2-Span Cast-in-Place Post-Tensioned Concrete Box Girder Bridge [CIPPTCBGB] Example

This example illustrates the design of a two span cast-in-place post-tensioned concrete box girder bridge. The bridge has spans of 118 feet and 130 feet. The bridge has zero skew. Standard ADOT 42-inch F-shape barriers will be used resulting in a bridge configuration of 1'-7" barrier, 12'-0" outside shoulder, two 12'-0" lanes, a 6'-0" inside shoulder and a 1'-7" barrier. The overall out-to-out width of the bridge is 45'-2". A plan view and typical section of the bridge are shown in Figures 1 and 2.

The following legend is used for the references shown in the left-hand column:

[2.2.2]	LRFD Specification Article Number
[2.2.2-1]	LRFD Specification Table or Equation Number
[C2.2.2]	LRFD Specification Commentary
[A2.2.2]	LRFD Specification Appendix
[BDG]	ADOT LRFD Bridge Design Guideline

Bridge Geometry

Span lengths	118.00, 130.00 ft
Bridge width	45.17 ft
Roadway width	42.00 ft
Superstructure depth	5.50 ft
Web spacing	7.75 ft
Web thickness	12.00 in
Top slab thickness	8.00 in
Bottom slab thickness	6.00 in
Deck overhang	2.63 ft

Minimum Requirements

[Table 2.5.2.6.3-1]

The minimum span to depth ratio for a multi-span bridge should be taken as 0.040 resulting in a minimum depth of (0.040)(130) = 5.20 feet. Use 5'-6"

[9.7.1.1]
[BDG]

The minimum top slab thickness shall be as shown in the LRFD Bridge Design Guidelines. For a centerline spacing of 7.75 feet, the effective length is 6.75 feet resulting in a minimum thickness of 8.00 inches. The minimum overhang thickness is 9.00 inches, one inch thicker than the interior slab.

[5.14.1.5.1b]
[BDG]

The minimum bottom slab thickness shall be the larger of:

1/30 the clear web spacing = (6.75)(12) / 30 = 2.70 inches
6.0 inches

[C5.14.1.5.1c]
[BDG]

The minimum thickness of the web shall be 12 inche

Concrete Deck Slab Minimum Requirements

Slab thickness	8.00 in
Top concrete cover	2.50 in
Bottom concrete cover	1.00 in
Wearing surface	0.50 in

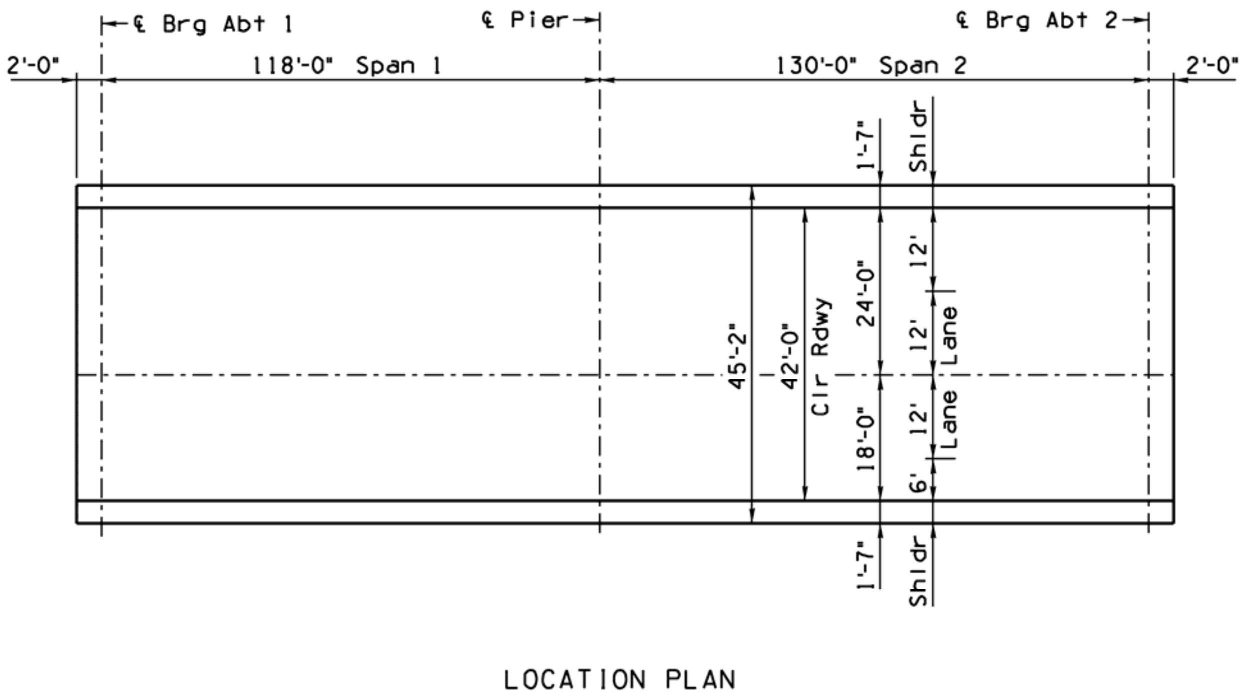


Figure 1

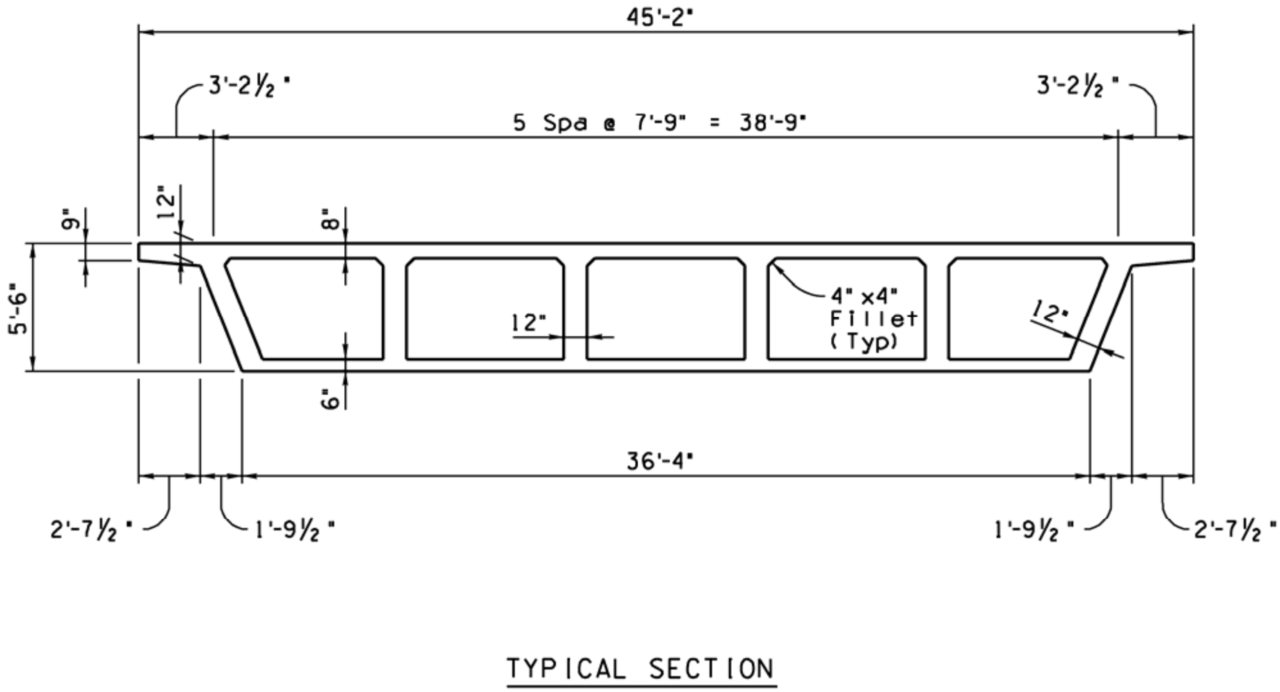

45'-2"
3'-2½"
3'-2½"
5 Spa @ 7'-9" = 38'-9"
9"
12"
8"
5'-6"
12"
4"×4"
Fillet
(Typ)
12"
6"
36'-4"
2'-7½"
1'-9½"
1'-9½"
2'-7½"
TYPICAL SECTION

Figure 2

Material Properties

Reinforcing Steel

[5.4.3.1] Yield Strength f_y = 60 ksi

[5.4.3.2] Modulus of Elasticity E_s = 29,000 ksi

Prestressing Strand

[Table 5.4.4.1-1] Low relaxation prestressing strands

[5.4.4.2]

0.6" diameter strand	A_{ps}	= 0.217 in^2
Tensile Strength	f_{pu}	= 270 ksi
Yield Strength	f_{py}	= 243 ksi
Modulus Elasticity	E_p	= 28500 ksi

Concrete

[5.4.2.1] The final and release concrete strengths are specified below:

[BDG]

Superstructure	Column & Drilled Shaft
f'_c = 4.5 ksi	f'_c = 3.5 ksi
f'_{ci} = 3.5 ksi	

[Table 3.5.1-1] Unit weight for normal weight concrete is listed below:

[C3.5.1]

Unit weight for computing E_c = 0.145 kcf
Unit weight for DL calculation = 0.150 kcf

[C5.4.2.4] The modulus of elasticity for normal weight concrete where the unit weight is 0.145 kcf may be taken as shown below:

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{4.5} = 3861\,ksi$$

$$E_{ci} = 1820\sqrt{f'_{ci}} = 1820\sqrt{3.5} = 3405\,ksi$$

[5.7.1] The modular ratio of reinforcing to concrete should be rounded to the nearest whole number.

$$n = \frac{29000}{3861} = 7.51 \text{ Use n = 8}$$

[5.7.2.2] β_1 = The ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone stress block.

