = M_u / P_u = (8.92)(12) / (2.655) = 40.32 in

The top layer of reinforcing yields and f_s = 60 ksi. The resulting force in the reinforcing is:

T_1 = (0.744)(60) = 44.64 k

Use the simplified method of analysis.

Simplified Method

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

$$\text{where } a = \frac{T_1 - P_u}{0.85 f'_c\, b} = \frac{44.64 - 2.655}{(0.85)\cdot(4.5)\cdot(12)} = 0.91 \text{ in}$$

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(5.69 - \frac{0.91}{2}\right) - (2.655)\cdot\left(\frac{8.50}{2} - \frac{0.91}{2}\right)\right] \div 12$$

φ M_n = 18.63 ft-k

Since φM_n = 18.63 ft-k > M_u = 8.92 ft-k, the deck has adequate strength at Location 3.

Since the axial and flexural strength of the deck at the three locations investigated exceeds the factored applied loads, the deck is adequately reinforced for Design Case I.

Design Case 2

Design Case 2: Vertical forces specified in Article A13.2 -
Extreme Event Load Combination II Limit State

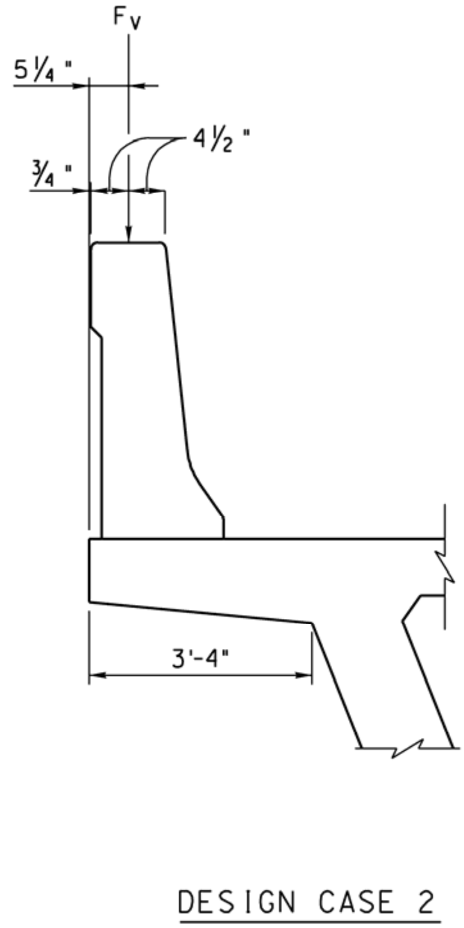


Figure 7

[Table A13.2-1]

This case represents a crashed vehicle resting on top of the barrier and is treated as an extreme event. The downward vertical force, F_v = 18.0 kips, is distributed over a length, F_l = 18.0 feet. The vehicle is assumed to be resting on top of the center of the barrier as shown in Figure 7.

At the face of exterior support:
DC Dead Loads = 1.70 ft-k
DW Dead Load = 0.05 ft-k

Vehicle
Collision = [18.0/18.0] [3.33 - (5.25 / 12)] = 2.89 ft-k

[A13.4.1]
Extreme Event II
[Table 3.4.1-1]

The load factor for dead load shall be taken as 1.0.

$M_u = 1.00(1.70) + 1.00(0.05) + 1.00(2.89) = 4.64$ ft-k

Flexural Resistance [5.7.3.2]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2} \right)$$

Try #5 reinforcing bars

d_s = 12.50 – 2.50 clr – 0.625 / 2 = 9.69 inches

Use #5 @ 5”, the same reinforcing required for the interior span and overhang Design Case 1.

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.744)\cdot(60)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(12)} = 1.179 \text{ in}$$

[5.7.3.2.3]

$a = \beta_1 c$ = (0.825)(1.179) = 0.97 inches

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{9.69}{1.179} - 1 \right) = 0.022$$

[5.5.4.2.1]

Since ε_T > 0.005 the member is tension controlled.

$$M_n = (0.744)\cdot(60)\cdot\left(9.69 - \frac{0.97}{2} \right) \div 12 = 34.24 \text{ ft-k}$$

[1.3.2.1]

φ = 1.00

$M_r = \phi M_n$ = (1.00)(34.24) = 34.24 ft-k

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the extreme limit state is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated this requirement.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification requires that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u.

$S_c = bh^2 / 6 = (12)(12.50)^2 / 6 = 312.5 \text{ in}^3$

$1.2M_{cr} = 1.2 f_r S_c$ = 1.2(0.785)(312.5) / 12 = 24.53 ft-k < M_r = 34.24 ft-k

Since the strength of the section exceeds 1.2 M_{cr}, the minimum reinforcing criteria is satisfied.

Design Case 3
[Article A13.4.1]

Design Case 3: The loads, specified in Article 3.6.1, that occupy the overhang – Load Combination Strength I Limit State

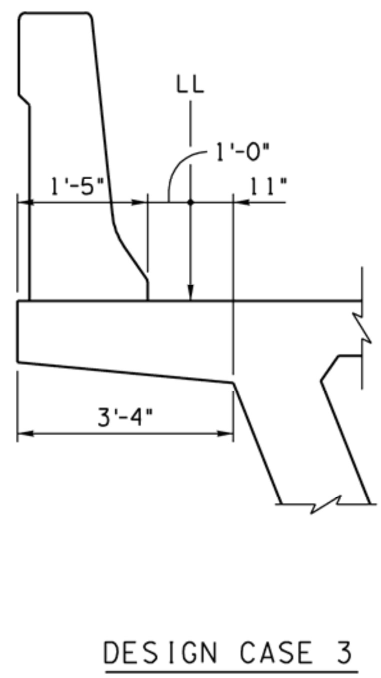


Figure 8

At the face of exterior support:

DC Dead Loads = 1.70 ft-k
DW Dead Load = 0.05 ft-k

LL Distribution
[BDG]

While the LRFD Specification allows use of a uniform load of 1.00 kip/ft for service limit state where the barrier is continuous ADOT does not. Therefore use the live load distribution for strength limit state for the service limit state also. For a cast-in-place concrete deck overhang, the width of the primary strip is 45.0 + 10.0 X where X equals the distance from the point of load to the support.

[Table 4.6.2.1.3-1]

Width Primary Strip (inches) = 45.0 + 10.0(0.917) = 54.17 in = 4.51 ft

IM
[Table 3.6.2.1-1]

Dynamic Load Allowance, IM

For all states other than fatigue and fracture limit state, IM = 33%.

Multiple Presence Factor
[Table 3.6.1.1.2-1]

The multiple presence factor must also be applied. Since one vehicle produces the critical load, m = 1.20.
LL + IM = [16.00(1.33)(1.20)(0.917)] / 4.51 = 5.19 ft-k

Strength I Limit State [Table 3.4.1-1]

M_u = 1.25(1.70) + 1.50(0.05) + 1.75(5.19) = 11.28 ft-k

The flexural resistance was previously calculated for Design Case 2. Since the member is tension controlled, φ = 0.90. Since M_r = (0.90)(34.24) = 30.82 ft-k is greater than M_u the deck is adequately reinforced for strength.

Service I Limit State

M_s = 1.0(1.70 + 0.05) + 1.0(5.19) = 6.94 ft-k

d_s = 12.50 – 2.50 clr – 0.625 / 2 = 9.69 in

A_s (#5 @ 5") = 0.744 in^2

Allowable Stress [BDG] [9.5.2]

The maximum allowable stress in a deck is limited to 24 ksi per the LRFD Bridge Practice Guidelines.

Determine stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.744}{(12)\cdot(9.69)} = 0.00640$$

np = 8(0.00640) = 0.0512

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.0512) + (0.0512)^2} - 0.0512 = 0.273$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.273}{3} = 0.909$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(6.94)\cdot(12)}{(0.744)\cdot(0.909)\cdot(9.69)} = 12.71\ ksi \le 24.0\ \text{ksi}$$

Since the applied stress is less than the allowable specified by ADOT, the service limit state requirement is satisfied.

Control of Cracking

For all concrete components in which tension in the cross section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in equation 5.7.3.4-1 shall be satisfied.

$f_{sa} = 0.80f_r = 0.80(0.24\sqrt{f'_c}) = 0.80(0.509) = 0.407$ ksi

S_b = (12.00)(12.00)^2 ÷ 6 = 288 in^3

$$f_s = \frac{M_s}{S_b} = \frac{(6.94)\cdot(12)}{288} = 0.289\ ksi < f_{sa} = 0.407\ \text{ksi}$$

Since the service limit state stress is less than the allowable, the control of cracking requirement is satisfied.

Bottom Slab Reinforcing [5.14.1.5.2b]

A minimum of 0.4% reinforcement shall be placed in the longitudinal direction in the bottom slab at a maximum 18 inch spacing.

A_s = (0.004)(6)(12) = 0.288 in^2 Use #5 @ 12"

A minimum of 0.5% reinforcement shall be placed in the transverse direction in the bottom slab at a maximum 18 inch spacing. The reinforcement shall extend to the exterior face of the outside web and be anchored by a 90° hook.

A_s = (0.005)(6)(12) = 0.360 in^2 Use #5 @ 9" (A_s = 0.413 in^2)

Temperature & Shrinkage Reinforcing [5.10.8]

Temperature and shrinkage reinforcement requirements were changed in the 2006 Interim Revisions. The required area reinforcement for the section follows:

$$A_s \geq \frac{1.30bh}{2 \cdot (b+h) f_y}$$

$$0.11 \leq A_s \leq 0.60$$

Exterior Web

$$A_s \geq \frac{1.30 \cdot (77.0) \cdot (12.0)}{2 \cdot (77.0 + 12.0) \cdot (60)} = 0.112 \text{ in}^2/\text{ft}$$

Top Slab

$$A_s \geq \frac{1.30 \cdot (99.0) \cdot (8.50)}{2 \cdot (99.0 + 8.50) \cdot (60)} = 0.085 \text{ in}^2/\text{ft}$$

Bottom Slab

$$A_s \geq \frac{1.30 \cdot (99.0) \cdot (6.0)}{2 \cdot (99.0 + 6.0) \cdot (60)} = 0.061 \text{ in}^2/\text{ft}$$

Use #5 @ 12 in^2/ft minimum reinforcement.

Figure 9 shows the required reinforcing in the deck slab.

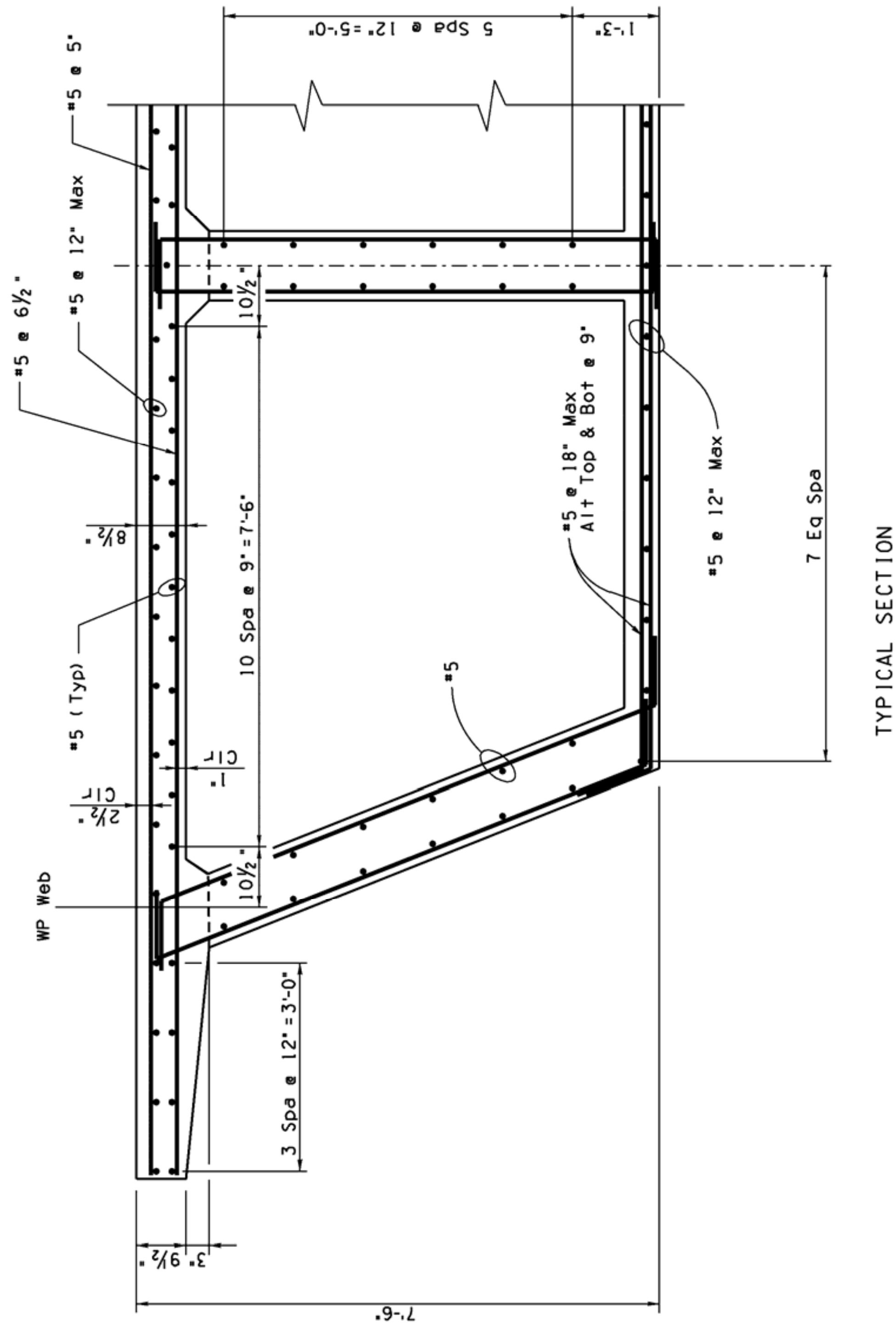


Figure 9

SUPERSTR DGN Flexure

Usual design practice is to determine moments and stresses at tenth points using computer programs. For this example using hand methods, only the critical locations will be investigated.

Section Properties

A separate set of calculations has determined the section properties. These are not shown since the LRFD Specification has not changed the method of calculation. The section properties have been calculated subtracting the ½-inch wearing surface from the top slab thickness. However, this wearing surface has been included in weight calculations. A summary of the gross section properties follows:

Section Properties

y_b	51.56	in
y_t	37.94	in
Inertia	13,309,829	in^4
Area	11,588	in^2

Dead Loads [3.5.1]

In LFRD design, the dead load must be separated between DC loads and DW loads since their load factors differ. The DC loads include the self-weight of the superstructure plus 10 psf for lost deck formwork, the intermediate diaphragm and barriers. The DW load includes the Future Wearing Surface and any utilities.

Loads and Midspan Moments

	Load	Units	Moment	Units
Superstructure	12.681	K/ft	40,579	Ft-k
Diaphragm	21.72	Kips	869	Ft-k
Barrier	0.710	K/ft	2272	Ft-k
FWS	1.050	K/ft	3360	Ft-k

Live Loads [3.6]

The HL-93 live load in the LRFD specification differs from the HS-20-44 load in the Standard Specifications. Standard design practice is to use a computer program to generate live loads due to the excessive time required by hand. However, for this example, the live load moment will be determined by hand calculations at the midspan to demonstrate how to apply the new live load.

Design Truck [3.6.1.2.2]

[3.6.1.2.3]

The HL-93 live load consists of a design lane load combined with either a design truck or a design tandem. The design lane load consists of a 0.640 k/ft uniform load placed to maximize the moment. The design truck looks like the HS-20-44 truck with a front axle of 8 kips, a 32 kip second axle, and a 32 kip third axle. The first two axles are spaced 14 feet apart while the second two axles are spaced from 14 to 30 feet apart. See Figure 10. The design tandem consists of two 25 kip axles separated by 4 feet. See Figure 11.

Design Lane Load [3.6.1.2.4]

The maximum moment at midspan from the design lane load is caused by loading the entire span. The force effects from the design lane load shall not be subject to a dynamic load allowance. At midspan the moment equals the following:

$$M_{lane} = w \cdot (l)^2 \div 8 = 0.640 \cdot (160)^2 \div 8 = 2048 \text{ ft-k}$$

Design Truck [3.6.1.2.2]

The maximum design truck moment results when the truck is located with the middle axle at midspan. The truck live load positioned for maximum moment at midspan is shown below:

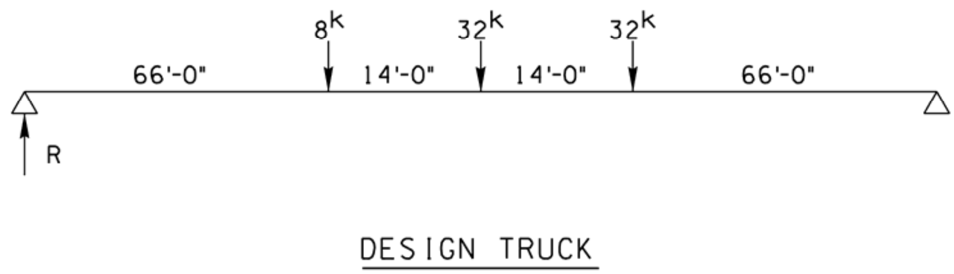


Figure 10

$$R = [8 \cdot (94) + 32 \cdot (80) + 32 \cdot (66)] \div 160 = 33.90 \text{ kips}$$
$$M_{truck} = 33.90 \cdot (80) - 8 \cdot (14) = 2600 \text{ ft-k}$$

Design Tandem [3.6.1.2.3]

The maximum design tandem moment results when the tandem is located with one of the axles at midspan. The tandem live load positioned for maximum moment is shown below:

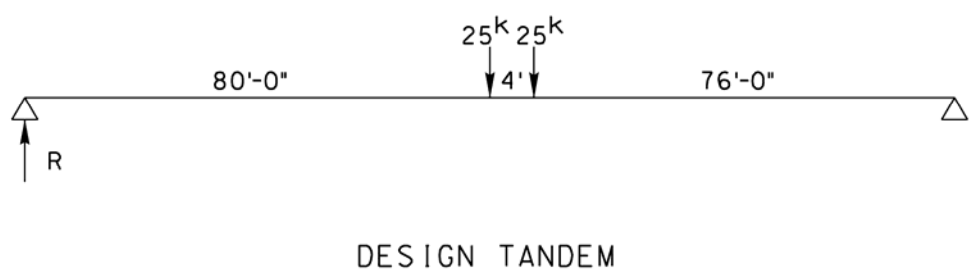


Figure 11

$$R = [25 \cdot (80) + 25 \cdot (76)] \div 160 = 24.38 \text{ kips}$$
$$M_{tandem} = 24.38 \cdot (80) = 1950 \text{ ft-k}$$

Application Design Live Loads [3.6.1.3]

By inspection the moment from the combination of design truck and design lane load is higher than the combination of design tandem and design lane load.