$$\beta_1 = 0.85 - 0.05 \cdot \left[\frac{f'_c - 4.0}{1.0}\right] = 0.85 - 0.05 \cdot \left[\frac{4.5 - 4.0}{1.0}\right] = 0.825$$

Modulus of Rupture

[5.4.2.6]

The modulus of rupture for normal weight concrete has two values. When used to calculate service level cracking, as specified in Article 5.7.3.4 for side reinforcing or in Article 5.7.3.6.2 for determination of deflections, the following equation should be used:

Service Level Cracking

$$f_r = 0.24\sqrt{f'_c}$$

For superstructure calculations:

$$f_r = 0.24\sqrt{4.5} = 0.509\ ksi$$

For substructure calculations:

$$f_r = 0.24\sqrt{3.5} = 0.449\ ksi$$

When the modulus of rupture is used to calculate the cracking moment of a member for determination of the minimum reinforcing requirement as specified in Article 5.7.3.3.2, the following equation should be used:

Minimum Reinforcing

$$f_r = 0.37\sqrt{f'_c}$$

For superstructure calculations:

$$f_r = 0.37\sqrt{4.5} = 0.785\ ksi$$

For substructure calculations:

$$f_r = 0.37\sqrt{3.5} = 0.692\ ksi$$

Limit States
[1.3.2]

In the LRFD Specification, the general equation for design is shown below:

$$\sum \eta_i \gamma_i Q_i \leq \varphi R_n = R_r$$

For loads for which a maximum value of γ_i is appropriate:

$$\eta_i = \eta_D \eta_R \eta_I \geq 0.95$$

For loads for which a minimum value of γ_i is appropriate:

$$\eta_i = \frac{1}{\eta_D \eta_R \eta_I} \leq 1.0$$

[1.3.3]

Ductility
For strength limit state for conventional design and details complying with the LRFD Specifications and for all other limit states:

[3.4.1]
[BDG]

$\eta_D = 1.00$

Redundancy
For the strength limit state for conventional levels of redundancy and for all other limit states:

[1.3.4]

$\eta_R = 1.0$

[1.3.5]

Operational Importance
For the strength limit state for typical bridges and for all other limit states:

$\eta_I = 1.0$

[3.4.1]
[BDG]

For an ordinary structure with conventional design and details and conventional levels of ductility, redundancy, and operational importance, it can be seen that $\eta_i = 1.0$ for all cases. Since multiplying by 1.0 will not change any answers, the load modifier η_i has not been included in this example.

For actual designs, the importance factor may be a value other than one. The importance factor should be selected in accordance with the ADOT LRFD Bridge Design Guidelines.

DECK DESIGN

[BDG]

As bridges age, decks are one of the first element to show signs of wear and tear. As such ADOT has modified some LRFD deck design criteria to reflect past performance of decks in Arizona. Section 9 of the Bridge Design Guidelines provides a thorough background and guidance on deck design.

Effective Length
[9.7.2.3]

[BDG]

ADOT Bridge Design Guidelines specify that deck design be based on the effective length rather than the centerline-to-centerline distance specified in the LRFD Specifications. The effective length for monolithic cast-in-place concrete is the clear distance between supports. For this example with a centerline-to-centerline web spacing of 7.75 feet and web width of 12 inches, the effective length is 6.75 feet. The resulting minimum deck slab thickness per ADOT guidelines is 8.00 inches.

Method of Analysis
[9.6.1]
[BDG]

In-depth rigorous analysis for deck design is not warranted for ordinary bridges. The empirical design method specified in AASHTO LRFD [9.7.2] is not allowed by ADOT Bridge Group. Therefore the approximate elastic methods specified in [4.6.2.1] will be used. Dead load analysis will be based on a strip method using the simplified moment equation of [$w S^2 / 10$], for both positive and negative moments, where "S" is the effective length.

Live Loads
[A4.1]

The unfactored live loads found in Appendix A4.1 will be used. Multiple presence and dynamic load allowance are included in the chart. Since ADOT bases deck design on the effective length, the chart should be entered under S equal to the effective length of 6.75 feet rather than the centerline-to-centerline distance of 7.75 feet. Since the effective length is used the correction for negative moment from centerline of the web to the design section should be zero. Entering the chart yields the following live load moments:

Pos M = 5.10 ft-k/ft
Neg M = -5.50 ft-k/ft (0 inches from centerline)

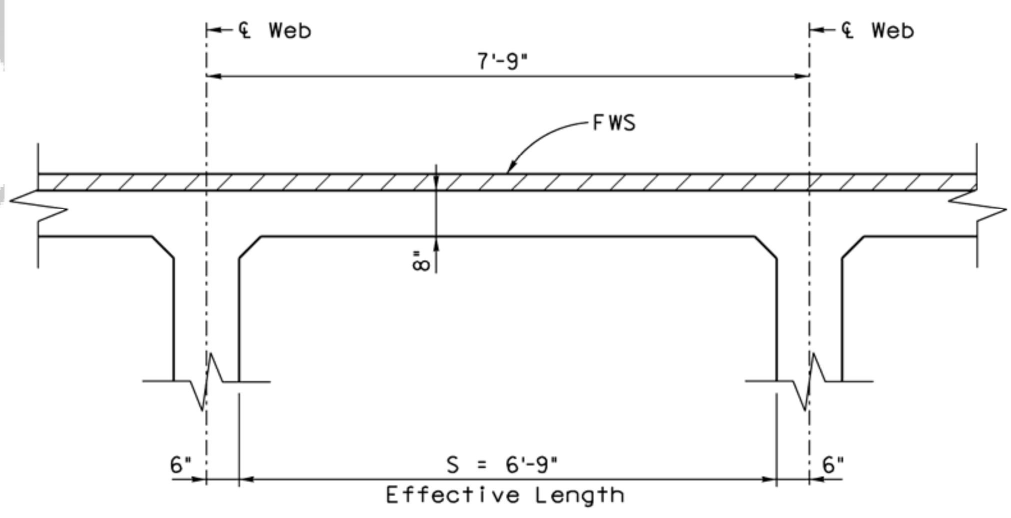


Figure 3

Positive Moment Design

A summary of positive moments follows:

DC Loads
Deck $0.150(8.00 / 12)(6.75)^2 \div 10$ = 0.46 ft-k

DW Loads
FWS $0.025(6.75)^2 \div 10$ = 0.11 ft-k

Vehicle
LL + IM = 5.10 ft-k

Service I Limit State [9.5.2] [BDG]

Deck design is normally controlled by the service limit state. The working stress in the deck is calculated by the standard methods used in the past. For this check Service I moments should be used.

$$M_s = 1.0 \cdot (M_{DC} + M_{DW}) + 1.0 \cdot (M_{LL+IM})$$

M_s = 1.0(0.46 + 0.11) + 1.0(5.10) = 5.67 ft-k

Try #5 reinforcing bars

d_s = 8.00 – 0.50 ws - 1 clr – 0.625 / 2 = 6.19 in

Determine approximate area reinforcing as follows:

$$A_s \approx \frac{M_s}{f_s j d_s} = \frac{(5.67)\cdot(12)}{(24.0)\cdot(0.9)\cdot(6.19)} = 0.509 \text{ in}^2$$

Try #5 @ 7 inches

A_s = (0.31)(12 / 7) = 0.531 in^2

Allowable Stress [9.5.2] [BDG]

The allowable stress for a deck under service loads is not limited by the LRFD Specifications. The 2006 Interim Revisions replaced the direct stress check with a maximum spacing requirement to control cracking. However, the maximum allowable stress in a deck is limited to 24 ksi per the LRFD Bridge Design Guidelines.

Determine stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.531}{(12)\cdot(6.19)} = 0.007149$$

np = 8(0.007149) = 0.05719

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.05719) + (0.05719)^2} - 0.05719 = 0.286$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.286}{3} = 0.905$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(5.67)\cdot(12)}{(0.531)\cdot(0.905)\cdot(6.19)} = 22.87 \text{ ksi} < 24 \text{ ksi}$$

Since the applied stress is less than 24 ksi, the LRFD Bridge Practice Guideline service limit state requirement is satisfied.

Control of Cracking [5.7.3.4]

For all concrete components in which the tension in the cross-section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in equation 5.7.3.4-1 shall be satisfied.

$f_{sa} = 0.80f_r = 0.80(0.24\sqrt{f'_c}) = 0.80(0.509) = 0.407$ ksi

$S_{cr} = (12.00)(7.50)^2 \div 6 = 112.5 \text{ in}^3$

$$f_s = \frac{M_s}{S_{cr}} = \frac{(5.67)\cdot(12)}{112.5} = 0.605\ ksi > f_{sa} = 0.407 \text{ ksi}$$

Since the service limit state stress exceeds the allowable, the spacing, s, of mild steel reinforcing in the layer closest to the tension force shall satisfy the following:

[5.7.3.4-1]

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

where

$\gamma_e = 0.75$ for Class 2 exposure condition for decks

$d_c = 1.0$ clear $+ 0.625 \div 2 = 1.31$ inches

f_s = 22.87 ksi

h_{net} = 7.50 inches

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{1.31}{0.7 \cdot (7.50 - 1.31)} = 1.30$$

$$s \le \frac{(700)\cdot(0.75)}{(1.30)\cdot(22.87)} - (2)\cdot(1.31) = 15.04 \text{ in}$$

Since the spacing of 7 inches is less than 15.04 the cracking criteria is satisfied.

Strength I Limit State [Table 3.4.1-1]

Factored moment for Strength I is as follows:

$$M_u = \gamma_{DC}(M_{DC}) + \gamma_{DW}(M_{DW}) + 1.75(M_{LL+IM})$$

$$M_u = 1.25 \cdot (0.46) + 1.50 \cdot (0.11) + 1.75 \cdot (5.10) = 9.67 \text{ ft-k}$$

Flexural Resistance [5.7.3] [5.7.3.2.2-1]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \phi M_n = \phi A_s f_y \left(d_s - \frac{a}{2} \right)$$

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.531)\cdot(60)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(12)} = 0.841 \text{ in}$$

[5.7.3.2.3]

$a = \beta_1 c = (0.825)(0.841) = 0.69$ in

The tensile strain must be calculated as follows:

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{6.19}{0.841} - 1 \right) = 0.019$$

[5.5.4.2.1]

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 0.90$.

$$M_r = (0.90)\cdot(0.531)\cdot(60)\cdot\left(6.19 - \frac{0.69}{2} \right) \div 12 = 13.97 \text{ ft-k}$$

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the strength limit state is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated this limit. Below a net tensile strain in the extreme tension steel of 0.005, the factored resistance is reduced as the tension reinforcement quantity increases. This reduction compensates for the decreasing ductility with increasing overstrength.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u. A conservative simplification for positive moments is to ignore the 0.5 inch wearing surface for this calculation. If this check is satisfied there are no further calculations required. If the criteria is not satisfied one check should be made with the wearing surface subtracted and one with the full section to determine which of the two is more critical.

S_c = (12.0)(8.00)^2 / 6 = 128 in^3

$$f_r = 0.37\sqrt{f'_c} = 0.785\ ksi$$

$$1.2M_{cr} = 1.2 f_r S_c = (1.2)\cdot(0.785)\cdot(128)\div 12 = 10.05 \text{ ft-k}$$

$$1.2M_{cr} = 10.05 \leq M_r = 13.97 \text{ ft-k}$$

$\therefore$ The minimum reinforcement limit is satisfied.

Fatigue Limit State [9.5.3] & [5.5.3.1]

Fatigue need not be investigated for concrete deck slabs in multi-girder applications.

The interior deck is adequately reinforced for positive moment using #5 @ 7 inches.

Distribution Reinforcement [9.7.3.2]

Reinforcement shall be placed in the secondary direction in the bottom of slabs as a percentage of the primary reinforcement for positive moments as follows:

$$\frac{220}{\sqrt{S}} < 67 \text{ percent maximum}$$

$$\frac{220}{\sqrt{6.75}} = 85 \; percent$$

Use 67% Maximum.

$A_s = 0.67(0.531) = 0.356 \text{ in}^2$

Use #5 @ 10” $\Rightarrow$ $A_s = 0.372 \text{ in}^2$

The LRFD Specification does not allow for a reduction of this reinforcing in the outer quarter of the span as was allowed in the Standard Specifications.

Skewed Decks [9.7.1.3] [BPG]

For bridges with skews less than 20 degrees, the ADOT LRFD Bridge Design Guidelines specifies that the primary reinforcing shall be placed parallel to the skew. For the zero degree skew in this example, the transverse deck reinforcing is placed normal to the webs.

Negative Moment Design

A summary of negative moments follows:

DC Loads

Deck 0.150(8.00 / 12)$(6.75)^2 \div 10$ = -0.46 ft-k

DW Loads

FWS $0.025(6.75)^2 \div 10$ = -0.11 ft-k

Vehicle

LL + IM = -5.50 ft-k

Service I Limit State [9.5.2] [BDG] [Table 3.4.1-1]

Deck design is normally controlled by the service limit state. The working stress in the deck is calculated by the standard methods used in the past. For this check Service I moments should be used.

$$M_s = 1.0(M_{DC} + M_{DW}) + 1.0(M_{LL+IM})$$

M_s = 1.0(0.46 + 0.11) + 1.0(5.50) = 6.07 ft-k

Try #5 reinforcing bars

d_s = 8.00 – 2.50 clear – 0.625 / 2 = 5.19 inches

Determine approximate area reinforcing as follows:

$$A_s \approx \frac{M_s}{f_s j d_s} = \frac{(6.07)\cdot(12)}{(24.0)\cdot(0.9)\cdot(5.19)} = 0.650 \text{ in}^2$$

Try #5 @ 5 ½ inches

A_s = (0.31)(12 / 5.50) = 0.676 in^2

Allowable Stress

Determine stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.676}{(12)\cdot(5.19)} = 0.01085$$

np = 8(0.01085) = 0.08680

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.08680) + (0.08680)^2} - 0.08680 = 0.339$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.339}{3} = 0.887$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(6.07)\cdot(12)}{(0.676)\cdot(0.887)\cdot(5.19)} = 23.41\,ksi \le 24.0\,\text{ksi}$$

[9.5.2]
[BDG]

Since the applied stress is less than the allowable specified in the LRFD Bridge Design Guidelines, the service limit state stress requirement is satisfied.

Control of Cracking
[5.7.3.4]

The deck must be checked for control of cracking. For all concrete components in which the tension in the cross section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in Equation 5.7.3.4-1 shall be satisfied.

$f_{sa} = 0.80 f_r = 0.80(0.24\sqrt{f'_c}) = 0.80(0.509) = 0.407$ ksi

$S_{cr} = (12.00)(7.50)^2 \div 6 = 112.5\ \text{in}^3$

$$f_s = \frac{M_s}{S_{cr}} = \frac{(6.07)\cdot(12)}{112.5} = 0.647\ ksi > f_{sa} = 0.407\ \text{ksi}$$

Since the service limit state stress exceeds the allowable, the spacing, s, of mild steel reinforcing in the layer closest to the tension force shall satisfy the following:

[5.7.3.4-1]

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

$\gamma_e = 0.75$ for Class 2 exposure condition for decks

$d_c = 2.50$ clear $+ 0.625 \div 2 = 2.81$ inches

$f_s = 23.41$ ksi

h = 8.00 inches

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{2.81}{0.7\cdot(8.00 - 2.81)} = 1.77$$

$$s \le \frac{(700)\cdot(0.75)}{(1.77)\cdot(23.41)} - (2)\cdot(2.81) = 7.05\,\text{in}$$

Since the spacing of 5.50 inches is less than 7.05 the cracking criteria is satisfied.

Strength I Limit State [3.4.1]

Factored moment for Strength I is as follows:

$$M_u = \gamma_{DC}(M_{DC}) + \gamma_{DW}(M_{DW}) + 1.75(M_{LL+IM})$$

$$M_u = 1.25 \cdot (0.46) + 1.50 \cdot (0.11) + 1.75 \cdot (5.50) = 10.37 \text{ ft-k}$$

Flexural Resistance [5.7.3]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right)$$

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.676) \cdot (60)}{(0.85) \cdot (4.5) \cdot (0.825) \cdot (12)} = 1.071 \text{ in}$$

[5.7.3.2.3]

a = β_1c = (0.825)(1.071) = 0.88 inches

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{5.19}{1.071} - 1 \right) = 0.012$$

[5.5.4.2.1]

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 0.90$.

$$M_r = (0.90) \cdot (0.676) \cdot (60) \cdot \left(5.19 - \frac{0.88}{2} \right) \div 12 = 14.45 \text{ ft-k}$$

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the strength limit state is satisfied.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u. The most critical cracking load for negative moment will be caused by ignoring the 0.5 inch wearing surface and considering the full depth of the section.

S_c = 12.0(8.0)^2 ÷ 6 = 128 in^3

$$1.2M_{cr} = 1.2 f_r S_c = (1.2) \cdot (0.785) \cdot (128) \div 12 = 10.05 \text{ ft-k}$$

$$1.2M_{cr} = 10.05 \le M_r = 14.45 \text{ ft-k}$$

$\therefore$ The minimum reinforcement limit is satisfied.

Fatigue Limit State [9.5.3] & [5.5.3.1]

Fatigue need not be investigated for concrete deck slabs in multi-girder applications.

The interior deck is adequately reinforced for negative moment using #5 @ 5 ½ inches.

Shear [C4.6.2.1.6]

Past practice has been not to check shear in typical decks. For a standard concrete deck shear need not be investigated.

OVERHANG DESIGN
[Appendix A13]
[Article A13.4.1]
Design Case 1

The overhang shall be designed for the three design cases described below:

Design Case 1: Transverse forces specified in [Table A13.2-1]
Extreme Event Limit Combination II Limit State

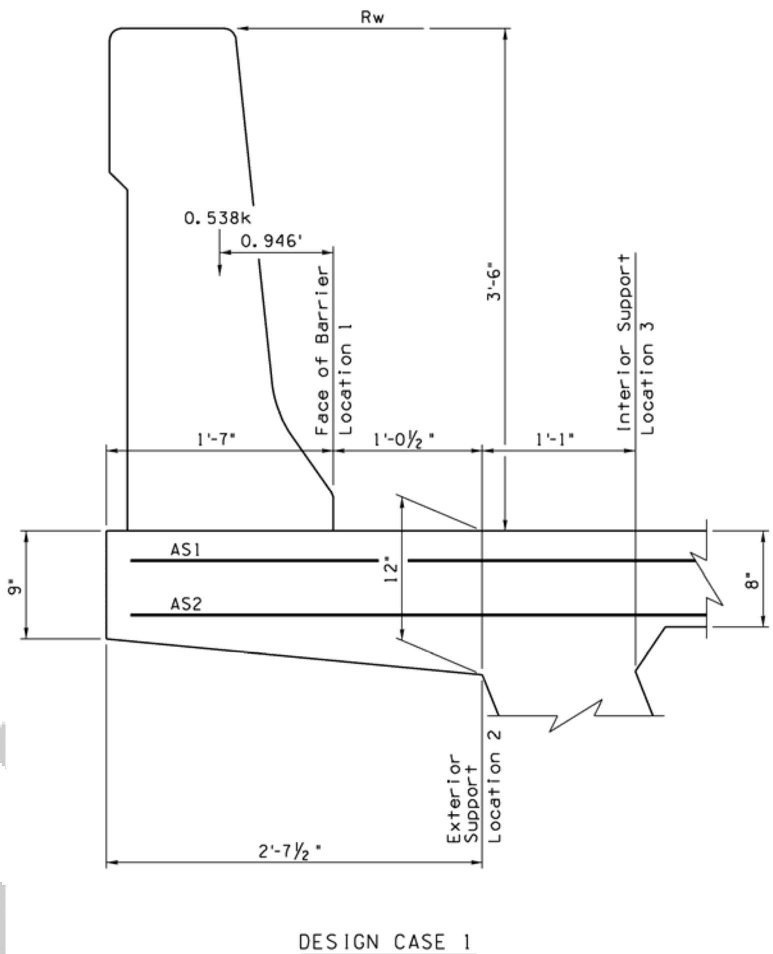


Figure 4

The deck overhang must be designed to resist the forces from a railing collision using the forces given in Section 13, Appendix A. TL-5 rail will be used with a 42 inch height. A summary of the design forces is shown below:

[

Design Forces		Units
F_t, Transverse	124.0	Kips
F_l, Longitudinal	41.0	Kips
F_v, Vertical Down	80.0	Kips
L_t and L_l	8.0	Feet
L_v	40.0	Feet
H_e Minimum	42.0	Inch

The philosophy behind the overhang analysis is that the deck should be stronger than the barrier. This ensures that any damage will be done to the barrier which is easier to repair and that the assumptions made in the barrier analysis are valid. The forces in the barrier must be known to analyze the deck.