Live Load Distribution [4.6.2.2.1]

Table 4.6.2.2.1-1]

The LRFD Specification has made major changes to the live load distribution factors. However, for a cast-in-place concrete box girder bridge a unit design is allowed by multiplying the interior distribution factor by the number of webs. From Table 4.6.2.2.1-1, a cast-in-place concrete multicell box is classified as a typical cross section type (d). The live load distribution factor in the table is valid when all the variables are within the range of applicability as shown below:

[Table 4.6.2.2.2b-1]
[Cross section d]

	Applicable Range	
N_c = number of cells	$N_c \geq 3$	$N_c = 4$ ok
S = web spacing (ft)	$7.0 \leq S \leq 13.0$	S = 9.25 ok
L = span length of beam (ft)	$60 \leq L \leq 240$	L = 160 ok

Since the range of applicability is satisfied the moment live load distribution factor for an interior web for one lane loaded is as follows:

$$LL\ Distribution = \left(1.75 + \frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}$$

$$LL\ Distribution = \left(1.75 + \frac{9.25}{3.6}\right)\left(\frac{1}{160}\right)^{0.35}\left(\frac{1}{4}\right)^{0.45} = 0.392$$

The distribution factor for two or more lanes loaded is:

$$LL\ Distribution = \left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25} = \left(\frac{13}{4}\right)^{0.3}\left(\frac{9.25}{5.8}\right)\left(\frac{1}{160}\right)^{0.25} = 0.639$$

LL Distribution = (0.639)(5 webs) = 3.195

Skew Effects
[Table 4.6.2.2.2e]

For skewed bridges the live load may be reduced. The skew effect factor in the table is valid when all the variables are within the range of applicability as shown below:

	Applicable Range	
θ = skew angle (degrees)	$0° \leq \theta \leq 60°$	θ = 15° ok

Since the range of applicability is satisfied the skew effect factor for a typical cross section type (d) is shown below:

$1.05 - 0.25 \tan\theta = 1.05 - 0.25 \tan(15) = 0.983 \leq 1.0$

Dynamic Load Allowance [Table 3.6.2.1-1]

The dynamic load allowance IM equals 33% for the strength and service limit states.

Application Design Live Loads [3.6.1.3]

Dynamic load allowance does not apply to the design lane load. The maximum live load plus dynamic load allowance including distribution factor and skew effect at midspan equals the following:

LL + IM = [2048 + 2600(1.33)](3.195)(0.983) = 17,293 ft-k

Load Combinations [Table 3.4.1-1]

The LRFD Specification has made major changes to the group load combinations. There are several limit states that must be considered in design. Limit states for this problem are as follows:

DC = 40,579 + 869 + 2272 = 43,720 ft-k

STRENGTH I – Basic load combination relating to the normal vehicular use of the bridge without wind.

M_u = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)
M_u = 1.25(43,720) + 1.50(3360) + 1.75(17,293) = 89,953 ft-k

SERVICE I – Load combination relating to normal operational use of the bridge including wind loads to control crack width in reinforced concrete structures.

M_s = 1.0(DC + DW) + 1.00(LL + IM)
M_s = 1.0(43,720 + 3360) = 47,080 ft-k DL only
M_s = 1.0(43,720 + 3360) + 1.0(17,293) = 64,373 ft-k

SERVICE III – Load combination relating only to tension in prestressed concrete superstructures with the objective of crack control.

M_s = 1.0(DC + DW) + 0.80(LL + IM)
M_s = 1.0(43,720 + 3360) + 0.80(17,293) = 60,914 ft-k

Prestress Design

The design of a post-tensioned concrete bridge involves making assumptions, calculating results, comparing the results to the assumptions and reiterating the process until convergence. To limit the number of pages of calculations, the reiterative portion of this process has been eliminated by using final answers as starting assumptions. While actual design will involve iteration, the design usually converges very quickly.

Step 1 – Assume Cable Path

The first step in design is to assume a cable path as shown in Figure 12. The location of the cg at the ends is very important for the anchor zone design. Placing the cg at the neutral axis results in a uniform stress distribution at the ends but the top tendons will probably be too high to have sufficient top edge clearance. Placing the cable path near the geometric center of the section is usually a good compromise. At the midspan the cable path should be as low as possible. However, care must be taken to ensure that the cable path can be physically located where assumed. A check on the cg at the ends and at midspan is required once the area of prestressing steel is determined.

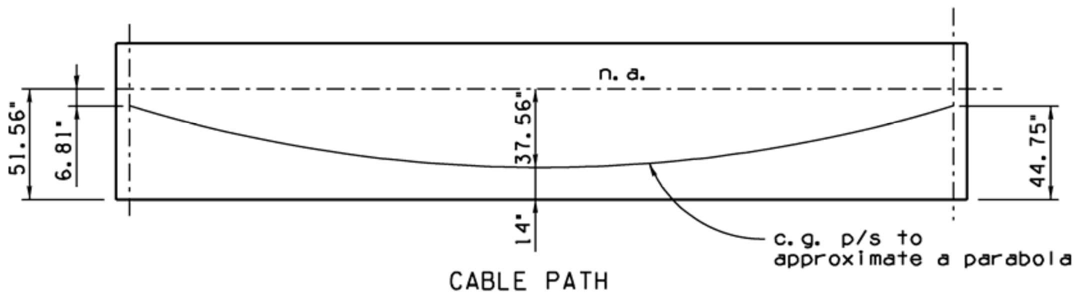


Figure 12 – Cable Path

Friction Losses
[BDG]
[5.9.5]

Step 2 – Calculate Friction and Anchor Set Losses

Total losses in prestress are due to instantaneous and time-dependent losses. The instantaneous losses include friction loss, anchor set loss and elastic shortening loss. Time-dependent losses include shrinkage, creep and relaxation. When the strands are pulled through the ducts, losses occur. Some loss is due to a uniform friction along the length of the path and some is due to angle changes in the cable path. Figure 13 is a diagram showing the friction losses, anchor set losses, elastic shortening losses and time dependent losses. Standard friction coefficients for post-tensioning tendons are as follows:

$k = 0.0002$
$\mu = 0.25$

.

The angle change for half the structure is shown below:

$$\alpha_{mid} = \frac{2 \cdot (44.75 - 14) \div 12}{80} = 0.064063 \text{ radians}$$

The angle change at the non-jacking end of the structure will be double that at the midspan due to symmetry. The friction loss calculation at the non-jacking end is shown below:

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(Kx + \mu\alpha)}\right)$$

$$kx + \mu\alpha = (0.0002)(160) + (0.25)(2)(0.064063) = 0.06403$$

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(0.06403)}\right) = 0.0620 f_{pj} = 0.0620(0.74)(270) = 12.39 \text{ ksi}$$

where $f_{pj} = 0.74\ f_{pu}$ based on allowable stress in strand (Step 6).

At midspan the friction loss based on an assumed linear friction loss equals half the end loss or $0.0310\ f_{pj} = 0.0310(0.74)(270) = 6.19$ ksi.

Anchor Set Losses [BDG]

The anchor set loss is determined by standard equations. The derivation, based on straight line losses and similar triangles, will not be shown here. The anchor set may be taken as 0.375 inches for normal structures. The structure will be jacked from one end only.

The anchor set length equals the dimension X shown below:

$$X = \sqrt{\frac{E_p(\Delta L)L}{12\Delta f_{pF}}} = \sqrt{\frac{28500 \cdot (0.375) \cdot 160}{12 \cdot (12.39)}} = 107.24 \text{ ft}$$

$$\Delta f_{pA} = \frac{2\Delta f_{pF} X}{L} = \frac{2 \cdot (12.39) \cdot 107.24}{160} = 16.61 \text{ ksi}$$

Anchor Set Loss At End:

$$\Delta f_{pA} = \frac{16.61}{0.74 \cdot (270)} = 0.0831 f_{pj}$$

Anchor Set Loss At Midspan:

$$\Delta f_{pA} = 0.0831 f_{pj} \cdot \frac{(107.24 - 80)}{107.24} = 0.0211 f_{pj}$$

$$\Delta f_{pA} = 0.0211(0.74)(270) = 4.22 \text{ ksi}$$

The force coefficient is the coefficient that when multiplied by the jacking stress results in the effective prestress stress in the strand including losses. The initial force coefficients including both friction and anchor set losses are as follows:

Jacking End	$FC_i = 1.0000 - 0.0831$	$= 0.9169\ f_{pj}$
Midspan	$FC_i = 1.0000 - 0.0310 - 0.0211$	$= 0.9479\ f_{pj}$
End Seating	$FC_i = 1.0000 - 0.0831 / 2$	$= 0.9585\ f_{pj}$
Non-Jacking End	$FC_i = 1.0000 - 0.0620$	$= 0.9380\ f_{pj}$

The stress in the prestress including both friction and anchor set losses are as follows:

Jacking End	$P_i = 0.74(270) - 16.61$	= 183.19 ksi
Midspan	$P_i = 0.74(270) - 6.19 - 4.22$	= 189.39 ksi
End Seating	$P_i = 0.74(270) - 16.61 / 2$	= 191.50 ksi
Non-Jacking End	$P_i = 0.74(270) - 12.39$	= 187.41 ksi

As an example of the use of these coefficients, at the jacking end the stress in the strand = (0.9169)(0.74)(270) = 183.19 ksi, the same as the value determined above.

Step 3 – Assume Prestress Losses

The next step is to assume total final losses from elastic shortening, shrinkage, creep and relaxation. In determining final stress, these losses are added to the friction and anchor set losses. The elastic shortening loss occurs immediately and is assumed to equal 4.38 ksi or 0.0219 f_{pj}. The time-dependent losses from shrinkage, creep, and relaxation are assumed to equal 24.93 ksi or 0.1248 f_{pj}. For this example final losses will be assumed equal to 29.31 ksi or 0.1467 f_{pj}. The final force coefficient at midspan including friction, anchor set, elastic shortening and time dependent losses is as follows:

$$FC_f = 0.9479\ f_{pj} - 0.1467\ f_{pj} = 0.8012\ f_{pj}$$

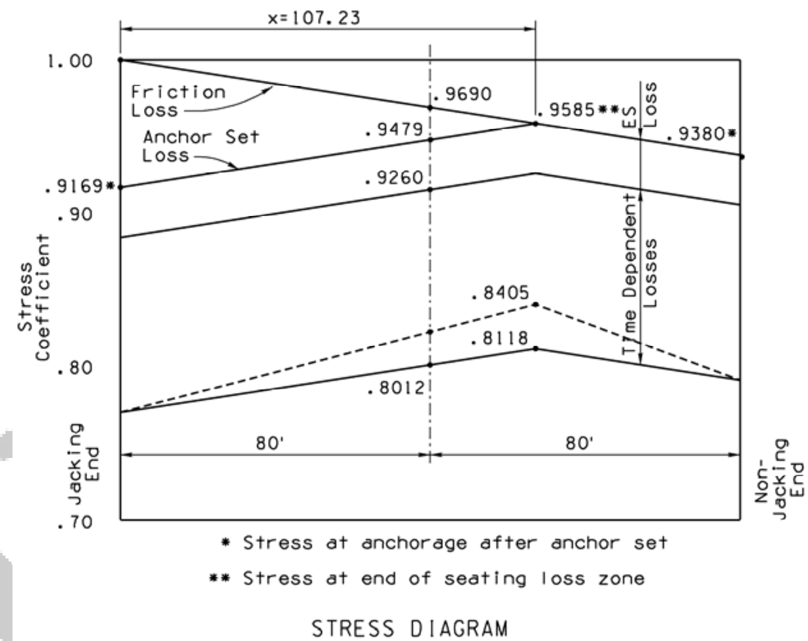


Figure 13 – Stress Diagram

Step 4 – Determine Area Prestressing Steel

The amount of prestressing steel required is controlled by the tension in the bottom fiber at midspan. The area of steel is calculated by assuming a final loss, using that loss to determine the required area of steel and then verifying the calculated losses. In addition to the Service III limit state, the tension

[BPG] reinforcing is also controlled by the requirement of zero tension from the effective prestress and all dead loads. This structure is to be constructed on soffit fill. Per the LRFD Bridge Practice Guidelines the allowable tension is:

[BPG]

$$Allowable\ Tension = 0.0948\sqrt{f'_c} = 0.0948\sqrt{4.5} = 0.201\ \text{ksi}$$

Basic Stress Equation:

$$\frac{P_j \cdot \left(FC_f\right)}{A} + \frac{P_j \cdot \left(e_m\right) \cdot \left(FC_f\right) \cdot y_b}{I} - \frac{\left(\sum \gamma M\right)\left(y_b\right)}{I} + \textit{Allowable Tension} = 0$$

Solving for the jacking force results in the following design equation:

$$P_j = \frac{\dfrac{\left(\sum \gamma M\right) \cdot \left(y_b\right)}{I} - \textit{Allowable Tension}}{\dfrac{\left(FC_f\right)}{A} + \dfrac{e_{m} \cdot \left(FC_f\right) \cdot y_b}{I}}$$

For Service III Limit State:

$$P_j = \frac{\dfrac{(60{,}914) \cdot 12 \cdot (51.56)}{13{,}309{,}829} - 0.201}{\dfrac{0.8012}{11{,}588} + \dfrac{(37.56) \cdot (0.8012) \cdot (51.56)}{13{,}309{,}829}} = 14{,}165 \text{ kips}$$

For Zero Tension under Effective Prestress and Dead Load:

$$P_j = \frac{\dfrac{(47{,}080) \cdot 12 \cdot (51.56)}{13{,}309{,}829} - 0}{\dfrac{0.8012}{11{,}588} + \dfrac{(37.56) \cdot (0.8012) \cdot (51.56)}{13{,}309{,}829}} = 11{,}784 \text{ kips}$$

[BPG]

The greater of the two above jacking forces is used to determine the strand area. For post-tensioned box girder bridges, design should be based on use of 0.6-inch diameter strand.

Required A_{ps} = (14,165) / [(0.74)(270)] = 70.896 in^2
Number of Strands = (70.896) / (0.217) = 327
A_{ps} = (0.217)(327) = 70.959 in^2
P_j = (70.959)(0.74)(270) = 14,178 kips

Use three tendons per web. This requires that each duct holds a maximum of 22 strands.

Step 5 – Calculate Prestress Losses

Elastic shortening losses require that the number of tendons in the bridge be known. The above estimate of 15 tendons will be used to calculate the elastic shortening losses. Elastic shortening losses can be calculated directly with a rather lengthy equation in lieu of a trial and error method. However, the equation for calculation of elastic shortening in the LRFD Commentary [C5.9.5.2.3b-1] is incorrect. The correct formula is shown below:

[5.9.5.2.3b]

[BDG]
[5.9.5]

$$\Delta f_{pES} = \frac{A_{ps}(FC_i) f_{pj}\left(I + A e_m^{\ 2}\right) - e_m M_g A}{A_{ps}\left(I + A e_m^{\ 2}\right) + \dfrac{A \cdot I \cdot E_{ci}}{E_p} \cdot \dfrac{2N}{(N-1)}}$$

This equation can be modified by dividing both the numerator and denominator by A and substituting r^2 for the ratio I / A. This version of the equation produces more manageable numbers.

$$\Delta f_{pES} = \frac{(FC_i) f_{pj} A_{ps}\left(r^2 + e_m^{\ 2}\right) - e_m M_g}{A_{ps}\left(r^2 + e_m^{\ 2}\right) + \dfrac{E_{ci} I}{E_p} \cdot \dfrac{2N}{N-1}}$$

$r^2 = I / A = 13{,}309{,}829 / 11588 = 1148.59 \text{ in}^2$

$$A_{ps}\left(r^2 + e_m^{\ 2}\right) = (70.959) \cdot \left(1148.59 + (37.56)^2\right) = 181{,}608$$

$$\frac{E_{ci} I}{E_p} \cdot \frac{2N}{(N-1)} = \frac{(3405) \cdot (13{,}309{,}829)}{28500} \cdot \frac{2 \cdot (15)}{(15-1)} = 3{,}407{,}516$$

$$\Delta f_{pES} = \frac{(0.9479) \cdot (0.74) \cdot (270) \cdot (181{,}608) - (37.56) \cdot (41448) \cdot (12)}{181{,}608 + 3{,}407{,}516}$$

$\Delta f_{pES} = 4.38$ ksi

Calculate f_{cgp} and verify the elastic shortening by substituting into [Eqn. 5.9.5.2.3b-1].

$$f_{cgp} = (70.959) \cdot (0.9479 \cdot 0.74 \cdot 270 - 4.38) \cdot \left(\frac{1}{11588} + \frac{(37.56)^2}{13{,}309{,}829}\right) - \frac{(41448) \cdot 12 \cdot (37.56)}{13{,}309{,}829} = 1.121 \text{ ksi}$$

[5.9.5.2.3b]

$$\Delta f_{pES} = \frac{N-1}{2N} \cdot \frac{E_p}{E_{ci}} \cdot f_{cgp} = \frac{15-1}{2 \cdot 15} \cdot \frac{28500}{3405} \cdot (1.121) = 4.38 \text{ ksi OK}$$

Refined Estimates Time-Dependent Losses [5.9.5.4] 2004

[BDG] [5.9.5]

The method of loss calculation contained in the 2006 LRFD Specification shall not be used for post-tensioned box girder bridges. The approximate method for time-dependent losses is only applicable for precast prestressed members. There are questions as to the applicability of the refined method for cast-in-place post-tensioned members. For this example, the refined method of loss determination contained in the 2004 LRFD Specification will be used. This is the preferred method of loss calculation specified in the LRFD Bridge Design Guidelines.

For Arizona, most locations have an average relative humidity of approximately 40%. The equation for shrinkage losses follows:

$$\Delta f_{pSR} = (13.5 - 0.123\ H) = 13.5 - (0.123)(40) = 8.58 \text{ ksi}$$

Creep Losses

The equation for creep follows:

$$\Delta f_{pCR} = 12.0\ f_{cgp} - 7.0\ \Delta f_{cdp} \geq 0$$

where f_{cgp} has been previously calculated in the determination of elastic shortening losses and Δf_{cdp} equals the change in concrete stress due to externally applied dead loads excluding self weight.

$$\Delta f_{cgp} = (2272 + 3360)(12)(37.56) / (13{,}309{,}829) = 0.191 \text{ ksi}$$

$$\Delta f_{pCR} = 12.0(1.121) - 7.0(0.191) = 12.12 \text{ ksi}$$

Relaxation Losses

For low relaxation strands, the relaxation in the prestressing strands equals 30% of the equation shown below:

$$\Delta f_{pR2} = 20.0 - 0.3\ \Delta f_{pF} - 0.4\ \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$$

where Δf_{pF} = the friction loss below 0.70 f_{pu} at the point under consideration.

At midspan the friction stress is $0.9479 f_{pj}$ or $(0.9479)(0.74) f_{pu} = 0.701\ f_{pu}$. Since this value is greater than 0.70, $\Delta f_{pF} = 0$.

$$\Delta f_{pR2} = 0.3[20.0 - 0.3(0) - 0.4(4.38) - 0.2(8.58 + 12.12)] = 4.23 \text{ ksi}$$

Final Losses

The final loss excluding friction and anchor set loss is:

Final Loss = 4.38 + 8.58 + 12.12 + 4.23 = 29.31 ksi

Since this equals the assumed loss of 29.31 ksi in Step 3, there is no need to make another design cycle.

The initial effective prestress force at midspan including friction, anchor set, and elastic shortening losses equals:

$$F_i = [(0.74)(270) – 6.19 - 4.22 – 4.38](70.959) = 13{,}128 \text{ kips}$$

The final effective prestress force at midspan including total losses equals:

$$F_f = [(0.74)(270) – 6.19 – 4.22 – 29.31](70.959) = 11{,}359 \text{ kips}$$

]
[BDG]
[5.9.3]

Step 6 – Check Allowable Stress in Strands

There are four limits for stress in prestressing strands. The first allowable limit is prior to seating. Bridge Group has modified the LRFD allowable of 0.90 f_{py} = (0.90)(0.90) f_{pu} = 0.81f_{pu} to a maximum of 0.78 f_{pu}.

(1) f_{pj} = 0.74 f_{pu}, < 0.78 f_{pu} OK.

The second stress limit is 0.70 f_{pu} at anchorages immediately after anchor set. At this time friction losses and anchor set losses have occurred.

At jacking end:
(2) Strand stress = 0.9169f_{pj} = (0.9169)(0.74)f_{pu} = 0.679 f_{pu} < 0.70 f_{pu}

At non-jacking end:
(2) Strand stress = 0.9380f_{pj} = (0.9380)(0.74)f_{pu} = 0.694 f_{pu} < 0.70 f_{pu}

The third stress limit to be checked occurs at the end of the seating loss zone immediately after anchor set (friction and elastic shortening losses).