R_w = total transverse resistance of the railing.
L_c = critical length of yield line failure. See Figures 5 and 6.

For impacts within a wall segment:

[A13.3.1-1]

$$R_w = \left(\frac{2}{2L_c - L_t}\right)\left(8M_b + 8M_w + \frac{M_c L_c^{\ 2}}{H}\right)$$

[A13.3.1-2]

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_w)}{M_c}}$$

[Section 9]
[BDG]

Required design values for the ADOT 42-inch F-shape barrier shown in SD 1.02 are published in the Bridge Design Guidelines and are repeated below:

M_b = 0.00 ft-k
M_c = 15.16 ft-k
M_w = 56.42 ft-k
R_w = 129.6 k

$$\mathrm{L_c} = \frac{8.00}{2} + \sqrt{\left(\frac{8.00}{2}\right)^2 + \frac{8\cdot(3.50)\cdot(0+56.42)}{15.16}} = 14.96\ \mathrm{ft}$$

Since the railing resistance to transverse load, R_w = 129.60 kips, is greater than the applied load, F_t = 124.00 kips, the railing is adequately designed for the test level specified.

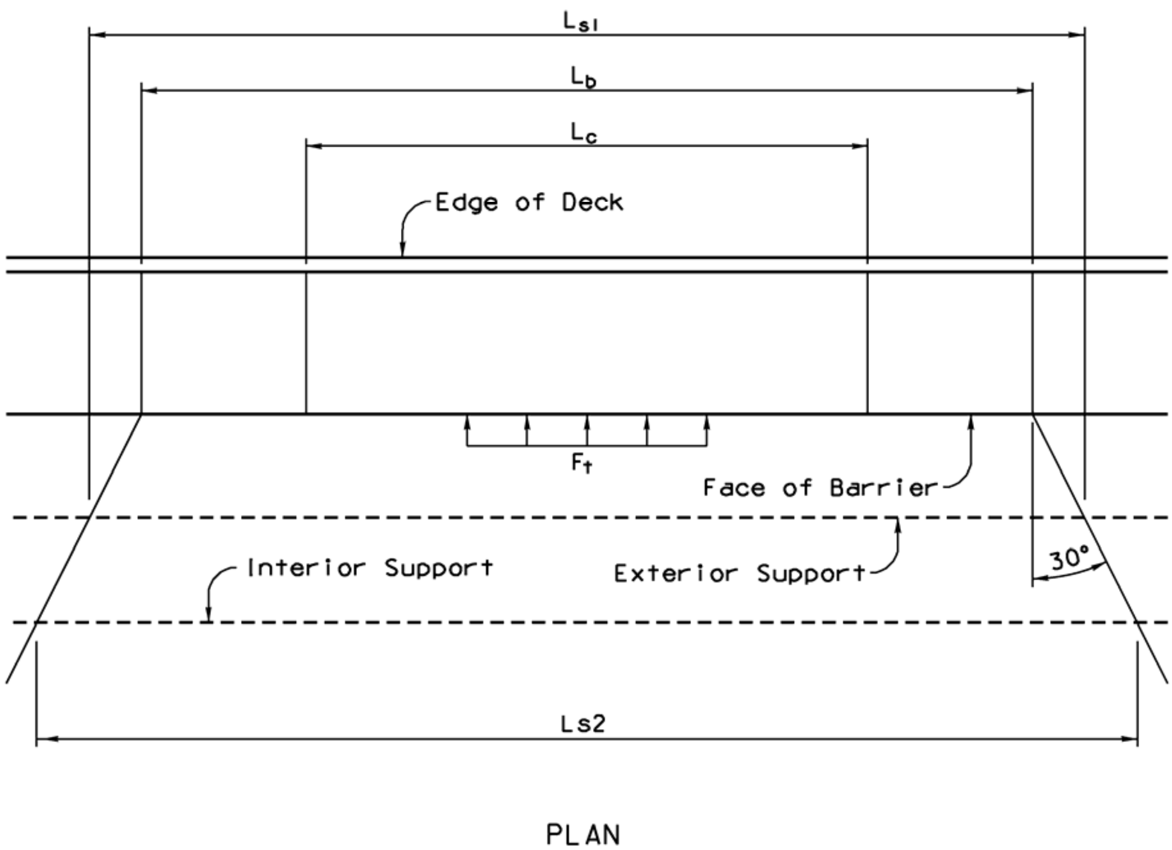


Figure 5

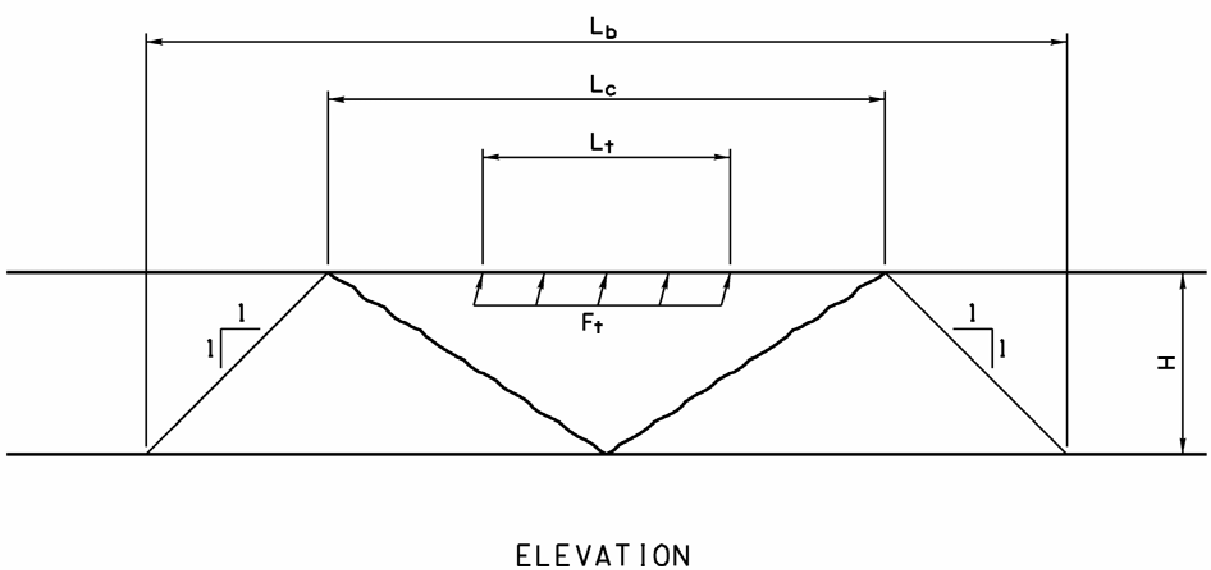


Figure 6

Barrier Connection To Deck

The strength of the attachment of the barrier to the deck must also be checked. The deck will only see the lesser of the strength of the barrier or the strength of the connection. For the 42 inch barrier, #5 reinforcing at 12 inches connects the barrier to the deck.

$A_s = 0.310 \text{ in}^2$

$d_s = 17.25 – 1 \frac{1}{2} \text{ clear} – 0.625 / 2 = 15.44 \text{ inches}$

Flexure

For a reinforcing bar not parallel to the compression face only the parallel component is considered. The #5 reinforcing is oriented at an angle of 26 degrees.

$$c = \frac{A_s f_y \cos\theta}{0.85 f'_c \beta_1 b} = \frac{(0.31)(60)\text{Cos}(26)}{0.85(4)(0.85)(12)} = 0.482 \text{ in}$$

$a = \beta_1 c = (0.85)(0.482) = 0.41 \text{ inches}$

$$M_n = A_s f_y \cos(\theta)\left(d_s - \frac{a}{2}\right) \text{ ft-k}$$

$$M_n = (0.310)(60)\cos(26)\left(15.44 - \frac{0.41}{2}\right)/12 = 21.22 \text{ ft-k}$$

$\varphi M_n = (1.00)(21.22) = 21.22$ ft-k

$\varphi P_u = (21.22)(12) \div (42) = \underline{6.063 \text{ k/ft}}$

Shear

The barrier to deck interface must also resist the factored collision load. The normal method of determining the strength is to use a shear friction analysis. However, in this case with the sloping reinforcing, the horizontal component of reinforcing force will also directly resist the horizontal force.

$R_n = A_s f_y \sin\theta = (0.310)(60) \sin(26) = 8.15$ k/ft

The strength of the connection is limited by the lesser of the shear or flexural strength. In this case, the resistance of the connection is controlled by flexure with a value equal to 6.063 k/ft.

The design of the deck overhang is complicated because both a bending moment and a tension force are applied. The problem can be solved using equilibrium and strain compatibility as described in Appendix A. In lieu of that more complex method a simpler method will be demonstrated here.

Face of Barrier
Location 1
Figure 4

The design horizontal force in the barrier is distributed over the length L_b equal to L_c plus twice the height of the barrier. See Figures 5 and 6.

L_b = 14.96 + 2(3.50) = 21.96 ft
P_u = 129.60 / 21.96 = 5.90k/ft < 6.06 k/ft per connection strength

Dimensions

h = 9.00 + (3.00) (1.583) / (2.625) = 10.81 in = 0.901 ft
d_1 = 10.81 – 2.50 clr – 0.625 / 2 = 8.00 in

Moment at Face of Barrier

Deck = $0.150(9.00 / 12)(1.58)^2 \div 2$ = 0.14 ft-k
$0.150(1.81 / 12)(1.58)^2 \div 6$ = <u>0.01 ft-k</u>
= 0.15 ft-k

[3.5.1]
[BDG]

Barrier = 0.538(0.946) = 0.51 ft-k

Collision = 5.90 [3.50 + (0.901) / 2] = 23.31 ft-k

[Appendix A 13]
[A13.4.1]

The load factor for dead load shall be taken as 1.0.

Extreme Event II
[Table3.4.1-1]

M_u = 1.00(0.15 + 0.51) + 1.00(23.31) = 23.97 ft-k

e = M_u / P_u = (23.97)(12) / (5.90) = 48.75 in

Assume the top layer of reinforcing yields and f_s = 60 ksi

Determine resulting force in the reinforcing (#5 @ 5 ½”):

T_1 = (0.676)(60) = 40.56 k

Simplified Method

The simplified method of analysis will be used. If only the top layer of reinforcing is considered in determining strength, the assumption can be made that the reinforcing will yield. By assuming the safety factor for axial tension is 1.0 the strength equation can be solved directly. This method will determine whether the section has adequate strength. However, the method does not consider the bottom layer of reinforcing, does not maintain the required constant eccentricity and does not determine the maximum strain. For an in-depth review of the development of this equation refer to Appendix A of these guidelines.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \frac{T_1 - P_u}{0.85 f'_c\, b} = \frac{40.56 - 5.90}{(0.85)\cdot(4.5)\cdot(12)} = 0.76$ in

$$\varphi M_n = (1.00)\cdot\left[(40.56)\cdot\left(8.00 - \frac{0.76}{2}\right) - (5.90)\cdot\left(\frac{10.81}{2} - \frac{0.76}{2}\right)\right] \div 12$$

φM_n = 23.28 ft-k

Since φM_n =23.28 < M_u =23.97 ft-k, the overhang does not have adequate strength. Note that the resulting eccentricity equals (23.28)(12) ÷ 5.90 = 47.35 inches compared to the actual eccentricity of 48.81 inches that is fixed by the constant deck thickness, barrier height and dead load moment. Reinforcing in deck should be increased since $\varphi M_n < M_u$.

Development Length [5.11.2]

The reinforcing must be properly developed from the barrier face towards the edge of deck. The available embedment length equals 19 inches minus 2 inches clear or 17 inches. For the #5 transverse reinforcing in the deck the required development length is as follows:

[5.11.2.1.1]

For No. 11 bar and smaller: $\dfrac{1.25 A_b f_y}{\sqrt{f'_c}} = \dfrac{(1.25)\cdot(0.31)\cdot(60)}{\sqrt{4.5}} = 10.96 \text{ in}$

But not less than $0.4\ d_b\ f_y = (0.4)(0.625)(60) = 15.00$ in

Since the available length is greater than the required length, the reinforcing is adequately developed using straight bars.

Exterior Support Location 2
Figure 4

The deck slab must also be evaluated at the exterior overhang support. At this location the design horizontal force is distributed over a length L_{s1} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the exterior support. See Figures 4, 5 and 6. Using a distribution of 30 degrees from the face of barrier to the exterior support results in the following:

L_{S1} = 14.96 + 2(3.50) + (2)tan(30)(1.04) = 23.16 ft
P_u = 129.60 / 23.16 = 5.60 k/ft

Dimensions

h = 12.00 in
d_1 = 12.00 – 2.50 clr – 0.625 / 2 = 9.19 in

Moment at Exterior Support

.

DC Loads

Deck	= 0.150(9.00 / 12)(2.63)^2 / 2	= 0.39 ft-k
	= 0.150(3.00 / 12)(2.63)^2 / 6	= 0.04 ft-k
Barrier	= 0.538(0.946 + 1.042)	= 1.07 ft-k
	DC	= 1.50 ft-k

DW Loads

FWS	= 0.025(1.04)^2 / 2	= 0.01 ft-k

Collision = 5.60[3.50 + (12.00 / 12) / 2] = 22.40 ft-k

The load factor for dead load shall be taken as 1.0.

[A13.4.1]

M_u = 1.00(1.50) + 1.00(0.01) + 1.00(22.40) = 23.91 ft-k

Extreme Event II
[3.4.1]

Solve for the strength of the deck section:

T_1 = (0.676)(60) = 40.56 k

$$a = \frac{T_1 - P_u}{0.85 f'_c\, b} = \frac{40.56 - 5.60}{(0.85)\cdot(4.5)\cdot(12)} = 0.76 \text{ in}$$

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

$$\varphi M_n = (1.00)\cdot\left[(40.56)\cdot\left(9.19 - \frac{0.76}{2}\right) - (5.60)\cdot\left(\frac{12.00}{2} - \frac{0.76}{2}\right)\right] \div 12$$

φM_n = 27.16 ft-k > M_u =23.91 ft-k $\therefore$ Section has adequate strength.

Design Case 1

Interior Support Location 3 Figure 4

The deck slab must also be evaluated at the interior point of support. At this location the design horizontal force is distributed over a length L_{s2} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the interior support. See Figures 4, 5 and 6. Using a distribution of 30 degree from the face of the barrier to the interior support results in the following:

L_{S2} = 14.96 + 2(3.50) + (2)tan(30)(2.13) = 24.42 ft
P_u = 129.60 / 24.42 = 5.31 k/ft

Dimensions
h = 8.00 in
d_1 = 8.00 – 2.50 clr – 0.625 / 2 = 5.19 in

Moment at Interior Support
For dead loads use the maximum negative moments for the interior cells used in the interior deck analysis
.

DC = 0.46 ft-k
DW = 0.11 ft-k
Collision = 5.31[3.50 + (8.00 / 12) / 2] = 20.36 ft-k

[A13.4.1]

The load factor for dead load shall be taken as 1.0.

Extreme Event II [3.4.1]

M_u = 1.00(0.46) + 1.00(0.11) + 1.00(20.36) = 20.93 ft-k

Solve for the strength of the deck section:

T_1 = (0.676)(60) = 40.56 k

$$a = \frac{T_1 - P_u}{0.85 f'_c\, b} = \frac{40.56 - 5.31}{(0.85)\cdot(4.5)\cdot(12)} = 0.77 \text{ in}$$

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

$$\varphi M_n = (1.00)\cdot\left[(40.56)\cdot\left(5.19 - \frac{0.77}{2}\right) - (5.31)\cdot\left(\frac{8.00}{2} - \frac{0.77}{2}\right)\right] \div 12$$

φM_n = 14.64 ft-k < M_u = 20.93 ft-k $\therefore$ Section has inadequate strength.

Deck reinforcement should be revised

[3.6.1]

The deck is not adequately reinforced for Design Case I.

[A13.4.1]
Extreme Event II
[3.4.1]

Design Case 2
[A13.4.1]

Design Case 2: Vertical forces specified in [A13.2]
Extreme Event Limit State

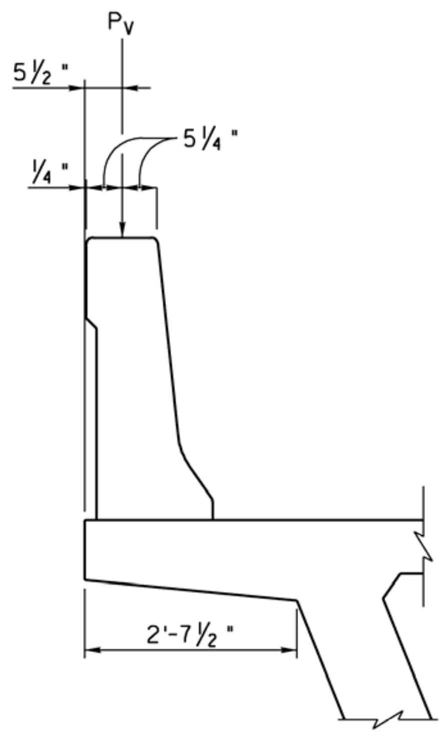


Figure 7

[A13.2-1]

This case represents a crashed vehicle on top of the barrier and is treated as an extreme event. The load is a downward vertical force of l8.0 kips distributed over a length of 18.0 feet. The vehicle is assumed to be resting on top of the center of the barrier. See Figure 7.

At the face of exterior support:

DC Dead Loads = 1.50 ft-k

DW Dead Load = 0.01 ft-k

Vehicle Collision = [80.0/40.0] [2.625 - (5.50/12)] = 4.34 ft-k

Minimum Reinforcing [5.7.3.3.2]

[5.4.2.6]

The load factor for dead load shall be taken as 1.0.

M_u = 1.00(1.50) + 1.00(0.01) + 1.00(4.34) = 5.85 ft-k

Flexural Resistance [5.7.3.2]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2} \right)$$

Try #5 reinforcing bars

d_s = 12.00 – 2.50 clr – 0.625 / 2 = 9.19 inches

Use #5 @ 5 ½", the same reinforcing required for the interior span and overhang Design Case 1.

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.676)\cdot(60)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(12)} = 1.071 \text{in}$$

[5.7.3.2.3]

a = β_1c = (0.825)(1.071) = 0.88 inches

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{9.19}{1.071} - 1 \right) = 0.023$$

[C 5.5.4.2.1]

Since ε_T > 0.005 the member is tension controlled.

$$M_n = (0.676)\cdot(60)\cdot\left(9.19 - \frac{0.88}{2} \right) \div 12 = 29.58 \text{ ft-k}$$

[1.3.2.1]

φ = 1.00

$M_r = \phi M_n = (1.00)(29.58) = 29.58$ ft-k

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the extreme limit state is satisfied.