(3) Strand stress = 0.9585f_{pj} = (0.9585)(0.74)f_{pu} = 0.709 f_{pu} < 0.74 f_{pu}

The fourth stress limit is a service limit state after all losses.

f_{pe} = 0.8012(0.74)f_{pu} = 0.593 f_{pu} after all losses

At service limit state composite dead load and live load plus dynamic allowance stresses are added to the strand stress since the strands are bonded through the grouting process.

$$f_{service} = \frac{(2272 + 3360 + 17293) \cdot 12 \cdot (37.56)}{13{,}309{,}829} \cdot \frac{28500}{3861} = 5.730 \text{ ksi}$$

Strand stress = 0.593 f_{pu} +(5.730) / (270)f_{pu} = 0.614 f_{pu}

(4) Strand stress = 0.614$_{pu}$ < 0.80 f_{py} = 0.80(0.90) f_{pu} = 0.720 f_{pu}

The actual maximum stress for this limit state may not occur at the midspan. Since the four criteria for stress in the strand are met, the jacking coefficient of 0.74 is satisfactory.

Step 7 – Verify Cable Path at Midspan

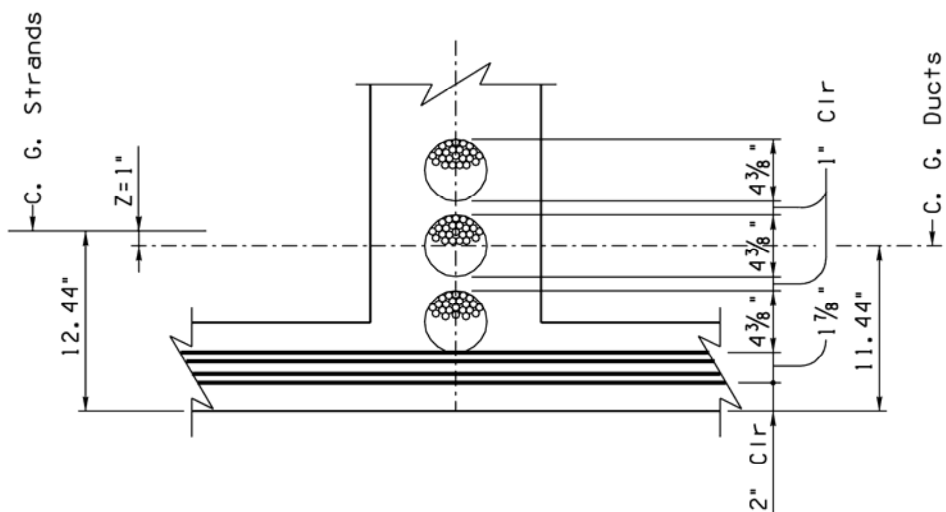
From previous calculations, each web will have three ducts large ducts holding a maximum 22 strands. The ducts must clear the three layers of #5 reinforcing in the bottom slab. From manufacturer's literature, the outside diameter of the duct will be 4 3/8". When the strands are pulled, they will rise at the midspan and not be located in the center of the duct. To estimate this effect, the variable Z is used. For ducts over 4 inch diameter, Z = 1 inch.

The calculation to determine the minimum cg of the prestressing strands follows:

cg ducts = 2" clr + 3(0.625) + 4.375 + 1" clr + 4.375 / 2 = 11.44 inches

cg strands = 11.44 + Z = 12.44 inches

Since there are many possible combinations of size of ducts and different suppliers, one should be conservative in estimating the cg of the strands. Therefore use 14 inches for the location of the cg of the strand at midspan. Since this is the initial assumed value our original assumption is valid.



C.G. STRANDS

Figure 14 – Strand CG

Initial Concrete [5.4.2.1] [BDG]

Step 8 – Determine Initial Concrete Strength

Once the amount of prestressing steel is determined from tension criteria, the resulting concrete stress and required concrete strength can be determined. Service I limit state is used to determine the concrete compressive stress. The concrete stress in compression before time dependent losses is limited to 0.60 f'_{ci}.

[BDG] [5.9.4]

$$Allowable\ Compression = 0.60 \cdot f'_{ci} = 0.60 \cdot (3.5) = 2.100\ ksi$$

The basic equation for stress in concrete follows:

$$f = P_{eff}\left[\frac{1}{A} \pm \frac{e_m y}{I}\right] + \frac{\sum(\gamma M)\cdot y}{I}$$

Bottom fiber at midspan

$$f_b = 13{,}128 \cdot \left[\frac{1}{11588} + \frac{(37.56)\cdot(51.56)}{13{,}309{,}829}\right] - \frac{(41448)\cdot(12)\cdot(51.56)}{13{,}309{,}829}$$

$$f_b = 3.043 - 1.927 = 1.116\ ksi \le 2.100\ ksi$$

Top fiber at midspan

$$f_t = 13{,}128 \cdot \left[\frac{1}{11588} - \frac{(37.56)\cdot(37.94)}{13{,}309{,}829}\right] + \frac{(41448)\cdot(12)\cdot(37.94)}{13{,}309{,}829}$$

$$f_t = -0.273 + 1.418 = 1.145\ ksi \le 2.100\ \text{ksi}$$

Bottom fiber at non-jacking end

Refer to the Stress Diagram in Figure 13.

P_{eff} = [(0.74)(270)(0.9380) – 4.38](70.959) = 12,988 kips

$$f_b = 12{,}988 \cdot \left[\frac{1}{11588} + \frac{(6.81)\cdot(51.56)}{13{,}309{,}829}\right] = 1.463\ ksi \ \le 2.100\ \text{ksi}$$

The initial concrete stresses are less than the allowable compressive stress. Therefore f'_{ci} = 3.5 ksi is acceptable. The initial concrete stress must also be checked in the design of the anchor zone. This check may control the required initial strength.

Step 9 – Temporary Tension at Ends

The ends of the structure should be checked to ensure that the end eccentricity has been limited so as to keep any tension within the allowable.

$$f_t = 12{,}988 \cdot \left[\frac{1}{11588} - \frac{(6.81)\cdot(37.94)}{13{,}309{,}829} \right] = 0.869\ ksi \geq 0$$

Since there is no tension at the ends, the criteria is met.

Final Concrete Strength

Step 10 – Determine Final Concrete Strength

The required final concrete strength is determined after all prestress losses. Service I load combination is used.

<u>Case I</u> – Permanent Loads plus Effective Prestress

Allowable Compression = 0.45 f'_c = (0.45)(4.5) = 2.025 ksi

$$f_t = 11{,}359 \cdot \left[\frac{1}{11588} - \frac{(37.56)\cdot(37.94)}{13{,}309{,}829} \right] + \frac{(47080)\cdot(12)\cdot(37.94)}{13{,}309{,}829}$$

$$f_t = -0.236 + 1.610 = 1.374\ \text{ksi} < 2.025\ \text{ksi}$$

<u>Case II</u> – One-half the Case I loads plus LL + IM

Allowable Compression = 0.40 f'_c = (0.40)(4.5) = 1.800 ksi

$$f_t = \frac{1}{2} \cdot [1.374] + \frac{(17{,}293)\cdot(12)\cdot(37.94)}{13{,}309{,}829}$$

f_t = 1.279 ksi < 1.800 ksi Allowable OK

<u>Case III</u> – Effective Prestress, Permanent Loads and Transient Loads

Allowable Compression = 0.60 φ_w f'_c = 0.60(0.963)(4.5) = 2.600 ksi

The reduction factor φ_w shall be taken equal to 1.0 when the wall slenderness ratio λ_w is not greater than 15. The critical ratio involves the bottom slab.

$$\lambda_w = \frac{X_u}{t} = \frac{(9.25 - 1.00)}{0.500} = 16.5 \geq 15$$

Since the ratio exceeds the allowable, the equivalent rectangular stress block cannot be used. However, a modification factor, φ_w, can be used when λ_w is less than 35.

[Article 5.7.4.7.2c]

$\varphi_w = 1.0 - 0.025(\lambda_w - 15) = 1.0 - 0.025(16.5 - 15) = 0.963$

$$f_t = 11{,}359 \cdot \left[\frac{1}{11588} - \frac{(37.56)\cdot(37.94)}{13{,}309{,}829}\right] + \frac{(64373)\cdot(12)\cdot(37.94)}{13{,}309{,}829}$$

$$f_t = -0.236 + 2.202 = 1.966 \leq 2.600 \text{ ksi}$$

Final Tension

Step 11 – Determine Final Tension in the Concrete

This step is not required since the number of strands was determined using the tension criteria. However, the amount of tension is required to satisfy the control of cracking criteria later.

Service III Limit State

$$f_b = 11{,}359 \cdot \left[\frac{1}{11588} + \frac{(37.56)\cdot(51.56)}{13{,}309{,}829}\right] - \frac{(60{,}914)\cdot(12)\cdot(51.56)}{13{,}309{,}829}$$

$f_b = 2.633 - 2.832 = -0.199$ ksi < -0.201 ksi

Service Limit State – Dead Load only

$$f_b = 11{,}359 \cdot \left[\frac{1}{11588} + \frac{(37.56)\cdot(51.56)}{13{,}309{,}829}\right] - \frac{(47{,}080)\cdot(12)\cdot(51.56)}{13{,}309{,}829}$$

$f_b = 2.633 - 2.189 = 0.444$ ksi > 0 ksi

Step 12 – Flexural Resistance

The flexural resistance of the structure must exceed the factored loads. Strength I loads should be compared to the flexural resistance.

$M_r = \varphi M_n < \sum\gamma M$

STRENGTH I: $\sum\gamma M$ = 89,953 ft-k

[BDG]
[5.5.4.2]

The resistance factor φ = 0.95 for flexure of cast-in-place prestressed concrete.

A_{ps} = (0.217)(327) = 70.959 in^2

d_s = 90.00 – 0.50 ws –14.00 = 75.50 in

[5.7.3.1.1]

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 2\left(1.04 - \frac{243}{270}\right) = 0.28$$

For a rectangular section without mild reinforcing steel:

$$c = \frac{A_{ps} f_{pu}}{0.85 f'_c \beta_1 b + kA_{ps}\frac{f_{pu}}{d_p}}$$

$$c = \frac{(70.959)\cdot(270)}{0.85\cdot(4.5)\cdot(0.825)\cdot(538) + 0.28\cdot(70.959)\cdot\frac{270}{75.50}} = 10.83 > t_{slab} = 8.00\text{”}$$

Since the depth of the stress block is greater than the slab thickness, the section must be treated as a T-section:

[5.7.3.1.1-3]

$$c = \frac{A_{ps} f_{pu} - 0.85 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w + kA_{ps}\frac{f_{pu}}{d_p}}$$

$$c = \frac{(70.959)\cdot(270) - 0.85\cdot(4.5)\cdot(538 - 61.85)\cdot(8.0)}{0.85\cdot(4.5)\cdot(0.825)\cdot(61.85) + (0.28)\cdot(70.959)\frac{270}{75.50}} = 17.24\ in$$

$$a = c\beta_1 = (17.24)\cdot(0.825) = 14.22\ in$$

Determine the tensile strain as follows:

$$\varepsilon_T = 0.003\left(\frac{d_T}{c} - 1\right) = 0.003 \cdot \left(\frac{75.50}{17.24} - 1\right) = 0.010$$

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 0.95$.

$$f_{ps} = (270) \cdot \left(1 - (0.28) \cdot \frac{17.24}{75.50}\right) = 252.74\ ksi$$

[5.7.3.2.2-1]

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85 f'_c (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

$$M_n = (70.959) \cdot (252.74) \cdot \left(75.50 - \frac{14.22}{2}\right)$$

$$+ 0.85 \cdot (4.5) \cdot (538 - 61.85) \cdot (8.00) \cdot \left(\frac{14.22}{2} - \frac{8.00}{2}\right) = 1{,}271{,}832 \text{ in-k}$$

φM_n = 0.95(1,271,832) / 12 = 100,687 ft-k > 89,953 ft-k

$\therefore$ Section is adequate.

Step 12 - Limits for Reinforcement

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated the maximum reinforcing requirement replacing it with the strain limitations associated with the phi factors.

Minimum Reinforcing [5.7.3.3.2]

There is a minimum amount of reinforcement that must be provided in a section. The amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

1.2 M_{cr}
or
1.33 M_u

The cracking moment is determined on the basis of elastic stress distribution and the modulus of rupture of the concrete.

[5.7.3.3.2]

$$M_{cr} = S_c\left(f_r + f_{cpe}\right) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c f_r$$

Since the structure is designed for the monolithic section to resist all loads, S_{nc} should be substituted for S_c resulting in the second term equaling zero.

$$S_c = S_{nc} = \frac{I}{y_b} = \frac{13{,}309{,}829}{51.56} = 258{,}143 \text{ in}^3$$

$$f_{cpe} = P_f\left[\frac{1}{A} + \frac{e_m y_b}{I}\right]$$

$$f_{cpe} = 11{,}359 \cdot \left[\frac{1}{11588} + \frac{(37.56)\cdot(51.56)}{13{,}309{,}829}\right] = 2.633\,ksi$$

M_{cr} = 258,143(0.785 + 2.633) / 12 = 73,528 ft-k

1.2 M_{cr} = 1.2(73,528) = 88,233 ft-k

The second criteria for determining minimum reinforcing is 1.33 times the factored moment required by the applicable strength load combinations. The critical combination for this structure is Strength I.

1.33 M_u = 1.33(89,953) = 119,637 ft-k

1.2 M_{cr} = 88,233 < 1.33 M_u = 119,637

Since 1.2 M_{cr} = 88,233 < φM_n = 100,687 ft-k the section is adequately reinforced and the minimum reinforcement limit is satisfied.

Control of Cracking [5.7.3.4]

The maximum service limit state load combination specified in Table 3.4.1-1 produces –0.199 ksi tension. Since this is less than 80 percent of the modulus of rupture = 0.80(0.509) = 0.407 ksi, the provisions of this article need not be satisfied.

Shear
[5.8]

The LRFD method of shear design is a complete change from the methods specified in the Standard Specifications and that used by ADOT. For this example an in-depth shear design will be performed at the critical location near the abutment

The critical shear is located a distance d_v from the support. The parabolic cable path complicates the determination of d_v. To simplify the issue use d_v = 0.72h for determining the critical location. d_v = 0.72(89.50) / 12 = 5.37 ft.

Step 1 – Determine Shear

The shear is determined by traditional methods as shown below:

Super	V_{crit} = 12.681(80 -5.37)	= 946.38 kips
Diaph	V_{crit} = 21.72 / 2	= 10.86 kips
Barrier	V_{crit} = 0.710(80 -5.37)	= 52.99 kips
FWS	V_{crit} = 1.050(80 -5.37)	= 78.36 kips

Lane	V_{crit} = 0.64(154.63)(154.63 ÷ 2) / 160	= 47.82 kips
Truck	V_{crit} = [32(154.63)+32(140.63)+8(126.63)] / 160	= 65.38 kips
Tandem	V_{crit} = [25(154.63)+25(150.63)] / 160	= 47.70 kips

Live Load Distribution
[4.6.2.2.1]

The live load distribution factor for shear will be determined based on the provisions for a whole width design. From Table 4.6.2.2.3a-1, a cast-in-place concrete multicell box is classified as a typical cross section type (d). The live load distribution factor for shear in the table is valid when all the variables are within the range of applicability as shown below:

N_c = number of cells	$N_c \geq 3$	N_c = 4 ok
S = web spacing (ft)	$6.0 \leq S \leq 13.0$	S = 9.25 ok
L = span length of beam (ft)	$20 \leq L \leq 240$	L = 160 ok
d = depth of member (in)	$35 \leq d \leq 110$	d = 89.5 ok

Since the range of applicability is satisfied the shear live load distribution factor for an interior web for one lane loaded is as follows:

[Table 4.6.2.2.3a-1]

$$LL\ Distribution = \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1} = \left(\frac{9.25}{9.5}\right)^{0.6}\left(\frac{89.5}{12.0\cdot(160)}\right)^{0.1} = 0.724$$

The distribution for two or more design lanes loaded is:

$$LL\ Distribution = \left(\frac{S}{7.3}\right)^{0.9}\left(\frac{d}{12.0L}\right)^{0.1} = \left(\frac{9.25}{7.3}\right)^{0.9}\left(\frac{89.5}{12.0\cdot(160)}\right)^{0.1} = 0.911$$

For a whole width bridge, LL Distribution = (0.911)(5 webs) = 4.555

Skew Effect

For skewed bridges, the shear shall be adjusted to account for the effects of the skew. The skew effect factor for shear in the table is valid when all the variables are within the range of applicability as shown below:

θ = skew angle (deg)	$0° \le \theta \le 60°$	$\theta = 15°$ ok
S = web spacing (ft)	$6.0 \le S \le 13.0$	S = 9.25 ok
L = span length of beam (ft)	$20 \le L \le 240$	L = 160 ok
d = depth of member (in)	$35 \le d \le 110$	d = 89.5 ok
N_c = number of cells	$N_c \ge 3$	$N_c = 4$ ok

[Table 4.6.2.2.3c-1]

Since the criteria for range of applicability is met, the skew correction factor shown in the table may be used. The factor is applied to all the webs.

$$CF = 1.0 + \left(0.25 + \frac{12.0L}{70d}\right)\tan\theta$$

$$CF = 1.0 + \left(0.25 + \frac{12.0 \cdot (160)}{70 \cdot (89.50)}\right)\tan(15) = 1.149$$

LL V_{crit} = [47.82+1.33(65.38)](4.555)(1.149) = 705.37 kips

Strength I Limit State

V_u = 1.25(946.38 + 10.86 + 52.99) + 1.50(78.36) + 1.75(705.37)
V_u = 2615 kips

Sectional Model [5.8.3]

Step 2 – Determine Analysis Model

The sectional model of analysis is appropriate for the design of typical bridge webs where the assumptions of traditional beam theory are valid. Where the distance from the point of zero shear to the face of the support is greater than 2d, the sectional model may be used. Otherwise, the strut-and-tie model should be used.

[5.8.1.1]

Point of Zero Shear to Face of Support = 80.00-2.00/cos(15) = 77.93 ft
2d = 2(7.50) = 15.00 ft < 77.93 ft $\therefore$ Sectional model may be used.

[5.8.2.9]

Step 3 – Shear Depth, d_v

The shear depth is the maximum of the following criteria:

1) $d_v = 0.9\, d_e$ where $d_e = \dfrac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = d_p$ when $A_s = 0$

y_{eq} = 14 + (44.75 – 14)[(80-5.37) / 80]^2 = 40.76 in
d_p = 89.50 – 40.76 = 48.74 in

$$d_v = 0.9d_p = 0.9(48.74) = 43.87 \text{ in}$$

2) d_v = 0.72h = 0.72(89.50) = 64.44 in

[C5.8.2.9-1]

$$3)\ d_v = \frac{M_n}{A_s f_y + A_{ps} f_{ps}}$$

Where for a T-Section at the critical section:

[5.7.3.1.1-3]

$$c = \frac{(70.959)\cdot(270) - 0.85\cdot(4.5)\cdot(538-61.85)\cdot(8.0)}{0.85\cdot(4.5)\cdot(0.825)\cdot(61.85) + (0.28)\cdot(70.959)\frac{270}{48.74}} = 15.03 \text{ in}$$

a = $c\beta_1$ = (15.03)(0.825) = 12.40 in

[5.7.3.1.1-1]

$$f_{ps} = (270)\cdot\left[1-(0.28)\cdot\frac{15.03}{48.74}\right] = 246.69 \text{ ksi}$$

[5.7.3.2.2-1]

$$M_n = (70.959)\cdot(246.69)\left(48.74 - \frac{12.40}{2}\right)$$

$$+0.85\cdot(4.5)\cdot(538-61.85)\cdot(8.00)\cdot\left(\frac{12.40}{2} - \frac{8.00}{2}\right) = 776{,}712 \text{ in-k}$$

M_n = (776,712) / 12 = 64,726 ft-k

$$d_v = \frac{(64{,}726)\cdot(12)}{(70.959)\cdot(246.69)} = 44.37 \text{ in}$$

Based on the above, the shear depth, d_v, equals 64.44 inches.