The LRFD Specification requires that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u.

LL Distribution [BDG]

$S_c = bh^2 / 6 = (12)(12.00)^2 / 6 = 288$ in^3

[Table 4.6.2.1.3-1]

$1.2M_{cr} = 1.2f_rS_c = 1.2(0.785)(288)/12 = 22.61$ ft-k $< M_r = 29.58$ ft-k

IM [3.6.2]

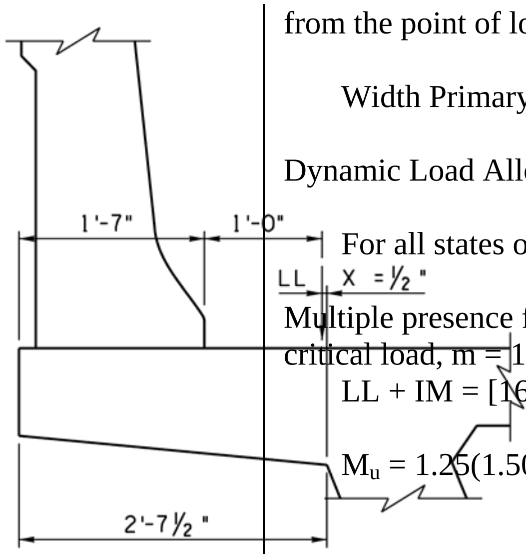
Since the strength of the section exceeds 1.2 M_{cr}, the minimum reinforcing criteria is satisfied.

Multiple Presence Factor [3.6.1.1.2-1]

Strength Limit State

Design Case 3

Design Case 3: The loads specified in [3.6.1] that occupy the overhang Strength and Service Limit State



DESIGN CASE 3

Figure 8

At the face of exterior support:

DC Dead Loads = 1.50 ft-k
DW Dead Loads = 0.01 ft-k

While the LRFD Specification allows use of a uniform load of 1.00 kip/ft for service limit state where the barrier is continuous ADOT does not. Therefore use the live load distribution for strength limit state for the service limit state also. For a cast-in-place concrete deck overhang, the width of the primary strip is 45.0 + 10.0 X where X equals the distance

from the point of load to the support.

Width Primary Strip (inches) = 45.0 + 10.0(0.042) = 45.42 in = 3.79 ft

Dynamic Load Allowance, IM

For all states other than fatigue and fracture limit state, IM = 33%.

Multiple presence factor must also be applied. Since one vehicle produces the critical load, m = 1.20.

$LL + IM = [16.00(1.33)(1.20)(0.042)] / 3.79 = 0.28$ ft-k

$M_u = 1.25(1.50) + 1.50(0.01) + 1.75(0.28) = 2.38$ ft-k

Service I Limit State [3.4.1]

Allowable Stress [BDG]

Control of Cracking

The flexural resistance was previously calculated for Design Case 2. Since the member is tension controlled, φ = 0.90. Since M_r = (0.90)(29.58) = 26.62 ft-k is greater than M_u the deck is adequately reinforced for strength.

M_s = 1.0(1.50 + 0.01) + 1.0(0.28) = 1.79 ft-k

d_s = 12.00 – 2.50 clr – 0.625 / 2 = 9.19 in

A_s (#5 @ 5 ½") = 0.676 in^2

The maximum allowable stress in a deck is limited to 24 ksi per the LRFD Bridge Design Guidelines.

Determine stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.676}{(12)\cdot(9.19)} = 0.00613$$

np = 8(0.00613) = 0.0490

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.0490) + (0.0490)^2} - 0.0490 = 0.268$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.268}{3} = 0.911$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(1.79)\cdot(12)}{(0.676)\cdot(0.911)\cdot(9.19)} = 3.80\ ksi \le 24.0\,\text{ksi}$$

Since the applied stress is less than the allowable specified by ADOT, the service limit state requirement is satisfied.

For all concrete components in which tension in the cross section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in Equation 5.7.3.4-1 shall be satisfied.

$f_{sa} = 0.80f_r = 0.80(0.24\sqrt{f'_c}) = 0.80(0.509) = 0.407$ ksi

$S_{cr} = (12.00)(11.50)^2 \div 6 = 264.5$ in^3

$$f_s = \frac{M_s}{S_{cr}} = \frac{(1.79)\cdot(12)}{264.5} = 0.081\ ksi < f_{sa} = 0.407\text{ ksi}$$

Bottom Slab Reinforcing [5.14.1.5.2b]

placed in the longitudinal direction in the bottom slab at a maximum 18 inch spacing.

Temperature & Shrinkage Reinforcing [5.10.8]

$A_s = (0.004)(6)(12) = 0.288 \text{ in}^2$ Use #5 @ 12”

A minimum of 0.5% reinforcement shall be placed in the transverse direction in the bottom slab at a maximum 18 inch spacing. The reinforcement shall extend to the exterior face of the outside web and be anchored by a 90° hook.

[5.10.8-1]

$A_s = (0.005)(6)(12) = 0.360 \text{ in}^2$ Use #5 @ 9” ($A_s = 0.413 \text{ in}^2$)

[5.10.8-2]

Temperature and shrinkage reinforcement requirements were changed in the 2006 Interim Revisions. The required area reinforcement for the section follows:

$$A_s \geq \frac{1.30bh}{2 \cdot (b+h) f_y}$$

$0.11 \leq A_s \leq 0.60$

Exterior Web

$b = (5.50 – 1.00)(12) \div \cos(21.80) = 58.2$ inches

$$A_s \geq \frac{1.30 \cdot (58.2) \cdot (12.0)}{2 \cdot (58.2 + 12.0) \cdot (60)} = 0.108 \text{ in}^2/\text{ft}$$

Top Slab

$B = 7.65(12) = 81.0$ inches

$$A_s \geq \frac{1.30 \cdot (81.0) \cdot (8.00)}{2 \cdot (81.0 + 8.00) \cdot (60)} = 0.079 \text{ in}^2/\text{ft}$$

Since the service limit state stress is less than the allowable, the control of cracking requirement is satisfied.

Bottom Slab

$$A_s \geq \frac{1.30 \cdot (81.0) \cdot (6.0)}{2 \cdot (81.0 + 6.0) \cdot (60)} = 0.060 \text{ in}^2/\text{ft}$$

Use #5 @ 12 in^2/ft minimum reinforcement.

A minimum of 0.4% reinforcement shall be

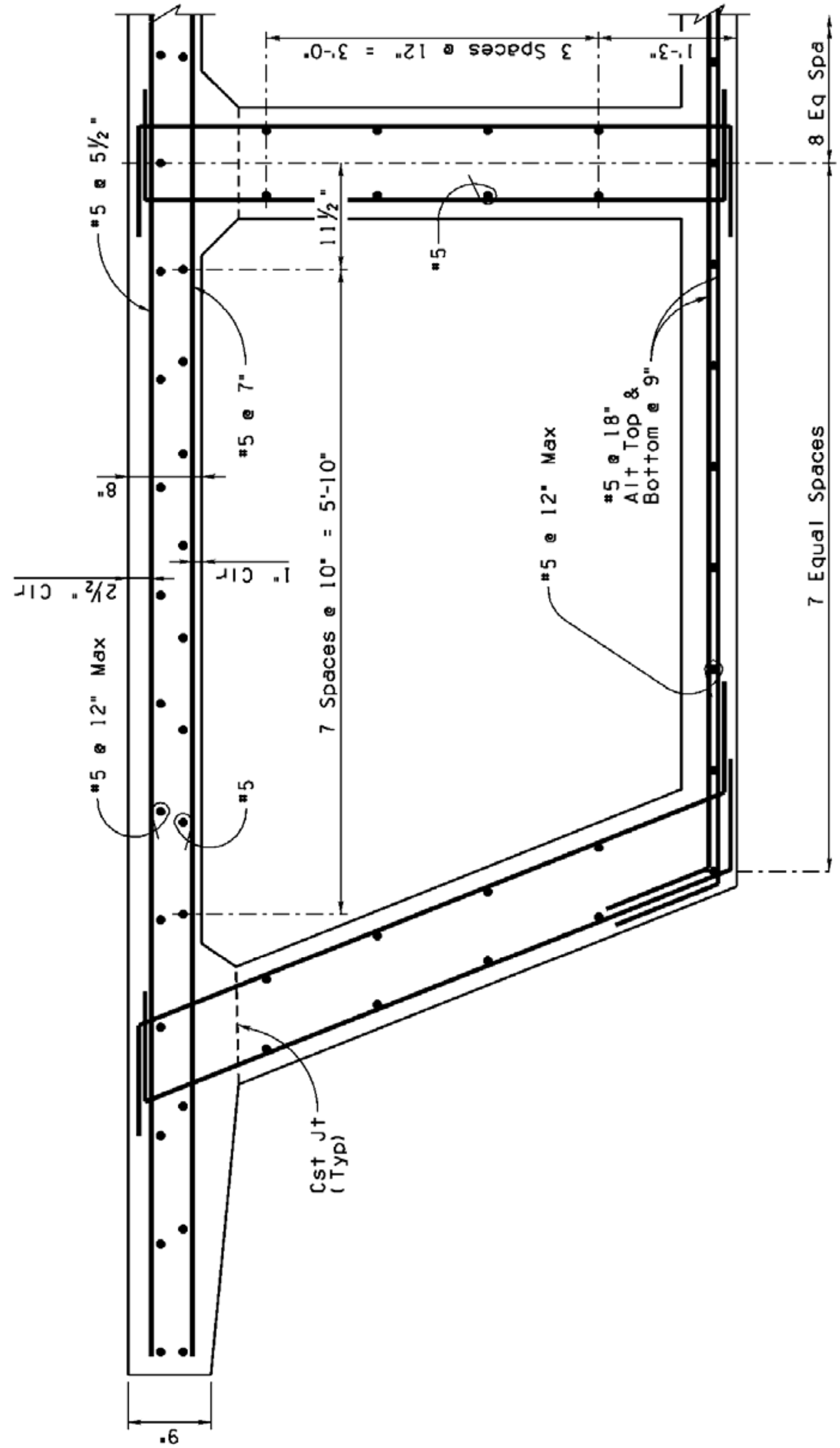


Figure 9

SUPERSTR DGN

subtracting the ½-inch wearing surface from the top slab thickness. However, this wearing surface has been included in weight calculations. The bridge has a uniform cross section except where the web flares from 12 inches to 18 inches starting 16 feet from the face of the abutment diaphragms. A summary of section properties follows:

Section Properties

Section Properties

	12" Web	18" Web	
yb	36.63	35.93	in
yt	28.87	29.57	in
Inertia	6,596,207	7,063,707	in^4
Area	10,741	12,660	in^2

The typical pier section is shown in Figure 10. To model the longitudinal frame (Figure 11) the equivalent length of the drilled shaft is required. This can be a complex problem. However, for a uniform layer of dense sand the problem can be simplified by using the relative stiffness factor T. The relative stiffness factor reflects the relative ratio of flexural rigidity of the shaft to stiffness of the soil. For a dense sand with n = 0.200 kci.