Step 4 – Calculate, V_p

Due to the cable curvature, some of the prestress force is in the upward vertical direction and directly resists the applied shear. At the jacking end:
From Figure 13:

f_{pe} = 0.9169 - 0.1467 = 0.7702 f_{pj}
P_j = (0.74)(270)(70.959) = 14,178 k

From Figure 12:

The cable path angle = α = 2[(44.75-14)/12] / 80[74.63 / 80] = 0.05976

V_p = (14,178)(0.7702)(0.05976) = 653 kips

Step 5 – Check Shear Width, b_v

[5.8.2.9] The LRFD Specification requires that web width be adjusted for the presence of voided or grouted ducts. For ungrouted ducts, 50% of the width should be subtracted from the gross width and for grouted ducts, 25% should be subtracted. When the structure is first prestressed, the ducts are ungrouted. For this condition of dead load and prestressing, the shear should be checked with the 50% reduction for ducts. For the final condition, the ducts are grouted and only the 25% reduction is required.

For ungrouted ducts under DC dead load of superstructure and diaphragm:

[5.8.3.3-2] $V_u <= \varphi V_n = \varphi(0.25f'_c b_v d_v + V_p)$

[5.8.2.9-1]

$$\text{Required } b_v = \frac{\dfrac{V_u}{\varphi} - V_p}{0.25 d_v f'_c} = \frac{\dfrac{1.25 \cdot (946.38 + 10.86)}{0.9} - 653}{(0.25) \cdot (64.44) \cdot (4.5)} = 9.33 \text{ inches}$$

Available b_v = 61.85 - 0.50(4.375)(5webs) = 50.91 inches, ok

For grouted ducts under full load:

$$\text{Required } b_v = \frac{\dfrac{2615}{0.9} - 653}{(0.25) \cdot (64.44) \cdot (4.5)} = 31.07 \text{ inches}$$

Available b_v = 61.85 – 0.25(4.375)(5 webs) = 56.38 inches, ok

For the remainder of the problem, the shear width will be 56.38 inches.

Step 6 – Evaluate Shear Stress

[5.8.2.9-1]

$$v_u = \frac{|V_u - \varphi V_p|}{\varphi b_v d_v} = \frac{|2615 - (0.90) \cdot (653)|}{0.90 \cdot (56.38) \cdot (64.44)} = 0.620 \text{ ksi}$$

$$\frac{v_u}{f'_c} = \frac{0.620}{4.5} = 0.138$$

Step 7 – Estimate Crack Angle θ

The LRFD method of shear design involves several cycles of iteration. The first step is to estimate a value of θ, the angle of inclination of diagonal compressive stress. Since the formula is not very sensitive to this estimate assume that θ = 26.5 degrees. This simplifies the equation somewhat by setting the coefficient 0.5 cotθ = 1.0.

Step 8 – Calculate strain, ε_x

[5.8.3.4.2]

There are two formulae for the calculation of strain for sections containing at least the minimum amount of transverse reinforcing. The first formula is used for positive values of strain, while the second formula is used for negative values.

Formula for ε_x for positive values:

[5.8.3.4.2-1]

$$\varepsilon_x = \left[\frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}}{2(E_s A_s + E_p A_{ps})} \right]$$

Formula for e_x for negative values:

[5.8.3.4.2-3]

$$\varepsilon_x = \left[\frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}}{2(E_c A_c + E_s A_s + E_p A_{ps})} \right]$$

where:

A_c = area of concrete on the flexural tension side of the member. The flexural tension side of the member is the half depth of the member that is on the side with flexural tension stress. For positive moment, the flexural tension side is the lower 89.50 ÷ 2 = 44.75 inches of the section.
A_c = (396)(6)+2(0.5)(6)(2.4) + (44.75 – 6.00)(61.85) = 4787 in^2

A_{ps} = area of prestressing steel on the flexural tension side of the member.
A_{ps} = 70.959 in^2

A_s = area of nonprestressed steel on the flexural tension side of the member. A_s = 0.

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete. For the usual levels of prestressing, a value of 0.7 f_{pu} will be appropriate.
f_{po} = 0.70(270) = 189 ksi

N_u = factored axial force taken as positive if tensile.
N_u = 0 kips

V_u = factored shear force.
V_u = 2615 kips

M_u = factored moment but not to be taken less than $V_u d_v$.

Super	M_{crit} = (12.681 / 2)(5.37)(154.63)	= 5265 ft-k
Diaph	M_{crit} = (10.86)(5.37)	= 58 ft-k
Barrier	M_{crit} = (0.710 / 2)(5.37)(154.63)	= 295 ft-k
FWS	M_{crit} = (1.050 / 2)(5.37)(154.63)	= 436 ft-k

Lane	M_{crit} = (47.82)(5.37) = 257 ft-k
Truck	M_{crit} = (65.38)(5.37) = 351 ft-k
Lane	M_{crit} = (47.70)(5.37) = 256 ft-k
LL	M_{crit} = [257 + 1.33(351)](3.195)(0.983) = 2273 ft-k

M_u = 1.25(5265 + 58 + 295) + 1.50(436) + 1.75(2273) = 11,654 ft-k

M_u = 11,654 ft-k < $V_u d_v$ = (2615)(64.44 / 12) = 14,043 ft-k

[5.8.3.4.2-1]

$$\varepsilon_x = \left[\frac{\frac{|14{,}043 \cdot 12|}{64.44} + 0 + 1.0 \cdot |2615 - 653| - (70.959)\cdot(189)}{2(29000 \cdot 0 + 28500 \cdot 70.959)}\right]$$

ε_x = -0.00218

Since the value is negative the second formula must be used.

$$\varepsilon_x = \left[\frac{\frac{|14{,}043 \cdot 12|}{64.44} + 0 + 1.0 \cdot |2615 - 653| - (70.959)\cdot(189)}{2(3861 \cdot 4787 + 29000 \cdot 0 + 28500 \cdot 70.959)}\right]$$

ε_x = -0.000215 = -0.215 x 1000

[5.8.3.4.2-1]

Now go into [Table 5.8.3.4.2-1] to read the values for θ and β. From the previously calculated value of v_u/f'_c =0.138, enter the ≤ 0.150 row and the ≤ -0.20 column. The new estimate for values is shown below:

θ = 21.6 degrees
β = 2.88

With the new value of θ, the strain must be recalculated.

$$\varepsilon_x = \left[\frac{\frac{|14{,}043 \cdot 12|}{64.44} + 0 + 0.5 \cdot |2615 - 653| \cot(21.6) - 70.959 \cdot (189)}{41{,}009{,}877} \right]$$

ε_x = -0.000203 = -0.203 x 1000

With this new estimate for strain, reenter the table and determine new values for θ and β. Since our new values are the same as assumed, our iterative portion of the design is complete.

Step 9 - Calculate Concrete Shear Strength, V_c

The nominal shear resistance from concrete, V_c, is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316\beta\sqrt{f'_c}\,b_v d_v$$

$$V_c = 0.0316 \cdot (2.88) \cdot \sqrt{4.5} \cdot (56.38) \cdot (64.44) = 701 \text{ kips}$$

Step 10 - Determine Required Vertical Reinforcement, V_s

[5.8.3.3-1]

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s} = \frac{A_v f_y d_v \cot\theta}{s} \text{ where } \alpha = 90°$$

[5.8.3.3-4]

$$V_u \le V_R = \phi V_n = \phi(V_c + V_s + V_p)$$

$$\frac{V_u}{\phi} - V_c - V_p = \frac{A_v f_y d_v \cot\theta}{s}$$

For #5 u-stirrups per web, A_v = (0.31)(2 legs)(5 webs) = 3.10 in^2

$$s = \frac{A_v f_y d_v \cot\theta}{\frac{V_u}{\phi} - V_c - V_p} = \frac{(3.10)\cdot(60)\cdot(64.44)\cdot\cot(21.6)}{\frac{2615}{0.90} - 701 - 653} = 19.5 \text{ in}$$

Minimum Transverse Reinforcing [5.8.2.5]

When transverse shear reinforcing is required, the minimum area of reinforcing must satisfy the following.

$$A_v \ge 0.0316\sqrt{f'_c}\frac{b_v s}{f_y}$$

Rearranging yields the maximum spacing for a given area of shear reinforcing:

$$s_{max} = \frac{A_v f_y}{0.0316\sqrt{f'_c}\, b_v} = \frac{(3.10)\cdot(60)}{(0.0316)\cdot\sqrt{4.5}\cdot(56.38)} = 49.2 \text{ in}$$

Maximum Spacing [5.8.2.7]

The specification also limits the maximum spacing of transverse reinforcing to:

When $v_u < 0.125\, f'_c$ max spacing equals 0.8 $d_v \le 24$ inches

When $v_u \ge 0.125\, f'_c$ max spacing equals 0.4 $d_v \le 12$ inches

From previous calculations $v_u = 0.620 \ge 0.125(4.5) = 0.563$ ksi. Therefore, the maximum spacing equals 0.4(64.44) = 25.78 inches but not greater than 12 inches.

Use #5 stirrups at 12 inch spacing

$$V_s = \frac{(3.10)\cdot(60)\cdot(64.44)\cot(21.6)}{12} = 2523 \text{ kips}$$

[5.8.3.3-1]

$V_n = V_c + V_s + V_p$

$V_n = [701 + 2523 + 653] = 3877$ kips

[5.8.3.3-2]

$V_n = 0.25f'_c b_v d_v + V_p$

$V_n = [0.25(4.5)(56.38)(64.44) + 653] = 4740$ kips

$\varphi V_n = (0.90)(3877) = 3489\text{ k} > 2615\text{ k}$

[5.8.3.5]

Step 11 - Longitudinal Reinforcement

In addition to vertical reinforcement, shear requires a minimum amount of longitudinal reinforcement. The requirement for longitudinal reinforcement follows:

$$A_s f_y + A_{ps} f_{ps} \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\varphi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

[5.8.3.5-1]

V_s shall not be taken greater than V_u / φ.

$V_s = 2523\text{ k} < V_u / \varphi = 2615 / 0.90 = 2906\text{ k}$

Only considering the prestressing steel yields the following:

$$(70.959)\cdot(246.69) \geq \frac{|14{,}043\cdot 12|}{(64.44)\cdot(0.95)} + \left(\left|\frac{2615}{0.90} - 653\right| - 0.5\cdot(2523)\right)\cot(21.6)$$

17,505 kips > 5256 kips

$\therefore$ The prestressing strands are adequate for longitudinal reinforcement without additional mild reinforcing. The longitudinal reinforcement must be checked at points where the longitudinal reinforcement is terminated or at locations where the shear reinforcing changes spacing.

The shear strength is the lesser of:

Interface Shear Transfer [5.8.4]

For cast-in-place box girder bridges, the deck is cast separately from the bottom slab and webs. Thus the shear transfer across this surface would appear to require investigation. In the past this was sometimes performed but was rarely a controlling criteria. The current specifications would appear to require analysis for interface shear across the horizontal joint at the top of the web since the concrete is poured across the joint at different times.

However, the method contained in the specification is more appropriate for precast girders with concrete decks poured after the member is erected. For a post-tensioned box girder bridge, the deck is poured prior to the prestressing. Once stressed the member acts as a unit with the vertical reinforcing providing adequate strength for horizontal shear. Application of the current specification to this problem resulted in a requirement for wider webs and additional vertical reinforcing.

In 2006 the Specification added a diagram and discussion concerning web-flange interfaces. This has traditionally not been a problem with the usual configuration of cast-in-place post-tensioned concrete box girder bridges used in Arizona. For single cell boxes or those with widely spaced webs the shear transfer mechanism should be investigated.

Based on the above discussion interface shear need not be checked for typical a cast-in-place post-tensioned concrete box girder bridge.

Post-Tensioned Anchor Zone [5.10.9] [5.10.9.3.1]

The design of anchor zone involves the strength limit state including factored jacking forces. Three design methods are provided in the LRFD Specification: strut-and-tie, refined elastic stress and approximate methods. The refined elastic stress method is very involved and not deemed appropriate for ordinary bridges. The approximate methods do not adequately consider the I-shape nature of the box and therefore may provide inaccurate answers. Therefore the strut-and-tie method will be used for analysis.

Step 1 – Define Geometry

The first step in the analysis process is to define the geometry of the anchor zone. Figure 15 below shows a plan view of the end diaphragm while Figure 16 shows an elevation view.

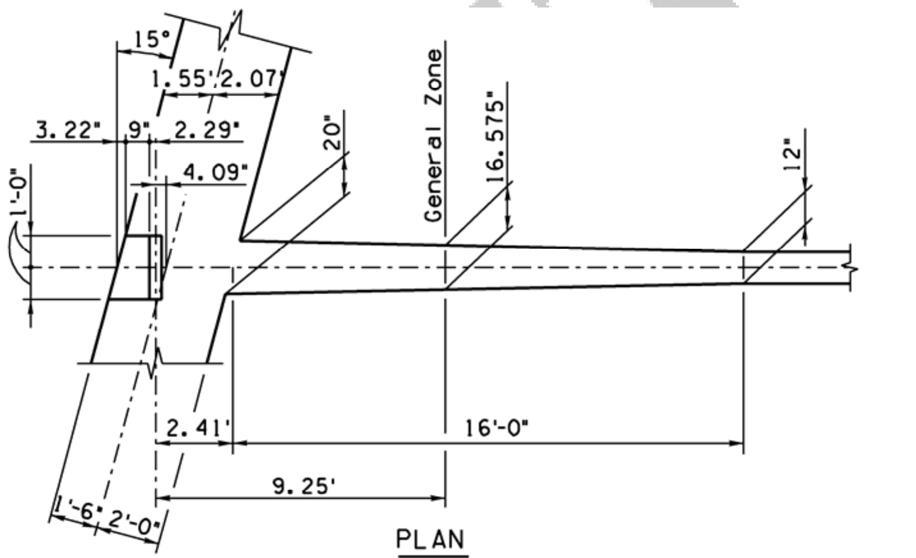


Figure 15 – Plan View Abutment Diaphragm

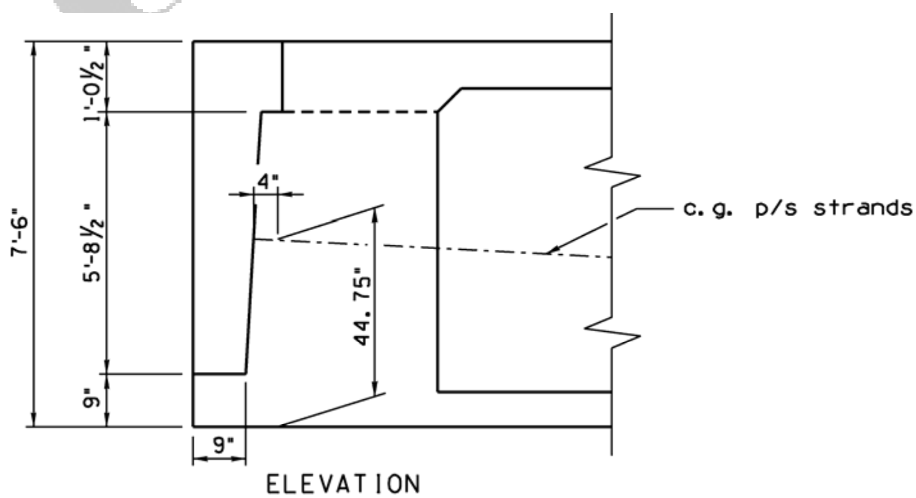


Figure 16 – Elevation View Abutment Diaphragm

The anchor zone design is based on the location of the actual anchorage devices. Since there is no direct relation between its location and the centerline of bearing of the abutment, one must be estimated. Based on the width of the diaphragm and the skew angle an approximate location of the bearings in relation to the centerline of bearing can be made. The calculated value is 4.09 inches but 4 inches will be assumed in the calculations.

Step 2 – Determine Anchorage Zone

The anchorage zone is geometrically defined as the volume of concrete through which the concentrated prestressing force at the anchorage device spreads to a more linear stress distribution across the entire cross-section at some distance from the anchorage device. Within this zone, the assumption that plane sections remain plane is not valid, requiring a different method of analysis. The anchorage zone may be taken as the maximum depth or width of the section but not larger than the longitudinal extent of the anchorage zone

]
[5.10.9.2.3

Step 3 – Determine Local Zone

The local zone is the rectangular prism of the concrete surrounding and immediately ahead of the anchorage device and any integral confining reinforcement. The local zone is the region of high compressive stresses immediately ahead of the anchorage device.

When the manufacturer has provided edge distance recommendations, the width and height shall be twice the edge distance. For the anchorage system required in this example, the minimum recommended edge distance is 10.50 inches. This produces a local zone with a width, height and length of 21 inches

[5.10.9.7.1]

When the manufacturer has not provided a minimum edge distance, the transverse dimension in each direction shall be taken as the greater of:

1. The bearing plate size plus twice the minimum cover.
2. The outer dimension of any required confining reinforcement plus the required concrete cover.

[5.12.3-1]

Based on the flexural design, 22 strands are required per duct based on usage of 0.6” diameter strands. From post-tensioning literature, the spirals for this system are 9.5 inches long with a 17 inch outside diameter. Adding two inch clearance to each side yields a local zone of 21 inches diameter. This produces an equivalent square of 18.61 inches.

The length of the local zone shall not be taken to be less than:

1. The maximum width of the local zone = 18.61”
2. The length of the anchorage device confining reinforcement = 9.5”

The length of the local zone shall not be taken as greater than 1.5 times the width of the local zone = 1.5(18.61) = 27.92 inches. The length of the local zone should be greater than 18.61 inches and less than 27.92 inches. For this problem a length of 18.61 inches will be used.

[5.10.9.2.2]

Step 4 – Determine General Zone

The general zone extent is the same as the anchorage zone. The general zone is the region subjected to tensile stresses due to spreading of the tendon force into the structure and includes the local zone.

The minimum general zone length is the maximum of the width (9.25 feet) or depth (7.50 feet). The maximum general zone length equals 1.5 times this value. Use a general zone length of 9.25 feet.

Step 5 – Determine Section Properties

The section properties are required at the end of the anchorage zone to allow for the determination of the stresses. At this location the web is flared requiring that the dimension between the anchorages and the centerline bearing be known. Based on the above calculations the anchorages can be assumed to be 4 inches behind the centerline. Based on an anchorage zone length of 9.25 feet, the width of the flared web can be determined at the end of the anchorage zone.

web = 12.00 + (20 - 12)(0.33 + 2.07 + 16.00 - 9.25)/(16) = 16.575 inches
$\sum$web = 16.575[2/cos(21.80) + 3] = 85.43 inches

For anchor zone design the ½ inch wearing surface has not been subtracted. The calculations for the section properties at the end of the anchor zone are not shown. A summary of the section properties follows:

Area	13,678	in^2
Inertia	14,688,357	in^4
y_b	51.373	in
y_t	38.627	in

Step 6 – Determine External Loads

[3.4.3.2]

For post-tensioning, a load factor of 1.2 is used. This is applied to the maximum stress in the strand that can be interpreted to be the jacking stress. The total jacking force is as follows:

$$P_u = (1.2)\cdot(0.74)\cdot(270)\cdot(327)\cdot(0.217) = 17{,}013 \text{ kips}$$

While the cable path follows a parabolic shape, in reality near the anchorage device, the path will be tangent. The anchorage device and trumpets are straight and must be installed as such. For this problem assume that the tangent segment length is 16 feet. This will require that the tendon path be located on an angle from the horizontal as follows:

$$y_{b16} = 14 + [44.75 - 14.00] \cdot \left(\frac{64}{80}\right)^2 = 33.68 \text{ in}$$

$$\alpha = \tan^{-1}\left[\frac{44.75 - 33.68}{(12)\cdot(16.00)}\right] = 3.2998 \text{ degrees}$$

The total tendon force must be divided into vertical and horizontal components as follows:

$$P_{uh} = (17{,}013) \cdot \cos(3.2998) = 16{,}985 \text{ kips}$$
$$P_{uv} = (17{,}013) \cdot \sin(3.2998) = 979 \text{ kips}$$

This force will be equally divided among the tendons at the anchorage end.

P_{uh} = 16,985 / 3 = 5661.67 kips
P_{uv} = 979 / 3 = 326.33 kips

To simplify the analysis but still maintain equilibrium, round these values as follows:

P_{uh} = 5662 kips top and bottom tendons
P_{uh} = 5661 kips middle tendon

P_{uv} = 326 kips top and bottom tendons
P_{uv} = 327 kips middle tendon

Step 7 – General Zone Stress Distribution

The stress at the end of the anchor zone is determined by classical methods. The stress on each structural shape is calculated to determine the forces acting on the various shapes. The eccentricity at the end of the anchor zone follows:

c.g. = 44.75 – [(44.75 – 33.68) / 16.00](9.25 – 4.00 / 12) = 38.581 in

$e_{genzone}$ = 51.373 – 38.581 = 12.792 inch

The stress acting on each interface is determined as follows

Top	[16985][1/13678-(12.792)(38.627)/(14688357)] = 0.67040 ksi
Soffit	[16985][1/13678-(12.792)(30.127)/(14688357)] = 0.79613 ksi
Overhang	[16985][1/13678-(12.792)(29.127)/(14688357)] = 0.81092 ksi
Fillet	[16985][1/13678-(12.792)(26.127)/(14688357)] = 0.85530 ksi
Top Bot	[16985][1/13678+(12.792)(45.373)/(14688357)] = 1.91294 ksi
Bottom	[16985][1/13678+(12.792)(51.373)/(14688357)] = 2.00169 ksi

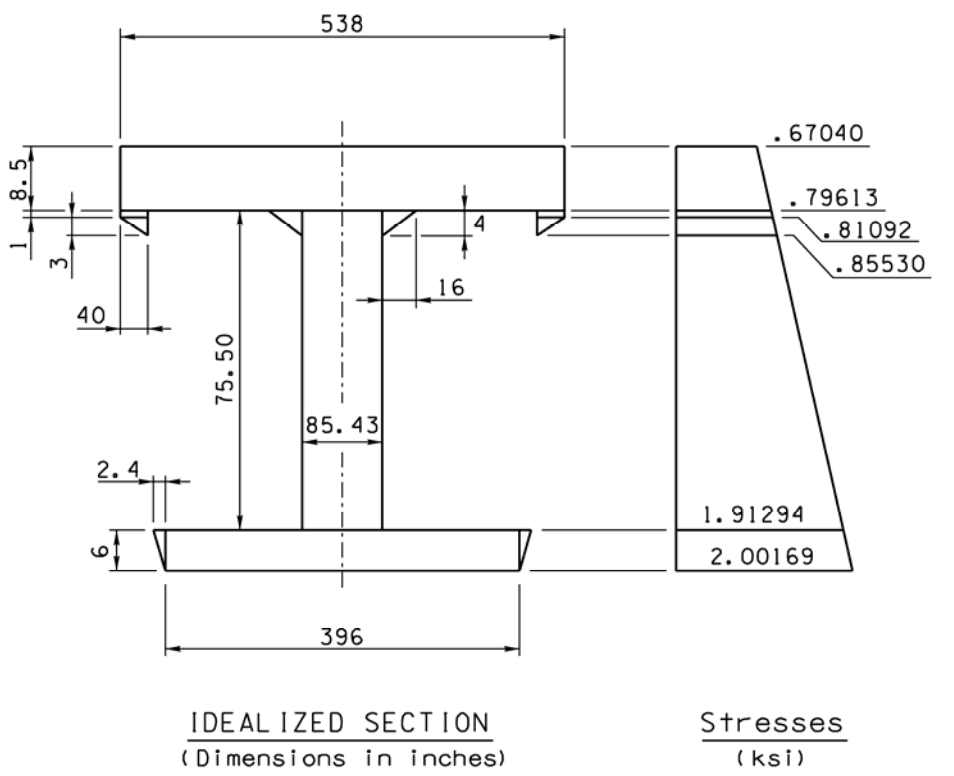


Figure 17

Step 8 – Determine Forces at End of Anchorage Zone

The stresses calculated in Step 7 must now be applied to the various shapes of the cross section to determine the magnitude of the force acting on each area and the location of the center gravity of the load. These forces must be combined into three groups: top slab, web and bottom slab with the top fillets included in the web force.

Determine the forces and center gravity resulting from the stress distribution acting on the individual member shapes.

Top Slab

Force 1 = [0.67040](8.50)(538.00) = 3065.74 k
Force 2 = [0.79613 – 0.67040](8.50)(538.00) / 2= 287.48 k
= 3353.22 k

CG = 90.00 - [(3065.74)(8.50/2) + (287.48)(8.50)(2/3)] / 3353.22
= 85.6285 in

Overhang

Force 3 = [0.79613](1.00)(80.00) = 63.69 k
Force 4 = [0.81092 – 0.79613](1.00)(80.00) / 2 = 0.59 k
= 64.28 k

c.g. = [63.69(1.00 / 2) + 0.59(1.00)(2/3)] ÷ 64.28 = 0.5015 in

Exterior Fillets

Force 5 = [0.81092](3.00)(80.00) / 2 = 97.31 k
Force 6 = [0.85530 – 0.81092](3.00)(80.00) / 6 = 1.78 k
= 99.09 k

c.g. = [97.31(3.00/3) + 1.78(3.00 / 2)] ÷ 99.09 = 1.0090 in

Interior Fillets

Force 7 = [0.79613](4.00)(32.00) / 2 = 50.95 k
Force 8 = [0.85530 – 0.79613](4.00)(32.00)/6 = 1.26 k
= 52.21 k

c.g. = [50.95(4.00/3) + 1.26(4.00 / 2)] ÷ 52.21 = 1.3494 in

Web

Force 9 = [0.79613](75.50)(85.43) = 5135.01 k
Force 10 = [1.91294– 0.79613](75.50)(85.43)/2= 3601.69 k
= 8736.70 k

c.g. = [5135.01(75.50/2) + 3601.69(75.50)(2/3)] ÷ 8736.70 = 42.9375 in

Combination of Fillets and Web

Force = 64.28 + 99.09 + 52.21 + 8736.70 = 8952.28 k

c.g. = [64.28(0.5015) + 99.09(1.00+1.0090) + 52.21(1.3494)
+ 8736.70(42.9375)] ÷ 8952.28 = 41.9372 in

CG = 90.00 - 8.50 – 41.9372 = 39.5628 in

<u>Bottom Slab</u>

Force 11 = [1.91294](6.00)(396.00) = 4545.15 k
Force 12 = [2.00169-1.91294](6.00)(396.00) / 2= 105.44 k
Force 13 = [1.91294](6.00)(4.80) / 2 = 27.55 k
Force 14 = [2.00169-1.91294](6.00)(4.80) / 6 = <u>0.43</u> k
= 4678.57 k

CG = [(4545.15)(6.00/2) + (105.44)(6.00/3) + (27.55)(6.00)(2/3)
+ (0.43)(6.00/2)] / (4678.57) = 2.9834 in

The sum of the forces from all the members is 3353.22 + 8952.28 + 4678.57 = 16,984.07 kips compared to the 16,985 kips horizontally applied load.

The accuracy and number of significant digits used may appear excessive. However, this was done in this example to demonstrate the validity of the method and demonstrate that statics is maintained.

Step 9 – Create Strut-and-Tie Model

Using the calculated center gravity as the y-coordinate, the strut-and-tie model can be created. Joints 1 to 3 are located at the end of the Anchorage Zone. Joints 4, 5 and 6 are located 0.5h from the post-tensioned anchorages.

Joint 5

y-Coord = 39.5628 + (111.000 - 45.000)(979) / 8952 = 46.7806

Joint 8

y-Coord = 44.750 + (4.00)tan(3.2998) = 44.9806

Joint 7

y-Coord = 44.9806 + (24.00)cos(3.2998) = 68.9408
x-Coord = (24.00)sin(3.2998) = 1.3815

Joint 9

y-Coord =44.9806 – (24.00)cos(3.2998) = 21.0204
x-Coord =-(24.00)sin(3.2998) = -1.3815

A summary of coordinates and applied forces follows:

Joint	x-Coord	y-Coord	Fx	Fy
1	111.0000	85.6285	-3353	
2	111.0000	39.5628	-8952	979
3	111.0000	2.9834	-4679	
4	45.0000	85.6285		
5	45.0000	46.7806		
6	45.0000	2.9834		
7	1.3815	68.9408	5662	-326
8	0.0000	44.9806	5661	-327
9	-1.3815	21.0204	5662	-326

A diagram showing the strut-and-tie model with the externally applied forces is shown in Figure 18.

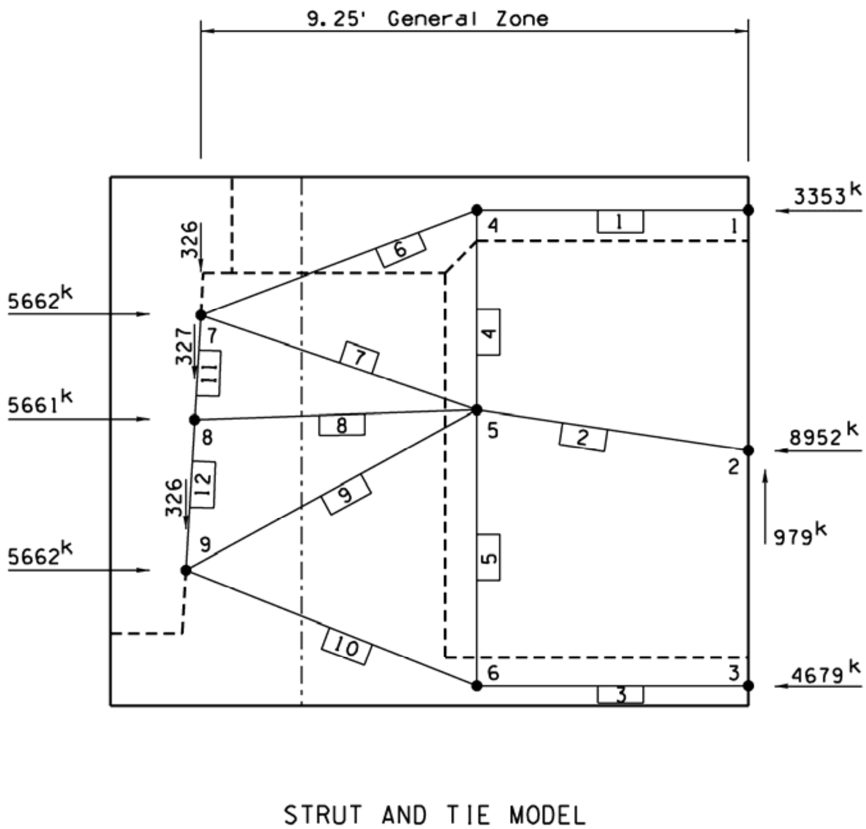


Figure 18 Strut-and-Tie Model

Step 10 – Solve for Member Forces

Member 1 and Member 3 carry the applied forces at the end of the anchorage zone. Member 2 has both a vertical and a horizontal component. The member forces are shown below:

F1 = 3353 kips
$F2 = [(8952)^2 + (979)^2]^{1/2} = 9005$ kips
F3 = 4679 kips

The remainder of the member forces must be calculated by equating the sum of the forces at a node equal to zero in both the vertical and horizontal directions.

Node 4

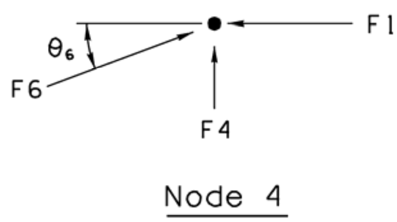


Figure 19

$\theta_6 = \tan^{-1}[(85.6285 - 68.9408) / (45.0000 - 1.3815)] = 20.9360$ degrees

$F6 = F1 / \cos \theta_6 = 3353 / \cos(20.9360) = 3590$ kips

$F4 = -F6 \sin \theta_6 = -3590 \sin(20.9360) = -1283$ kips

Node 6

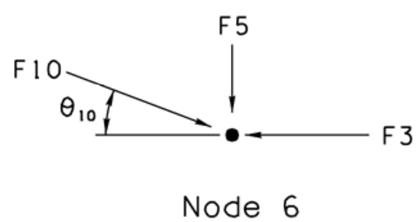


Figure 20

$\theta_{10} = \tan^{-1}[(21.0204 - 2.9834) / (45.0000 + 1.3815)] = 21.2502$ degrees

$F10 = F3 / \cos \theta_{10} = 4679 / \cos(21.2502) = 5020$ kips

$F5 = -F10 \sin \theta_{10} = -5020 \sin(21.2502) = -1820$ kips

Node 7

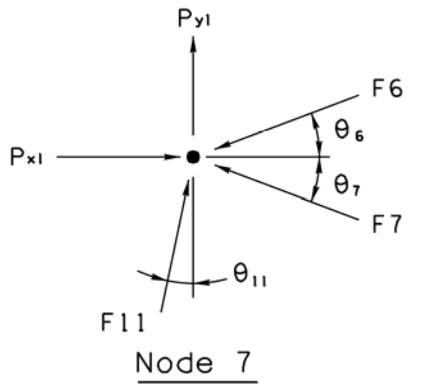


Figure 21

P_{x1} = 5662 k
P_{y1} = -326 k

F6 = 3590 k

θ_6 = 20.9360 degrees
$\theta_7 = \tan^{-1}[(68.9408 - 46.7806) / (45.0000 - 1.3815)]$ = 26.9327 degrees
θ_{11} = 3.2998 degrees

Sum Forces in x-direction

$P_{x1} - F6 \cos\theta_6 - F7 \cos\theta_7 + F11 \sin\theta_{11} = 0$

Sum Forces in y-direction

$P_{y1} - F6 \sin\theta_6 + F7 \sin\theta_7 + F11 \cos\theta_{11} = 0$

Solve the second equation for F11 and substitute into the first equation solving for F7:

$$F7 = \frac{P_{x1} - F6(\cos\theta_6 - \sin\theta_6 \tan\theta_{11}) - P_{y1}\tan\theta_{11}}{\cos\theta_7 + \sin\theta_7 \tan\theta_{11}} = 2617 \text{ k}$$

$$F11 = \frac{-P_{y1} + F6\sin\theta_6 - F7\sin\theta_7}{\cos\theta_{11}} = 424 \text{ k}$$

Node 9

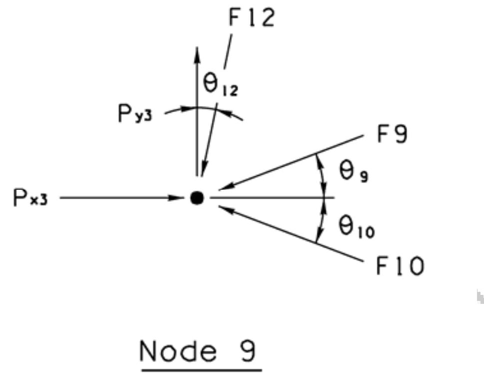


Figure 22

P_{x3} = 5662 k
P_{y3} = -326 k

F10 = 5020 k

$\theta_9 = \tan^{-1}[(46.7806 - 21.0204) / (45.0000 + 1.3815)] = 29.0477$ degrees
θ_{10} = 21.2502 degrees
θ_{12} = 3.2998 degrees

Sum Forces in x-direction

$P_{x3} - F9 \cos\theta_9 - F10 \cos\theta_{10} - F12 \sin\theta_{12} = 0$

Sum Forces in y-direction

$P_{y3} - F9 \sin\theta_9 + F10 \sin\theta_{10} - F12 \cos\theta_{12} = 0$

Solve for F12 in the second equation and substitute into the first equation to solve for F9:

$$F9 = \frac{P_{x3} - P_{y3}\tan\theta_{12} - F10(\cos\theta_{10} + \sin\theta_{10}\tan\theta_{12})}{\cos\theta_9 - \sin\theta_9\tan\theta_{12}} = 1060 \text{ k}$$

$$F12 = \frac{P_{y3} - F9\sin\theta_9 + F10\sin\theta_{10}}{\cos\theta_{12}} = 980 \text{ k}$$

Node 8

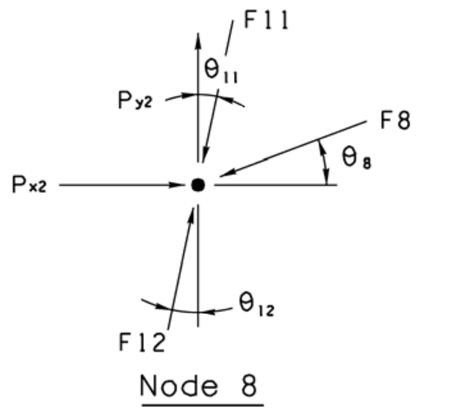


Figure 23

P_{x2} = 5661 k
P_{y2} = -327 k

F11 = 424 k
F12 = 980 k

$\theta_8 = \tan^{-1}[(46.7806 - 44.9806) / (45.000 + 0.000)] = 2.2906$ degrees
θ_{11} = 3.2998 degrees
θ_{12} = 3.2998 degrees

Sum Forces in x-direction

$$P_{x2} - F11 \sin\theta_{11} - F8\cos\theta_8 + F12\sin\theta_{12} = 0$$

$$F8 = \frac{P_{x2} - F11\sin\theta_{11} + F12\sin\theta_{12}}{\cos\theta_8} = 5698 \text{ k}$$

Sum Forces in y-direction for static check at joint

$$\sum F_y = P_{y2} - F11\cos\theta_{11} - F8\sin\theta_8 + F12\cos\theta_{12} = 0$$

$$\sum F_y = -327 - 424\cos(3.2998) - 5698\sin(2.2906) + 980\cos(3.2998) = 0.34 \approx 0$$

Node 5

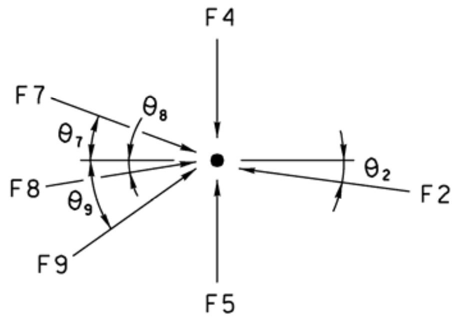


Figure 24

All member forces have been determined. Node 5 check is made to determine the accuracy of the solution.