[10.7.4.2]

$$T = \left(\frac{EI}{n}\right)^{\frac{1}{5}}$$

$$EI = (3405)\cdot\left(\frac{\pi \cdot (84)^4}{64}\right) = 8{,}321{,}500{,}000$$

$$T = \left(\frac{8{,}321{,}500{,}000}{0.200}\right)^{\frac{1}{5}} \div 12 = 11.08 \text{ ft}$$

For drilled shafts with an embedment depth at least three times T the equivalent length of the shaft may be taken as follows:

L_e = 1.8T = 1.8(11.08) = 19.95 ft

The top 5 feet of the column/shaft embedment is ignored as described in more detail in the substructure design. Since two feet of the column is embedded, the top three feet of the shaft is ignored for lateral support. Therefore the length of the shaft used in the structural analysis is as shown below:

L = 19.95 + 3.00 = 22.95 feet

The section properties have been calculated

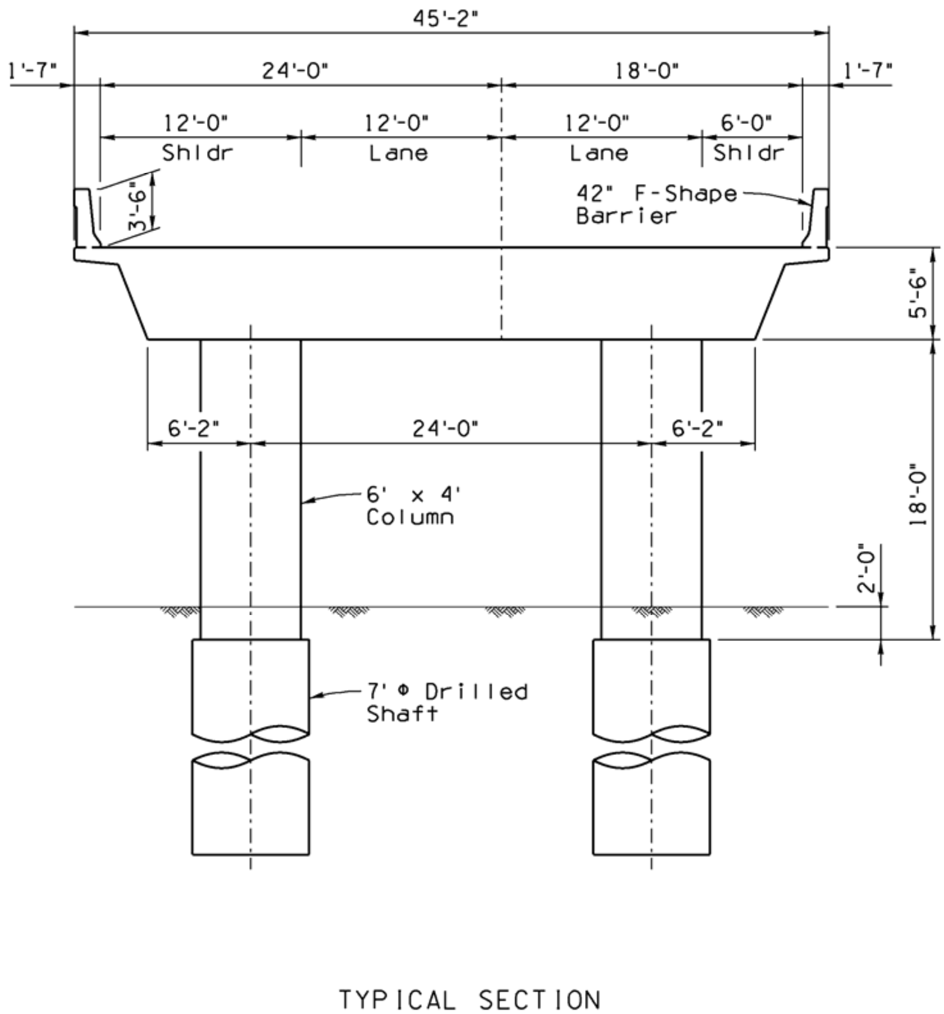


Figure 10

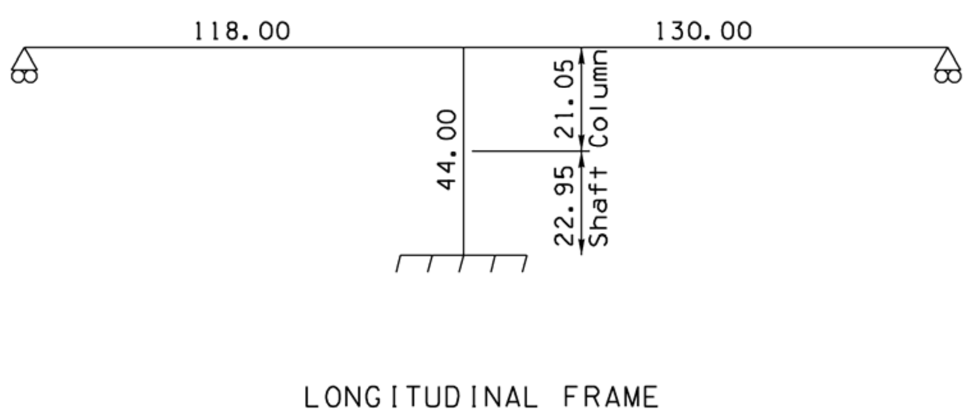


Figure 11

The equivalent length of the shaft is the length which when fixed at the base with the soil removed will produce the same deflection and rotation at the top. This can be verified by using a computer program such as L-Pile. For more complex soil types that are usually encountered, L-Pile or a similar soil structure interaction program may be used to determine the deflection at the top of the column/shaft. For a prismatic column/shaft the equivalent length can be solved for directly by using the following deflection formula:

$$\Delta = \frac{Pl^3}{3EI} \quad \text{or solving for } l \text{ yields:} \quad l = \sqrt[3]{\frac{3EI\Delta}{P}}$$

In the above formula, P is the applied load at the column top and Δ is the resulting deflection.

For a non prismatic member the length of the shaft can be determined by trial and error equating the two deflections. This length can then be used in the frame analysis to determine moments and shears at the top of the column. Values at the base will overestimate the true magnitude of the loads. These values can be used in the design especially if the forces are low and the 1 percent reinforcing requirement controls. Otherwise, the top moments and shears should be used as loads applied to a soil structure interaction program.

The longitudinal analysis was performed using a model as shown in Figure 11. For this example moments, shears and stresses were calculated at tenth points using computer software programs. A summary of the moments at critical locations is shown in Table 1.

Dead Load [3.5.1]

In LFRD design, the dead load must be separated between DC loads and DW loads since their load factors differ. The DC loads include uniform loads from the self-weight of the superstructure plus 0.010 ksf for lost deck formwork, and the barriers plus the concentrated load from the intermediate diaphragm. The DW load includes the 0.025 ksf Future Wearing Surface and any utilities.

Live Load [3.6]

The HL-93 live load in the LRFD specification differs from the HS-20-44 load in the Standard Specifications. For an in-depth discussion of live loads refer to Example 1. Computer software was used to generate the live loads for this problem. Values shown in Table 1 are for one vehicle but include dynamic load allowance.

Prestress Secondary Moment

Exact calculation of the secondary prestress moment is very difficult and time consuming since the moment is a function of the prestress force along the entire cable path not just at one point. The elastic shortening would have to be calculated at each point and the resulting secondary moment recalculated. Fortunately design does not require this level of refinement. The computer software automatically calculated the secondary moment based on uniform long-term losses along the span.

Live Load Distribution [4.6.2.2.1]

The LRFD Specification has made major changes to the live load distribution factors. However, for a cast-in-place concrete box girder bridge a unit design is allowed by multiplying the interior distribution factor by the number of webs. The live load distribution factor for moment for an interior web with one lane loaded is:

N_c = number of cells = 5
S = web spacing (ft) = 7.75 ft
L = span length of beam (ft). For negative moment use the average of the adjacent spans.