F2 = 9005 k
F4 = -1283 k
F5 = -1820 k
F7 = 2617 k
F8 = 5698 k
F9 = 1060 k

$\theta_2 = \tan^{-1}[(46.7806 - 39.5628) / (111.0000 - 45.0000)] = 6.2411$ degrees
$\theta_7 = 26.9327$ degrees
$\theta_8 = 2.2906$ degrees
$\theta_9 = 29.0477$ degrees

Sum Forces in x-direction for static check at joint

$$\sum F_x = F7 \cos\theta_7 + F8 \cos\theta_8 + F9 \cos\theta_9 - F2 \cos\theta_2 = 0$$

$$\sum F_x = 2617\cos(26.9327) + 5698\cos(2.2906) + 1060\cos(29.0477) - 9005\cos(6.2411) = 1.64 \text{ k} \approx 0$$

Sum Forces in y-direction for static check at joint

$$\sum F_y = -F7 \sin\theta_7 + F8 \sin\theta_8 + F9 \sin\theta_9 + F2 \sin\theta_2 - F4 + F5 = 0$$

$$\sum F_y = -2617\sin(26.9327) + 5698\sin(2.2906) + 1060\sin(29.0477) + 9005\sin(6.2411) + 1283 - 1820 = 0.99 \text{ k} \approx 0$$

[5.10.9.6.3]

Step 11 – Web Bursting Design

Determine the maximum vertical tensile force in the web. Member 5 has the largest tensile force of –1820 kips. Divide this force by the number of webs to obtain a force of –364.00 kips per web. For tension in steel in anchorage zones use φ = 1.0. Determine the required area of reinforcement.

[5.5.4.2.1]

$$A_s = \frac{F_{\max}}{\phi f_y} = \frac{364.00}{(1.00)\cdot(60)} = 6.07 \text{ in}^2$$

Try 7 - #6 stirrups. A_s = (0.44)(2)(7) = 6.16 in^2. Center these stirrups about the tie (Member 4 and 5) in the strut-and-tie model. The tie is located 45 inches from the anchorage or 41 inches from the centerline of bearing. Space the bursting stirrups at 7 inch spacing about the tie. This results in the first stirrup being 20 inches from the centerline of bearing or about 24 inches from the beginning of the anchorage. This leaves about 5 inches from the end of the local zone to the first stirrup.

[5.10.9.3.2]

Step 12 – Spalling Reinforcing

For multiple anchorages with a center-to-center spacing of less than 0.4 times the depth of the section, the spalling force shall not be taken to be less than 2 percent of the total factored tendon force. Since the strut-and-tie analysis did not reveal any tension between anchorages and our spacing of 24 inches is less than 0.4(90) = 36.0 inches, use the 2 percent criteria.

Spalling Force = 0.02(17,013) / (5 webs) = 68.05 kips per web.

$$A_s = \frac{T}{\varphi f_y} = \frac{68.05}{(1.00)\cdot(60)} = 1.13 \text{ in}^2$$

Use 4 - #5 rebar per web for spalling, yielding an A_s = 4(0.31) = 1.24 in^2.

Step 13 – Concrete Stresses

Concrete stresses in the local zone can be very high. The use of a spiral increases the allowable concrete stress in the local region with the designs verified by testing. The responsibility of this region is given to the post-tensioning device supplier.

However, at the local zone/general zone interface the concrete stresses must be checked. From the anchorage head to the interface, the stresses spread on a 1:3 angle. For a local zone 18.61 x 18.61 inches, the width of the interface is 18.61 + 2(18.61) / 3 = 31.02 inches. The height equals the spacing plus the spread = 2[(18.61) / 3 + (18.61) / 2 + 24.00] = 79.02 inches. The force per web equals 17,013 / 5 = 3403 kips

f_{ci} = [3403] / [(31.02) (79.02)] = 1.388 ksi

[5.5.4.2.1]

Allowable stress = 0.7 φ f'_{ci} = 0.7 (0.80) (3.5) = 1.960 ksi

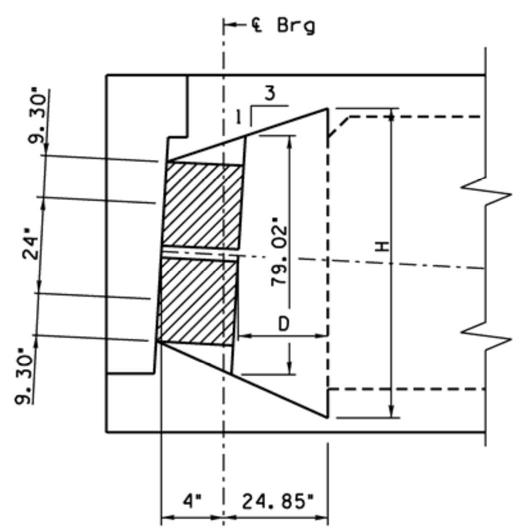
Discontinuity [5.10.9.3.2]

The concrete must also be checked at the end of the diaphragm where the width of the spread is limited to the width of the web member. The distance between the end of the local zone and the diaphragm equals:

D = 4.00 + 24.00 / cos(15) – 18.61 = 10.24 inches
H = [79.02 + 2(10.24) / 3] = 85.85 inches < 90 inches depth

f_{ci} = [3403] / [(20.00) (85.85)] = 1.982 ksi ≅ 1.960 ksi

Approximate 1% overstress is ok.



LOCAL ZONE/GENERAL ZONE INTERFACE

Figure 25

Step 14 – Top Slab Analysis

The top slab must also disperse the concentrated forces from the webs to the entire width of the slab. A strut-and-tie model (Figure 27) was created with one set of nodes at the web top slab interface 45 inches from the anchors and the other set half the web spacing away (55.50 inches). At the end of the general zone the stresses are uniformly distributed with nodes placed between the webs or the exterior web and the edge of deck. The force applied at each web equals the force in the top slab divided by the number of webs. The uniform load equals the top slab force divided by the web width.

Pweb = 3353 / 5 = 670.60 kips
Uniform = 3353 / 538.00 = 6.2323 kips/inch

The coordinate geometry of the top slab is shown below:

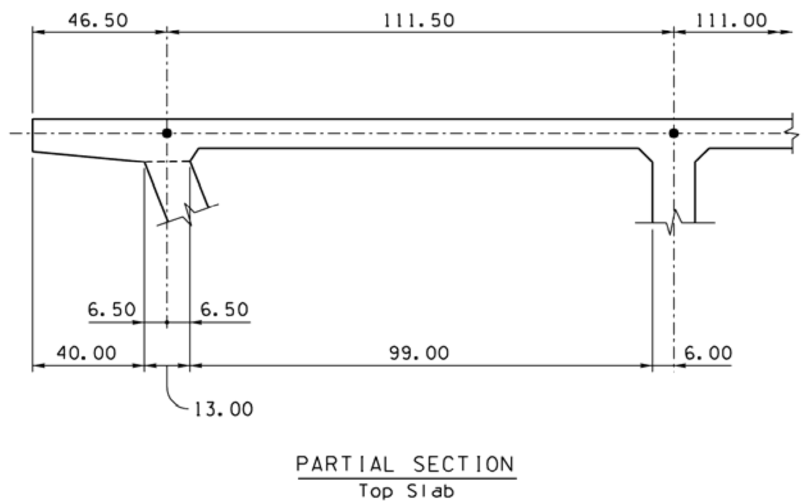
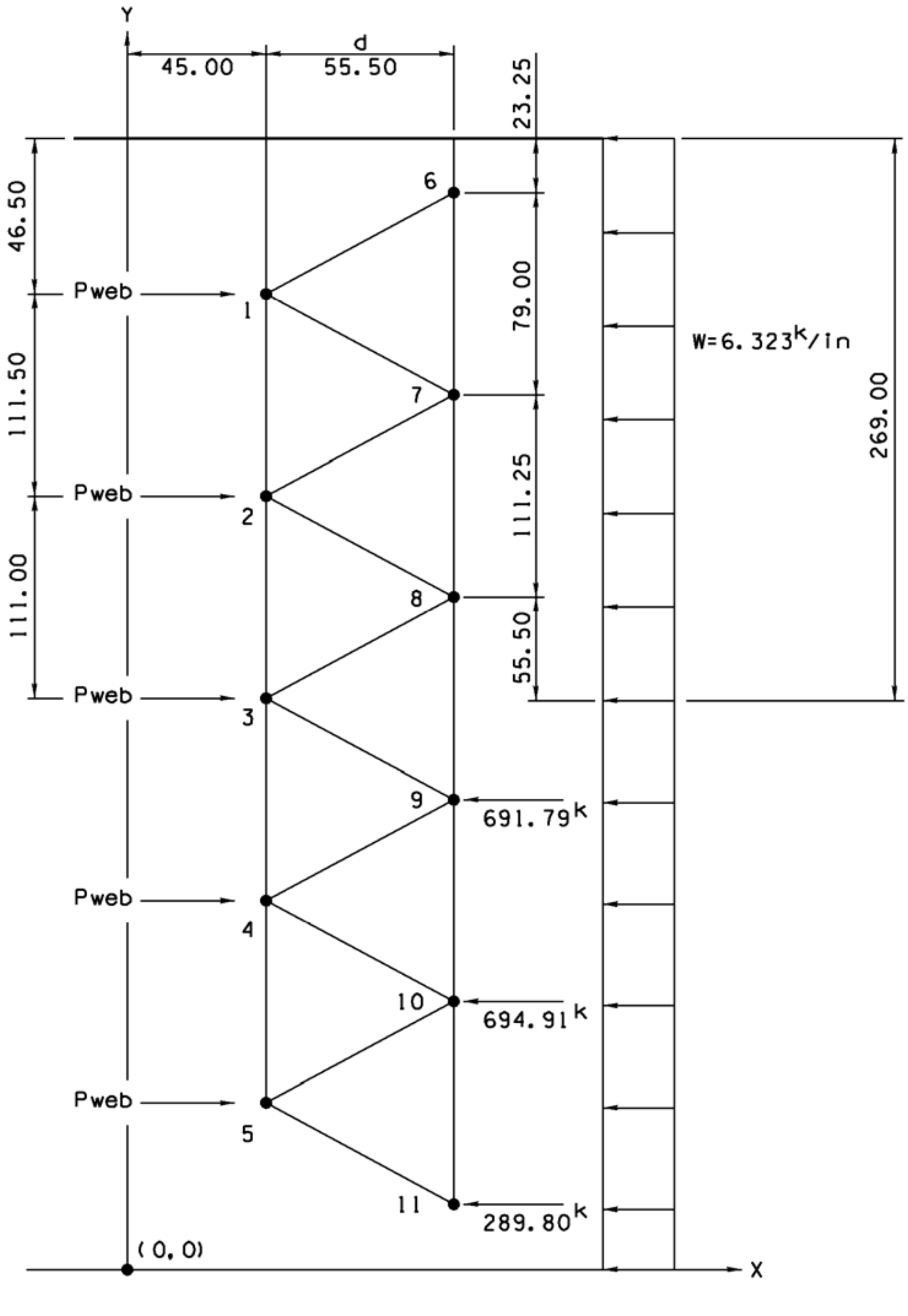


Figure 26

Joint coordinates and applied forces for the model are shown below:

Joint	x-Coord	y-Coord	Member Force
1	45.00	491.50	670.60
2	45.00	380.00	670.60
3	45.00	269.00	670.60
4	45.00	158.00	670.60
5	45.00	46.50	670.60
6	100.50	514.75	-289.80
7	100.50	435.75	-694.91
8	100.50	324.50	-691.79
9	100.50	213.50	-691.79
10	100.50	102.25	-694.91
11	100.50	23.25	-289.80



TOP SLAB

STRUT-AND-TIE MODEL

Figure 27

The forces applied at the joints due to the uniformly distributed force in the top slab equals the uniform load multiplied by the contributory area as follows:

P6 = P11 = 6.2323(46.50) = 289.80 k
P7 = P10 = 6.2323(111.50) = 694.91 k
P8 = P9 = 6.2323(111.00) = 691.79 k

The complete analysis of the strut-and-tie model is not shown here. The forces in each member can be determined by calculating the angles of the members and summing the forces in both the x and y directions at each node to determine the member forces.

A simple method to obtain the tension tie forces is to cut a section through a joint and sum the moments about the node. The member force is then the sum of the moments divided by the x-distance between the nodes. This method will be demonstrated below:

<u>First tie</u>

Sum forces about Joint 7:

F1 = [(289.80)(514.75 - 435.75) - (670.60)(491.50 - 435.75)] / 55.50
= -261.11 k

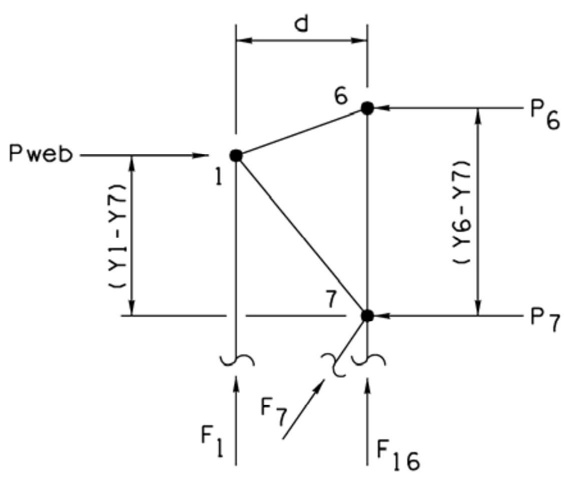


Figure 28

Sum forces about Joint 8:

F2 = [(289.80)(514.75 - 324.50) + (694.91)(435.75 - 324.50) –
(670.60)(491.50 - 324.50) – (670.60)(380.00 - 324.50)] / 55.50
= -302.08 k

Second Tie

Sum forces about Joint 1:

F15 = [-(289.80)(514.75 – 491.50)] / 55.50 = -121.40 k

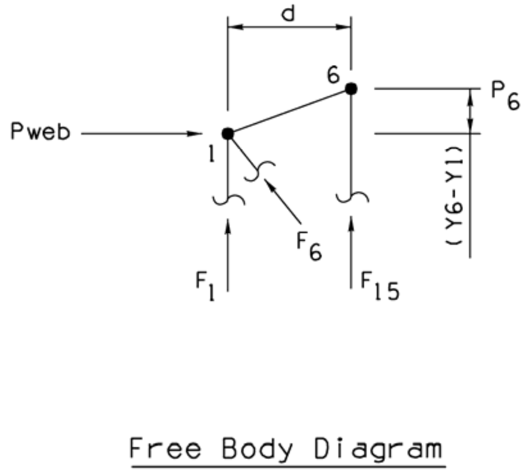


Figure 29

Sum forces about Joint 2:

F16 = [-(289.80)(514.75 – 380.00) - (694.91)(435.75 – 380.00)
+ (670.60)(491.50 – 380.00)] / 55.50 = -54.41 k

Sum forces about Joint 3:

F17 = [-(289.80)(514.75 – 269.00) - (694.91)(435.75 – 269.00) –
(691.79)(324.50 – 269.00) + (670.60)(491.50 – 269.00) +
(670.60)(380.00 – 269.00)] / 55.50 = -33.22 k

The first tie consists of forces F1 and F2, while the second tie consists of forces F15, F16 and F17. Both ties have tension forces with the required tensile reinforcement as follows:

First tie: A_s = 302.08 / [(1.00)(60)] = 5.03 in^2
Use 7 - #8 at 8 inches (A_s = 5.53 in^2)

Second tie: A_s = 121.40 / [(1.00)(60)] = 2.02 in^2
Use 5 - #6 at 8 inches (A_s = 2.20 in^2)

See Figure 32 for reinforcing location.

Step 15 – Bottom Slab Analysis

The bottom slab must also disperse the concentrated forces from the webs to the entire width of the slab. A strut-and-tie model (Figure 31) was created with one set of nodes at the web bottom slab interface 45 inches from the anchors and the other set half the web spacing away (55.50 inches). At the end of the general zone the stresses can be uniformly distributed with nodes placed between the webs or the exterior web and the edge of deck.

For the bottom slab with the sloping exterior web and no bottom overhang, the assumption of a uniformly distributed stress in the bottom slab is not reasonable over such a short distance. A better assumption is that the force from two exterior webs will be distributed from the edge of the slab for a distance to the midpoint between the second and third webs. The force applied at each web equals the force in the bottom slab divided by the number of webs. The force applied at the other joints equals the uniform load multiplied by the appropriate distance. Due to the sloping face, the bottom slab is assumed to be a rectangle with a width of 396.00 + 2.40 = 398.50 inches. See Figure 30.

P_{web} = 4679 / 5 = 935.80 kips

Exterior Width = 1.20 + 87.00 + 111.00 / 2 = 143.70 in
Exterior Uniform = (2)(935.80) / 143.70 = 13.0244 kips/inch

Interior Uniform = (935.80) / 111.00 = 8.4306 kips/inch

The forces applied to the joints due to the uniformly distributed force in the bottom slab equals the uniform load multiplied by the corresponding area as follows:

P6 = P9 = (13.0244)(88.20) = -1148.75 k
P7 = P8 = (13.0244)(111.00 / 2) + (8.4306)(111.00 / 2) = -1190.75 k

Calculations for the y-coordinates for the two exterior webs are shown below:

y-Coordinate
Jt. 6: y = 538.00 – 71.00 + 1.20 – 88.20 / 2 = 424.10
Jt. 9: y = 71.00 – 1.20 + 88.20 / 2 = 113.90

To maintain equilibrium the first interior joint must be located at the center of gravity of the assumed load. Summing moments about Joint 3 yields:

H = [13.0244(55.50)(83.25) + 8.4306(55.50)(27.75)] / 1190.75 = 61.44
Jt. 7: y = 269.00 + 61.44 = 330.44
Jt. 8: y = 269.00 – 61.44 = 207.56

Joint coordinates and applied forces for the model are shown below:

Joint	x-Coord	y-Coord	Member Force
1	45.00	459.27	935.80
2	45.00	380.00	935.80
3	45.00	269.00	935.80
4	45.00	158.00	935.80
5	45.00	78.73	935.80
6	100.50	424.10	-1148.75
7	100.50	330.44	-1190.75
8	100.50	207.56	-1190.75
9	100.50	113.90	-1148.75

Using the wider web at the end of the general zone is conservative but helpful. The diagram used to determine the coordinates for the exterior webs is as follows:

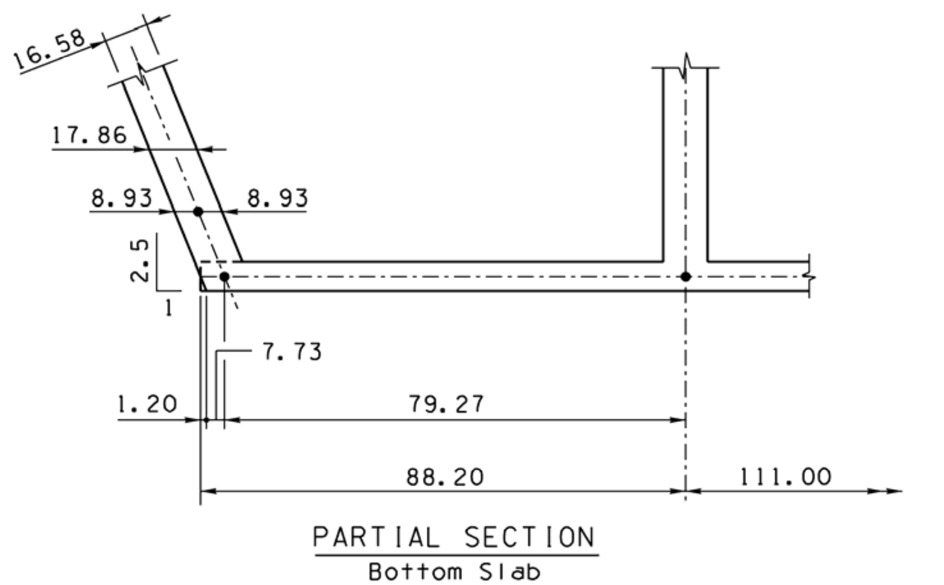
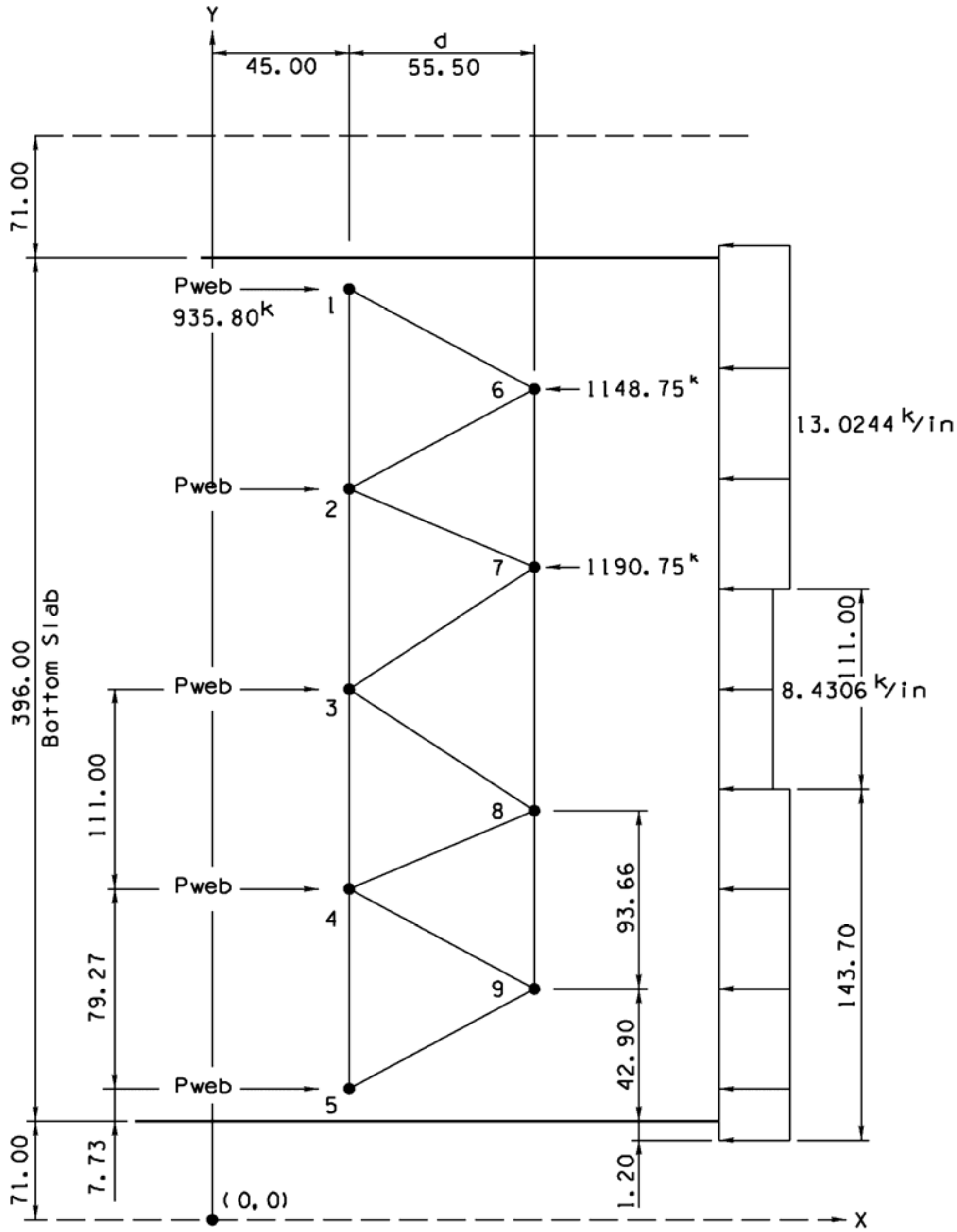


Figure 30



BOTTOM SLAB

STRUT-AND-TIE MODEL

Figure 31

The complete analysis of the strut-and-tie model is not shown here. The force in each member can be determined by calculating the angles of the members and summing the forces in both the x and y directions at each node to determine the member forces. A simple method to obtain the tension tie forces is to cut a section through a joint and sum moments dividing by the x-distance between the nodes. This method will be demonstrated below:

First tie

Sum forces about Joint 6:

F1 = [-(935.80)(459.27 – 424.10)] / 55.50 = -593.01 k

Sum forces about Joint 7:

F2 = [-(935.80)(459.27 – 330.44) - (935.80)(380.00 – 330.44) + (1148.75)(424.10 – 330.44)] / 55.50 = -1069.29 k

Second Tie

Sum forces about Joint 2:

F13 = [(935.80)(459.27 – 380.00) - (1148.75)(424.10 – 380.00)] / 55.50 = 423.80 k

Sum forces about Joint 3

F14 = [(935.80)(459.27 – 269.00) + (935.80)(380.00 – 269.00) – (1148.75)(424.10 – 269.00) - (1190.75)(330.44 – 269.00] / 55.50 = 551.31 k

Only the first tie has tension forces. Calculate the required tensile reinforcement for this tie:

A_s = 1069.29 / [(1.00)(60)] = 17.82 in^2

Use 9 - #9 bundles at 7 inches (A_s = 18.00 in^2).

Space the bars symmetrically about the center of the tie that is located 45 inches from the anchorage plates or 45.00 – 4.00 = 41.00 inches from the centerline of bearing of the abutment.

If a uniform stress distribution is assumed for the bottom slab, the resulting required area of reinforcing is 23.00 in^2. This increase in required reinforcing due to the assumed uniform stress distribution will increase further for a wider bridge.

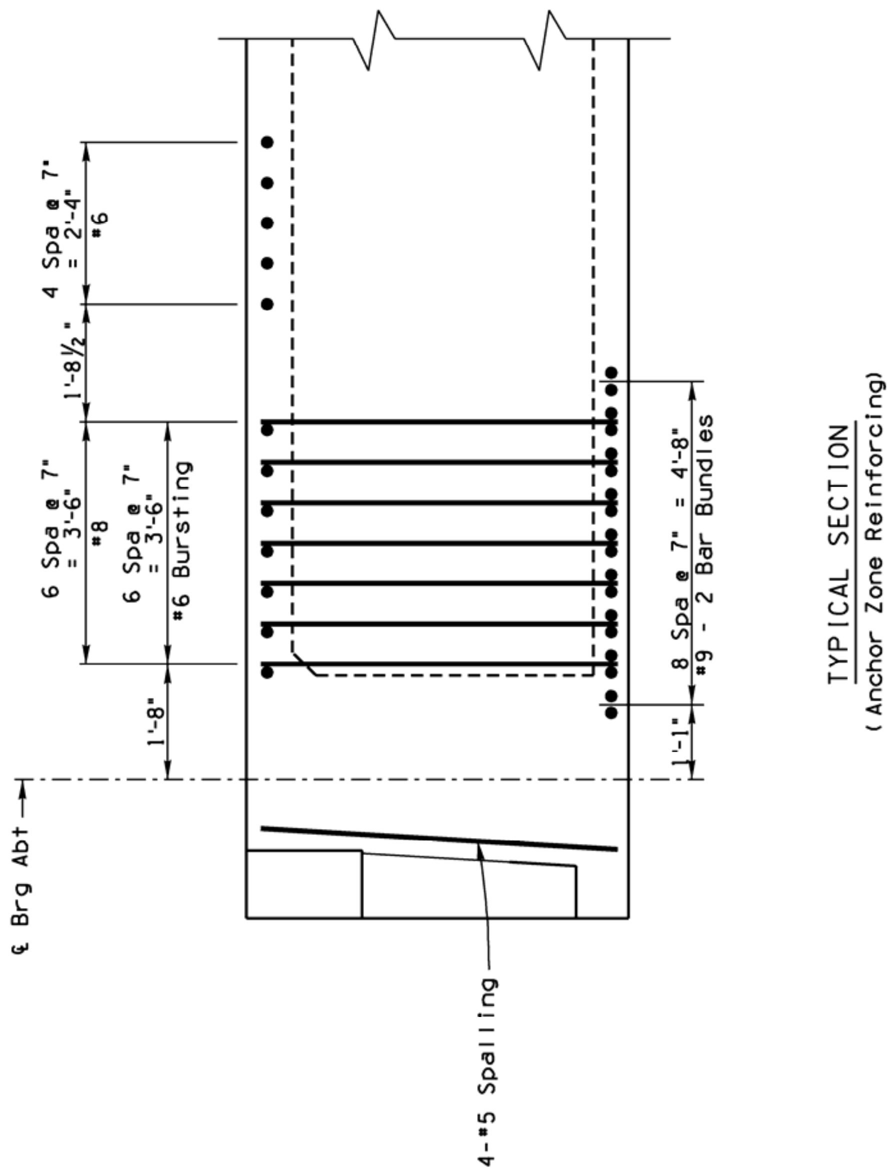

4 Spa @ 7" = 2'-4" #6
1'-8½"
6 Spa @ 7" = 3'-6" #8
6 Spa @ 7" = 3'-6" #6 Bursting
1'-8"
℄ Brg Abt
4-#5 Spalling
8 Spa @ 7" = 4'-8" #9 - 2 Bar Bundles
1'-1"
TYPICAL SECTION
(Anchor Zone Reinforcing)

Figure 32

Elastomeric Bearing Pad [14.7.6]

The bearings must be designed to resist the compressive force while allowing for horizontal movement and rotation. Steel reinforced elastomeric bearings are preferred over fabric reinforced bearings due to their greater strength. Two methods of analysis are allowed: Method A and Method B. Method B requires more analysis and testing but results in larger compressive capacities. Method A is preferred for ordinary applications due to the less stringent test requirements. Method A will be used for this example.

A 14 inch by 28 inch by 2 inch steel reinforced elastomeric bearings will be used. Due to the initial and long term prestress shortening, the elastomeric bearings will be greased. The shortening due to prestress will be assumed to be taken by sliding on the greased surface so the bearings need only be designed for temperature movements.

Step 1 – Determine Loads

DC dead loads

Super	12.681(80.00 + 1.50 / cos(15))	=1034.17 k
Flared Webs	0.150(41.23/12)(16.00)(6.292)(0.5)	= 25.94 k
Interior Diaph	21.72 / 2	= 10.86 k
Abutment Diaph	0.150(184.77)(3.50 / cos(15))	= 100.43 k
Barrier	0.710(80.00 + 1.50 / cos(15))	= 57.90 k
		1229.30 k

DW dead loads

FWS	1.050(80.00 + 1.50 / cos(15))	= 85.63 k

LL

Lane	0.640(80.00+ 1.50 / cos(15))	= 52.19 k
Truck	32 + 32(146)/(160) + 8(132)/(160)	= 67.80 k
Tandem	25 + 25(156)/(160)	= 49.38 k

LL = (52.19 + 67.80)(4.555)(1.149) = 627.99 k

Due to the torsional stiffness of box girder bridges, the large solid diaphragm, and the compressible bearings, the dead load and live load will be equally distributed to all bearings. While the live load can be eccentrically applied, the use of the full width live load distribution factors will account for this.

Service I

DLmin = [1.00(1229.30)] / [5 bearings] = 245.86 k

DLmax = [1.00(1229.30) + 1.00(85.63)] / [5 bearings] = 262.99 k

DL + LL = [1.00(1229.30) + 1.00(85.63) + (1.00)(627.99)] / [5 bearings]
= 388.58 k

Thermal Movements [14.4.2]

Step 2 – Thermal Movement

The minimum thermal movements are computed from extreme temperatures specified in [3.12.2]. Two methods are allowed for determining temperature movement: Procedure A and Procedure B. Procedure A is the historical method and will be used in this example. For concrete in a moderate temperature zone the range of temperature is 10 degree to 80 degree F. The coefficient of thermal expansion for concrete is 0.000006/ °F.

[5.4..2.2]

$$\Delta_s = \alpha L \Delta_T = 0.000006(160)(12)(80\text{-}10) = 0.806 \text{ in}$$

Step 3 – Material Properties

[14.7.6.2]
[14.7.5.2]

The elastomer will have a shear modulus of 0.130 ksi and a durometer hardness of 55. The elastomer will be specified explicitly by its shear modulus so the shear modulus at 73° F will be used as the basis for design. The specified shear modulus of 0.130 is within the 0.080 to 0.175 ksi limits. The corresponding durometer hardness of 55 is within the allowable range of 50 to 60.

Step 4 – Shape Factor

The shape factor of a layer of an elastomeric bearing is taken as the plan area of the layer divided by the area of perimeter free to bulge. For a rectangular bearing the shape factor is as follows:

$$S = \frac{LW}{2h_{ri}(L+W)} = \frac{(14)\cdot(28)}{2\cdot(0.4253)\cdot(14+28)} = 11$$

where:
L = length of the bearing parallel to the longitudinal axis.
W = width of the bearing in the transverse direction.
h_{ri} = thickness of i^{th} elastomeric layer.

In accordance with the ADOT Standard Specifications for Road and Bridge Construction, the steel shims are 14 gage (0.0747 inches). For half-inch interior layers the effective thickness of neoprene is 0.500-0.0747 = 0.4253 inches. The exterior layers will be half that of the interior layers or 0.21625 inches.

Step 5 – Compressive Stress

Elastomeric bearings are designed for strength using service loads with a load factor of 1.00 and a strength reduction factor, ϕ = 1.00.

The required area of the bearing must limit the compressive stress as follows:

$$\sigma_s = \frac{DL + LL}{WL} = \frac{388.58}{(14)\cdot(28)} = 0.991 \text{ ksi}$$

The allowable stress is limited by the following:

σ_s < 1.0 G S =1.0(0.130)(11) = 1.43 ksi
σ_s < 1.0 ksi

Step 6 – Compressive Strain

The compressive strain is used to calculate the deflection of the bearing. The strain has a nonlinear relation to the stress so the charts provided in the specification must be used. For a durometer hardness of 55, values for strain must be interpolated between the chart for 50 durometer and the chart for 60 durometer. A summary of strains follows:

DL min Stress = [245.86] / [(14)(28)] = 0.627 ksi

DL max Stress = [262.99] / [(14)(28)] = 0.671 ksi

DL + LL = [388.58] / [(14)(28)] = 0.991 ksi

[C14.7.5.3.3-1]

	DL min	DL max	DL + LL
50 Durometer	0.028	0.029	0.041
60 Durometer	0.024	0.025	0.037
55 Durometer	0.026	0.027	0.039

Step 7 – Compressive Deflection

[14.7.6.3-3]

The compressive deflection for the various load levels follows:

$$\delta_{DL\min} = \sum \varepsilon_i h_{ri} = 0.026 \cdot (0.4253) \cdot (4) = 0.044 \text{ in}$$

$$\delta_{DL\max} = \sum \varepsilon_i h_{ri} = 0.027 \cdot (0.4253) \cdot (4) = 0.046 \text{ in}$$

$$\delta_{DL+LL} = \sum \varepsilon_i h_{ri} = 0.039 \cdot (0.4253) \cdot (4) = 0.066 \text{ in}$$

[14.7.5.2-1]

The elastomer will creep over time due to dead load. For a durometer hardness of 55, the creep is 30%. The creep deflection follows:

δ_{creep} = (0.046)(0.30) = 0.014 in

[C14.7.5.3.3] Total deflection equals 0.066 + 0.014 = 0.080 inches < 0.125 inch maximum relative deflection across a joint.

Step 8 – Shear Deformation

Shear deformation is limited to one half the maximum service limit state applied deformation. The thickness of the elastomer equals:

h_{rt} = (2 ext layers)(0.21265) + (3 int layers)(0.4253) = 1.701 in

[14.7.6.3.4-1]

h_{rt} = 1.70 > 2 Δ_s = 2(0.806) = 1.61 in, ok

Step 9 – Shear Force

When the elastomeric pad deforms, shear forces develop that must be resisted by the substructure. The shear force is a function of the stiffness of the material, size of the pad and amount of deformation. Since these bearings are greased to take the post-tensioning deformation, a shear force will develop based on the coefficient of friction. The coefficient of friction for PTFE surfaces is about 0.08 while that for the neoprene against concrete is 0.20. For a greased surface assume a coefficient of friction of 0.10.

[14.6.3.1-1] $H_s = \mu P_u$ = (0.10)(262.99) = 26.30 k / pad

[14.6.3.1-2]

$$H_u = GA\frac{\Delta_u}{h_{rt}} = (0.130)\cdot(14)\cdot(28)\cdot\frac{0.806}{1.701} = 24.15 \text{ k / pad}$$

The substructure must be designed to resist the larger of these two forces

[14.4.2.1]

Bearing design for rotation is based on unfactored service rotations calculated using computer software as follows:

DL min = 0.006502 radians
DL max = 0.007005 radians
PS = -0.005665 radians
LL = 0.002008 radians

The general formula for rotation capacity is:

[BDG] 14.4.2.1

$\theta_s = (\theta_{DL} + \theta_{PS} + \theta_{LL} + 0.005)$
$\theta_{s\ DLmin} = (0.006502 - 0.005665 + 0.005) = 0.00584$ radians
$\theta_{s\ DL\ max} = (0.007005 - 0.005665 + 0.005) = 0.00634$ radians
$\theta_{s\ DL+LL} = (0.007005 - 0.005665 + 0.002008 + 0.005) = 0.00838$ radians

Steel reinforced elastomeric bearings are quite flexible in compressive loading and as a consequence very large strains are tolerated. The method of rotation calculation currently in the Specification shall not be used. Instead the method contained in the pre-1997 Standard Specifications shall be used.

Step 10 – Rotation

[BDG] 14.7.6.3.5

Pre-1997 Rotation Criteria

$\theta_s < 2\delta_c / L$

Min DL
$\theta_s = 0.00584 < 2(0.044) / 14 = 0.00629$

Max DL
$\theta_s = 0.00634 < 2(0.046) / 14 = 0.00657$

DL + LL
$\theta_s = 0.00838 < 2(0.066) / 14 = 0.00943$

Step 11 – Stability

[14.7.6.3.6] To ensure stability the total thickness of the pad shall not exceed the following:

T = 2.00 in < L / 3 = 14.00 / 3 = 4.67 in

T = 2.00 in < W / 3 = 28.00 / 3 = 9.33 in

Step 12 – Reinforcement

[14.7.6.3.7] The ADOT Standard Specifications require a minimum 14 gage thickness for the reinforcement. The thickness of the reinforcement in the bearing is limited to the following:

For Service Limit State:

$$h_s = 0.0747 \geq \frac{3h_{max}\sigma_s}{F_y} = \frac{3\cdot(0.4253)\cdot(0.991)}{36.0} = 0.0351\,\text{in}$$

For Fatigue Limit State:

$$h_s = 0.0747 \geq \frac{2.0h_{max}\sigma_L}{\Delta F_{TH}} = \frac{2.0\cdot(0.4253)\cdot(0.320)}{24.0} = 0.0113\ \text{in}$$

where:

[6.6.1.2.5-3] ΔF_{TH} = constant amplitude fatigue threshold = 24.0 for a Category A Detail.

Step 13 - Anchorage

[14.7.6.4] If the factored shear force sustained by the deformed pad at the strength limit state exceeds one-fifth of the minimum vertical force, V_{sd}, due to permanent loads the pad shall be secured against horizontal movement.

V_u = 1.20(26.10) = 31.32k

V_{sd} = (245.86) / 5 = 49.17 k

Since the criteria is met, the pad does not have to be secured against horizontal movement.

Appendix A

Precise Overhang Design

Tension and Flexure [5.7.6.2]

The solution of the deck design problem involves determining the resistance of the deck overhang to a combination of tension and flexure. Members subjected to eccentric tension loading, which induces both tensile and compressive stresses in the cross section, shall be proportioned in accordance with the provisions of Article 5.7.2.

[5.7.2]

Assumptions for a valid analysis for an extreme event limit state are contained in Article 5.7.2. Factored resistance of concrete components shall be based on the conditions of equilibrium and strain compatibility and the following:

Strain is directly proportional to the distance from the neutral axis.

For unconfined concrete, the maximum usable strain at the extreme concrete compressive fiber is not greater than 0.003.

The stress in the reinforcement is based on a stress-strain curve of the steel or on an approved mathematical representation.

Tensile strength of the concrete is ignored.

The concrete compressive stress-strain distribution is assumed to be a rectangular stress block in accordance with Article 5.7.2.2.

The development of the reinforcing is considered.

[1.3.2.1]

While the article specifies the use of the reduction factors in Article 5.5.4.2, that requirement only applies to a strength limit state analysis. For an extreme event limit state, the resistance factor shall be taken as 1.0.

The above assumptions as shown in Figures A-1, A-2 and A-3 were used in the development of the equations for resistance of a deck from tension and flexure that occur with a vehicular collision with a traffic railing.

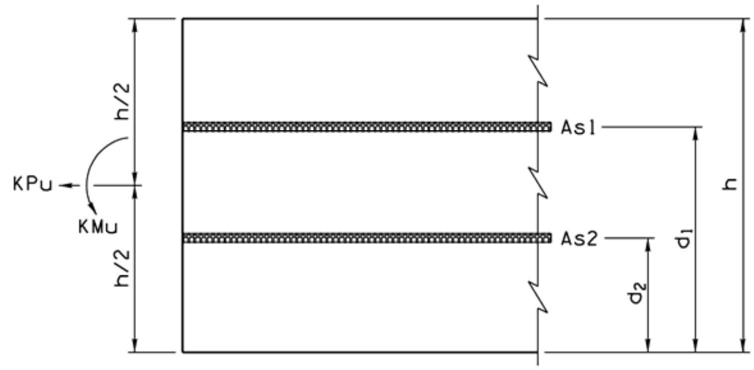


Figure A-1

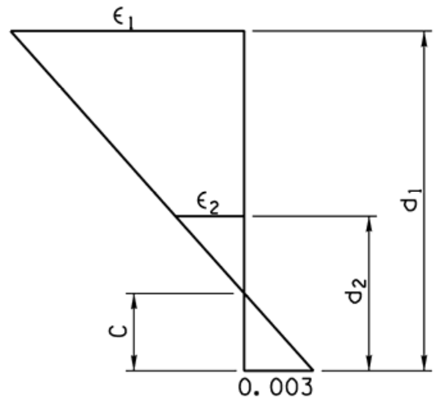


Figure A-2

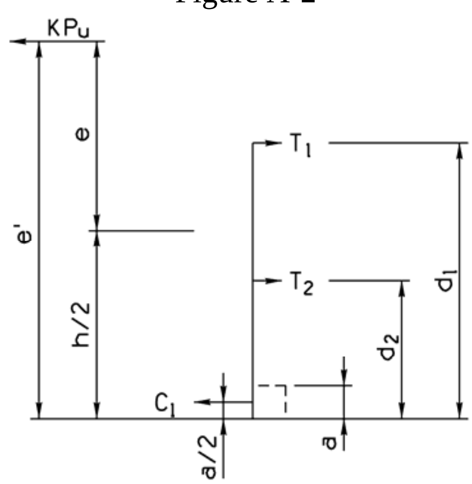


Figure A-3

The design of the deck overhang is complicated because both a bending moment and a tension force are applied. The problem can be solved using equilibrium and strain compatibility. The following trial and error approach may be used:

1. Assume a stress in the reinforcing
2. Determine force in reinforcing
3. Solve for k, the safety factor
4. Determine values for 'a' and 'c'
5. Determine corresponding strain
6. Determine stress in the reinforcing
7. Compare to assumed value and repeat if necessary

Face of Barrier
Location 1
Figure 4

The design horizontal force in the barrier is distributed over the length L_b equal to L_c plus twice the height of the barrier. See Figures 5 and 6.

L_b = 11.86 + 2(2.67) = 17.20 ft
P_u = 54.83 / 17.20 = 3.188 k/ft < 3.261 k/ft per connection strength.

Dimensions

h = 9.50 + (3.00) (1.42) / (3.33) = 10.78 in
d_1 = 10.78 – 2.50 clr – 0.625 / 2 = 7.97 in
d_2 = 10.78 – 8.50 + 1.00 clr + 0.625 / 2 = 3.59 in

Moment at Face of Barrier

Deck = $0.150(9.50 / 12)(1.42)^2 \div 2$ = 0.12 ft-k
$0.150(1.28 / 12)(1.42)^2 \div 6$ = $\underline{0.01 \text{ ft-k}}$
= 0.13 ft-k

Barrier = 0.355(0.817) = 0.29 ft-k

Collision = 3.188[2.67 + (10.78/12) / 2] = 9.94 ft-k

[A13.4.1]
Extreme Event II
[3.4.1]

The load factor for dead load shall be taken as 1.0.

M_u = 1.00(0.13 + 0.29) + 1.00(9.94) = 10.36 ft-k

e = M_u / P_u = (10.36)(12) / (3.188) = 39.00 in

1. Assume Stress

Assume both layers of reinforcing yield and f_s = 60 ksi

2. Determine Forces

Determine resulting forces in the reinforcing:

T_1 = (0.744)(60) = 44.64 k
T_2 = (0.572)(60) = 34.32 k

Strength Equation

Solving the equations of equilibrium by summing the forces on the section and summing the moments about the soffit and setting them equal to zero yields the following two equations. See Figure A-3.

Sum forces in horizontal direction

Eqn 1: $-kP_u + T_1 + T_2 - C_1 = 0$ where $C_1 = 0.85f'_c ab$

Sum of moments

Eqn 2: $kP_u(e') - T_1(d_1) - T_2(d_2) + C_1(a/2) = 0$

Solving the above equations for k, the ratio of strength to applied force and moment, results in a quadratic equation with the following coefficients:

$$A = \frac{P_u^{\ 2}}{1.70 f'_c\, b}$$

$$B = P_u\left(e + \frac{h}{2} - \frac{T_1 + T_2}{0.85 f'_c\, b}\right)$$

$$C = -T_1 d_1 - T_2 d_2 + \frac{(T_1 + T_2)^2}{1.70 f'_c\, b}$$

Solution

Substituting in specific values yields:

$$A = \frac{(3.188)^2}{1.70 \cdot (4.5) \cdot (12)} = 0.110712$$

$$B = (3.188) \cdot \left(39.00 + \frac{10.78}{2} - \frac{(44.64 + 34.32)}{0.85 \cdot (4.5) \cdot (12)}\right) = 136.0311$$

$$C = -(44.64) \cdot (7.97) - (34.32) \cdot (3.59) + \frac{(44.64 + 34.32)^2}{1.70 \cdot (4.5) \cdot (12)} = -411.0737$$

Solution of the quadratic equation yields the value k, the safety factor.

$$k = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

3. Determine k Safety Factor

$$k = \frac{-136.0311 + \sqrt{(136.0311)^2 - 4 \cdot (0.110712) \cdot (-411.0737)}}{2 \cdot (0.110712)} = 3.015$$

Since the value of k is greater than one the deck is adequately reinforced at this location.

4. Determine 'a' and 'c'

Calculate the depth of the compression block from Eqn 1. See Figure A-3.

$$a = \frac{(T_1 + T_2 - kP_u)}{0.85 f'_c\, b}$$

$$a = \frac{(44.64 + 34.32 - (3.015)\cdot(3.188))}{0.85\cdot(4.5)\cdot(12)} = 1.511 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{1.511}{0.825} = 1.83 \text{ in}$$

5. Strains
6. Stresses

Determine the resulting strain in the two layers of reinforcing. See Figure A-2.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

$\varepsilon_1 = 0.003(d_1 / c-1) = 0.003(7.97 / 1.83 - 1) = 0.01007$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

$\varepsilon_2 = 0.003(d_2 / c - 1) = 0.003(3.59 / 1.83 - 1) = 0.00289$
Since $\varepsilon_2 > \varepsilon_y$ the bottom layer yields and $f_{s2} = 60$ ksi

7. Verify Assumption

Since both layers of reinforcing yield the assumptions made in the analysis are valid.

Maximum Strain

The LRFD Specification does not have an upper limit on the amount of strain in a reinforcing bar. ASTM does require that smaller diameter rebar have a minimum elongation at tensile strength of 8 percent. This appears to be a reasonable upper limit for an extreme event state where $\varphi = 1.00$. For this example the strain of 1.0 percent is well below this limit.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section. This step is not necessary for design but is included for educational purposes.

$\varphi P_n = \varphi k P_u = \varphi[T_1 + T_2 - 0.85 f'_c ba]$
$\varphi P_n = 1.0[44.64 + 34.32 - 0.85(4.5)(12.0)(1.511)] = 9.61$ k

Solve for equilibrium from Figure A-3 by substituting M_n for $kP_u e$ and taking moments about the center of the compression block:

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) + T_2\left(d_2 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (44.64)\cdot\left(7.97 - \frac{1.51}{2}\right) + (34.32)\cdot\left(3.59 - \frac{1.51}{2}\right)$$

$$-(3.015)\cdot(3.188)\cdot\left(\frac{10.78}{2} - \frac{1.51}{2}\right) = 374.82 \text{ in-k}$$

φM_n = (1.00)(374.82) / 12 = 31.24 ft-k

The factor of safety for flexure is 31.24 / 10.36 = 3.015 approximately the same as for axial strength of 9.61 / 3.188 = 3.014. Thus this method provides a strength in tension and flexure with the same safety factor.

Simplified Method

A simplified method of analysis is available. If only the top layer of reinforcing is considered in determining strength, the assumption can be made that the reinforcing will yield. By assuming the safety factor for axial tension is 1.0 the strength equation can be solved directly. This method will determine whether the section has adequate strength. However the method does not consider the bottom layer of reinforcing, does not maintain the required constant eccentricity and does not determine the maximum strain.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{44.64 - 3.188}{(0.85)\cdot(4.5)\cdot(12)} = 0.90$ in

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(7.97 - \frac{0.90}{2}\right) - (3.188)\cdot\left(\frac{10.78}{2} - \frac{0.90}{2}\right)\right] \div 12$$

φM_n = 26.66 ft-k

Since φ $M_n > M_u$ the overhang has adequate strength. Note that the resulting eccentricity equals (26.66)(12) ÷ 3.188 = 100.35 inches compared to the actual eccentricity of 39.00 inches that is fixed by the geometry of the deck thickness and barrier height.

Independent analysis using the more complex method but considering only the top layer of reinforcing results in a flexural strength equal to 24.69 ft-k. Thus it would appear that the simplified analysis method produces non-conservative results. However, the simplified method uses a safety factor of 1.0 for axial load leaving more resistance for flexure. As the applied load approaches the ultimate strength the two methods will produce the same result.

Exterior Support Location 2 Figure 4

The deck slab must also be evaluated at the exterior overhang support. At this location the design horizontal force is distributed over a length L_{s1} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the exterior support. See Figures 4, 5 and 6. Using a distribution of 30 degrees from the face of barrier to the exterior support results in the following:

L_{S1} = 11.86 + 2(2.67) + (2)[tan(30)](1.92) = 19.42 ft
P_u = 54.83 / 19.42 = 2.823 k/ft

Dimensions

h = 12.50 in
d_1 = 12.50 – 2.50 clr – 0.625 / 2 = 9.69 in
d_2 = 4.00 + 1.00 clr + 0.625 / 2 = 5.31 in

Moment at Exterior Support

.

DC Loads			
Deck	$= 0.150(9.50 / 12)(3.33)^2 / 2$		= 0.66 ft-k
	$= 0.150(3.00 / 12)(3.33)^2 / 6$		= 0.07 ft-k
Barrier	= 0.355(0.817 + 1.917)		= 0.97 ft-k
		DC	= 1.70 ft-k
DW Loads			
FWS	$= 0.025(1.917)^2 / 2$		= 0.05 ft-k
Collision	= 2.823[2.67 + (12.50 / 12) / 2]		= 9.01 ft-k

[A13.4.1]

The load factor for dead load shall be taken as 1.0.

Extreme Event II [3.4.1]

M_u = 1.00(1.70) + 1.00(0.05) + 1.00(9.01) = 10.76 ft-k

e = M_u / P_u = (10.76)(12) / (2.823) = 45.74 in

1. Assume Stress

Assume both layers of reinforcing yield and f_s = 60 ksi

2. Determine Forces

Determine resulting forces in the reinforcing:

T_1 = (0.744)(60) = 44.64 k
T_2 = (0.572)(60) = 34.32 k

$T_1 + T_2$ = 44.64 + 34.32 = 78.96 k

Solution

Using the previously derived equations for safety factor yields the following:

$$A = \frac{(2.823)^2}{1.70 \cdot (4.5) \cdot (12)} = 0.086812$$

$$B = (2.823) \cdot \left(45.74 + \frac{12.50}{2} - \frac{(78.96)}{0.85 \cdot (4.5) \cdot (12)} \right) = 141.9115$$

$$C = -(44.64) \cdot (9.69) - (34.32) \cdot (5.31) + \frac{(78.96)^2}{1.70 \cdot (4.5) \cdot (12)} = -546.8849$$

Solution of the quadratic equation yields the value k, the safety factor.

3. Determine k Safety Factor

$$k = \frac{-141.9115 + \sqrt{(141.9115)^2 - 4 \cdot (0.086812) \cdot (-546.8849)}}{2 \cdot (0.086812)} = 3.845$$

Since the value of k is greater than one, the deck is adequately reinforced at this location.

4. Determine ‘a’ and ‘c’

Calculate the depth of the compression block.

$$a = \frac{(T_1 + T_2 - kP_u)}{0.85 f'_c b}$$

$$a = \frac{(78.96 - (3.845) \cdot (2.823))}{0.85 \cdot (4.5) \cdot (12)} = 1.484 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{1.484}{0.825} = 1.799 \text{ in}$$

5. Strains
6. Stress

Determine the resulting strain in the two layers of reinforcing. See Figure 8.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

$\varepsilon_1 = 0.003(d_1 / c - 1) = 0.003(9.69 / 1.799 - 1) = 0.01316$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

$\varepsilon_2 = 0.003(d_2 / c - 1) = 0.003(5.31 / 1.799 - 1) = 0.00585$
Since $\varepsilon_2 > \varepsilon_y$ the bottom layer yields and $f_{s2} = 60$ ksi

7. Verify Assumption

Since both layers of reinforcing yield the assumptions made in the analysis are valid.

Maximum Strain

The maximum strain of 1.3 percent is less than the ADOT limit of 8 percent and is therefore satisfactory.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section.

$$\varphi P_n = \varphi k P_u = \varphi[T_1 + T_2 - 0.85f'_c ba]$$
$$\varphi P_n = 1.0[44.64 + 34.32 - 0.85(4.5)(12.0)(1.484)] = 10.84 \text{ k}$$

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) + T_2\left(d_2 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (44.64)\cdot\left(9.69 - \frac{1.484}{2}\right) + (34.32)\cdot\left(5.31 - \frac{1.484}{2}\right)$$

$$-(3.845)\cdot(2.823)\cdot\left(\frac{12.50}{2} - \frac{1.484}{2}\right) = 496.43 \text{ in-k}$$

$$\varphi M_n = (1.00)(496.43) / 12 = 41.37 \text{ ft-k}$$

The factor of safety for flexure is 41.37 / 10.76 = 3.845 approximately the same as for axial strength of 10.84 / 2.823 = 3.840.

Simplified Method

A simplified method of analysis is available based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{44.64 - 2.823}{(0.85)\cdot(4.5)\cdot(12)} = 0.91$ in

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(9.69 - \frac{0.91}{2}\right) - (2.823)\cdot\left(\frac{12.50}{2} - \frac{0.91}{2}\right)\right] \div 12$$

$$\varphi M_n = 32.99 \text{ ft-k}$$

Interior Support Location 3 Figure 4

The deck slab must also be evaluated at the interior point of support. For this thinner slab the bottom reinforcing will be near the neutral axis and will not be effective. Only the top layer will be considered. At this location the design horizontal force is distributed over a length L_{s2} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the interior support. See Figures 4, 5 and 6. Using a distribution of 30 degree from the face of the barrier to the interior support results in the following:

L_{S2} = 11.86 + 2(2.67) + (2)[tan(30)](2.99) = 20.65 ft
P_u = 54.83 / 20.65 = 2.655 k/ft

Dimensions
h = 8.50 in
d_1 = 8.50 – 2.50 clr – 0.625 / 2 = 5.69 in

Moment at Interior Support
For dead loads use the maximum negative moments for the interior cells used in the interior deck analysis
.
DC = 0.72 ft-k
DW = 0.17 ft-k
Collision = 2.655[2.67 + (8.50 / 12) / 2] = 8.03 ft-k

[A13.4.1] Extreme Event II [3.4.1]

The load factor for dead load shall be taken as 1.0.
M_u = 1.00(0.72) + 1.00(0.17) + 1.00(8.03) = 8.92 ft-k
e = M_u / P_u = (8.92)(12) / (2.655) = 40.32 in

1. Assume Stress

Assume the top layer of reinforcing yields and f_s = 60 ksi

2. Determine Force

Determine resulting force in the reinforcing:

T_1 = (0.744)(60) = 44.64 k

Using the previously derived equations for safety factor yields the following:

$$A = \frac{(2.655)^2}{1.70 \cdot (4.5) \cdot (12)} = 0.076787$$

$$B = (2.655) \cdot \left(40.32 + \frac{8.50}{2} - \frac{(44.64)}{0.85 \cdot (4.5) \cdot (12)} \right) = 115.7512$$

$$C = -(44.64) \cdot (5.69) + \frac{(44.64)^2}{1.70 \cdot (4.5) \cdot (12)} = -232.2943$$

3. Determine k Safety Factor

Solution of the quadratic equation yields the value k, the safety factor.

$$k = \frac{-115.7512 + \sqrt{(115.7512)^2 - 4 \cdot (0.076787) \cdot (-232.2943)}}{2 \cdot (0.076787)} = 2.004$$

Since the value of k is greater than one, the deck is adequately reinforced at this location.

4. Determine 'a' and 'c'

Calculate the depth of the compression block.

$$a = \frac{(T_1 - kP_u)}{0.85 f'_c\, b}$$

$$a = \frac{(44.64 - (2.004) \cdot (2.655))}{0.85 \cdot (4.5) \cdot (12)} = 0.857 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{0.857}{0.825} = 1.038 \text{ in}$$

5. Strain

Determine the resulting strain in the top layer of reinforcing. See Figure 8.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

6. Stress

$\varepsilon_1 = 0.003(d_1 / c - 1) = 0.003(5.69 / 1.038 - 1) = 0.01345$

Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

7. Verify Assumption

Since the top layer of reinforcing yields the assumption made in the analysis is valid.

Maximum Strain

The maximum strain of 1.3 percent is less than the ADOT limit of 8 percent and is therefore satisfactory.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section.

$\varphi P_n = \varphi k P_u = \varphi[T_1 – 0.85f'_c ba]$

$\varphi P_n = 1.0[44.64 – 0.85(4.5)(12.0)(0.857)] = 5.30$ k

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (44.64)\cdot\left(5.69 - \frac{0.857}{2}\right) - (2.004)\cdot(2.655)\cdot\left(\frac{8.50}{2} - \frac{0.857}{2}\right)$$

M_n = 214.54 in-k

φM_n = (1.00)(214.54) / 12 = 17.88 ft-k

The factor of safety for flexure is 17.88 / 8.92 = 2.004 approximately the same as for axial strength of 5.30 / 2.655 = 1.996.

Simplified Method

A simplified method of analysis is available based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{44.64 - 2.655}{(0.85)\cdot(4.5)\cdot(12)} = 0.91$ in

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(5.69 - \frac{0.91}{2}\right) - (2.655)\cdot\left(\frac{8.50}{2} - \frac{0.91}{2}\right)\right] \div 12$$

φM_n = 18.63 ft-k