[4.6.2.2.2b-1]

$$LL\ Distribution = \left(1.75 + \frac{S}{3.6}\right)\left(\frac{1}{L}\right)^{0.35}\left(\frac{1}{N_c}\right)^{0.45}$$

$$\text{Span 1}\ LL\ Distribution = \left(1.75 + \frac{7.75}{3.6}\right)\left(\frac{1}{118}\right)^{0.35}\left(\frac{1}{5}\right)^{0.45} = 0.356$$

$$\text{Span 2}\ LL\ Distribution = \left(1.75 + \frac{7.75}{3.6}\right)\left(\frac{1}{130}\right)^{0.35}\left(\frac{1}{5}\right)^{0.45} = 0.344$$

$$\text{Negative}\ LL\ Distribution = \left(1.75 + \frac{7.75}{3.6}\right)\left(\frac{1}{124}\right)^{0.35}\left(\frac{1}{5}\right)^{0.45} = 0.350$$

(consider average span)

The distribution factor for moment with two or more lanes loaded is:

$$LL\ Distribution = \left(\frac{13}{N_c}\right)^{0.3}\left(\frac{S}{5.8}\right)\left(\frac{1}{L}\right)^{0.25}$$

$$\text{Span 1}\ LL\ Distribution = \left(\frac{13}{5}\right)^{0.3}\left(\frac{7.75}{5.8}\right)\left(\frac{1}{118}\right)^{0.25} = 0.540$$

$$\text{Span 2}\ LL\ Distribution = \left(\frac{13}{5}\right)^{0.3}\left(\frac{7.75}{5.8}\right)\left(\frac{1}{130}\right)^{0.25} = 0.527$$

$$\text{Negative}\ LL\ Distribution = \left(\frac{13}{5}\right)^{0.3}\left(\frac{7.75}{5.8}\right)\left(\frac{1}{124}\right)^{0.25} = 0.533$$

Span 1 LL Distribution = (0.540)(6 webs) = 3.240
Span 2 LL Distribution = (0.527)(6 webs) = 3.162
Negative LL Distribution = (0.533)(6 webs) = 3.198

Skew Reduction

Since the bridge is right angle the live load skew reduction factor is not applied.

Dynamic Load Allowance [3.6.2.1-1]

The dynamic load allowance IM equals 33% for strength and service limit states.

Dynamic load allowance applies to the truck or tandem but not to the design lane load. The dynamic load allowance has been included in the summation of live loads for one vehicle.

A summary of unfactored moments at critical locations from the computer program follows. The negative moments at the pier shown below have been reduced from the centerline moments.

Unfactored Moments (ft-k)

	0.4 Span 1	1.0 Span 1	0.0 Span 2	0.6 Span 2
Superstructure	10990	-20545	-20909	15044
Barrier	975	-1834	-1867	1340
DC	11965	-22379	-22776	16384
DW (FWS)	951	-1790	-1822	1308
Vehicle LL+IM	2677	-2614	-2736	3023
LL Distribution	3.240	3.198	3.198	3.162
Total LL + IM	8673	-8360	-8750	9559
Secondary M	2497	6243	7041	2817

Table 1

The LRFD Specification has made major changes to the group load combinations contained in [Table 3.4.1-1]. There are several limit states that must be considered in design of the superstructure. The secondary moments caused by the prestressing must be considered in the Strength Limit States. These moments are permanent loads classified as locked-in erection stresses (EL) with a load factor = 1.0. Limit states for this problem are as follows:

STRENGTH I – Basic load combination relating to the normal vehicular use of the bridge without wind.

M_u = 1.25(DC) + 1.50(DW) + 1.75(LL + IM) + 1.0(EL)

Span 1
0.4 M_u = 1.25(11,965) + 1.5(951) + 1.75(8673) + 1.0(2497) = 34,058 ft-k
1.0 M_u = 1.25(-22,379) + 1.5(-1790) + 1.75(-8360) + 1.0(6243)
= -39,046 ft-k
Span 2
0.0 M_u = 1.25(-22,776) + 1.5(-1822) + 1.75(-8750) + 1.0(7041)
= -39,475 ft-k
0.6 M_u = 1.25(16,384) + 1.5(1308) + 1.75(9559) + 1.0(2817) = 41,987 ft-k

[C5.7.1]

For service limit states the secondary moment is considered as part of the resisting prestress force and will be included as a resistance rather than a load. This simplifies the calculations since the secondary moment is a function of the amount of prestressing.

[Table 3.4.1-1]

SERVICE I – Load combination relating to normal operational use of the bridge including wind loads to control crack width in reinforced concrete structures.

M_s = 1.0(DC + DW) + 1.0(LL + IM)

Span 1
0.4 M_s = 1.0(11,965 + 951) + 1.0(8673) = 21,589 ft-k
1.0 M_s = 1.0(-22,379 -1790) + 1.0(-8360) = -32,529 ft-k

Span 2
0.0 M_s = 1.0(-22,776 -1822) + 1.0(-8750) = -33,348 ft-k
0.6 M_s = 1.0(16,384 + 1308) + 1.0(9559) = 27,251 ft-k

SERVICE III – Load combination relating only to tension in prestressed concrete superstructures with the objective of crack control.

M_s = 1.0(DC + DW) + 0.80(LL + IM)

Span 1
0.4 M_s = 1.0(11,965 + 951) + 0.8(8673) = 19,854 ft-k
1.0 M_s = 1.0(-22,379 -1790) + 0.8(-8360) = -30,857 ft-k

Span 2
0.0 M_s = 1.0(-22,776 -1822) + 0.8(-8750) = -31,598 ft-k
0.6 M_s = 1.0(16,384 + 1308) + 0.8(9559) = 25,339 ft-k

Prestress Design

The design of a post-tensioned concrete bridge involves making assumptions, calculating results, comparing the results to the assumptions and reiterating the process until convergence. Computer software was used to achieve this. Note that only 199 strands are required but 200 strands are specified to allow for a symmetric pattern. A summary of results follows:

$f_{pj} = 0.77\ f_{pu}$
No. Strands = 200 (0.6" φ strands)
$A_{ps} = (0.217)(200) = 43.400\ in^2$
$P_j = (0.77)(270)(43.400) = 9023\ k$

Step 1 – Assume Cable Path

The first step in design is to assume a cable path as shown in Figure 12. The location of the center of gravity at the ends is very important for the anchor zone design. Placing the center of gravity at the neutral axis results in a uniform stress distribution at the ends but the top tendons will probably be too high to have sufficient top edge clearance. Placing the cable path near the geometric center of the section is usually a good compromise. For the long span the cable path should be as low as possible. For the shorter span the cable path is often raised to prevent upward deflection growth. At the pier the cable path should be located as high as possible consistent with deck and pier cap reinforcing requirements. Care must be taken to ensure that the cable path can be physically located where assumed. A check on the center of gravity at the critical locations is required once the area of prestressing steel is determined.

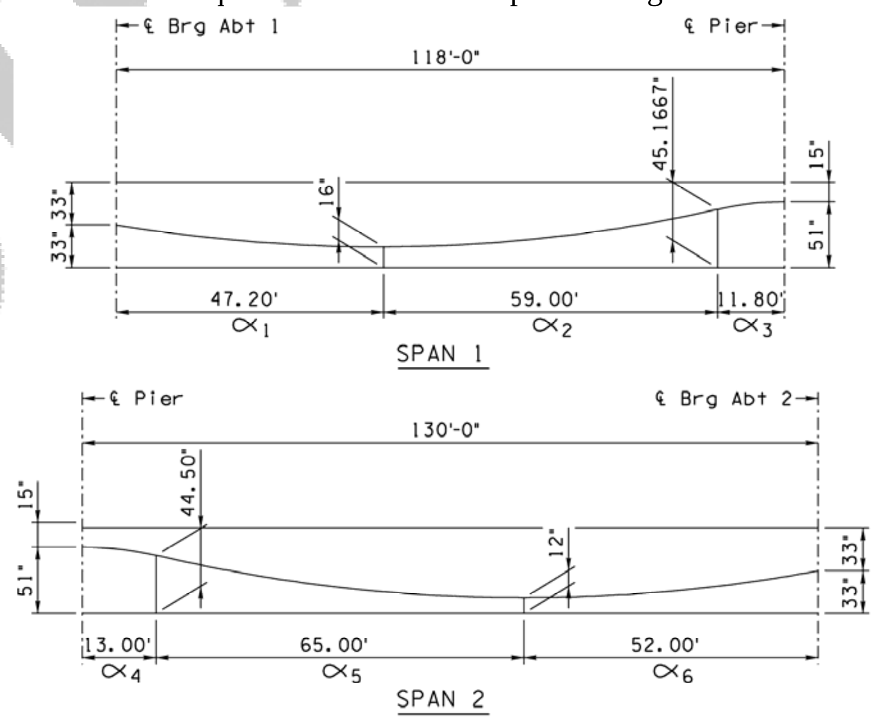


Figure 12 – Cable Path

Step 2 – Verify Cable Path

[C5.9.1.6]

For 200 strands use two tendons per web. One duct will hold 17 strands and the other 16 strands for four webs. The other two webs will have 17 strands in each duct. The ducts must clear the three layers of #5 reinforcing in the bottom slab. From manufacturers literature, the outside diameter of the duct will be 4 3/8” maximum. When the strands are pulled, they will rise at the low points and lower at the high points and not be located in the center of the duct. To estimate this effect, the variable Z is used. For ducts over 4 inch diameter, Z = 1 inch.

At the low point:

Duct c.g. = 4.375”+ 0.500” = 4.88 inches
c.g. ducts = 2 clr + 3(0.625) + 4.88 = 8.76 inches
Strand c.g. = [16(4.375)÷2 + 17(4.375 + 1 + 4.375÷2)]÷33 = 4.96 inches
c.g. strands = 8.76 + (4.96 – 4.88) + Z = 9.84 inches

At the pier: (Allow 2” for #11 pier cap reinforcing)

cg ducts = 7.00 + 2.00 + 4.88 = 13.88 inches
Strand c.g. = [17(4.375)÷2 + 16(4.375 + 1 + 4.375÷2)]÷33 = 4.79 inches
cg strands = 13.88 + (4.79 – 4.88) + Z = 14.79 inches

Since there are many possible combinations of size of ducts and different suppliers, one should be conservative in estimating the cg of the strands. Therefore use 12 inches for the location of the cg of the strands at low point of span 2 and 15 inches for the location of the cg of the strands at the pier.

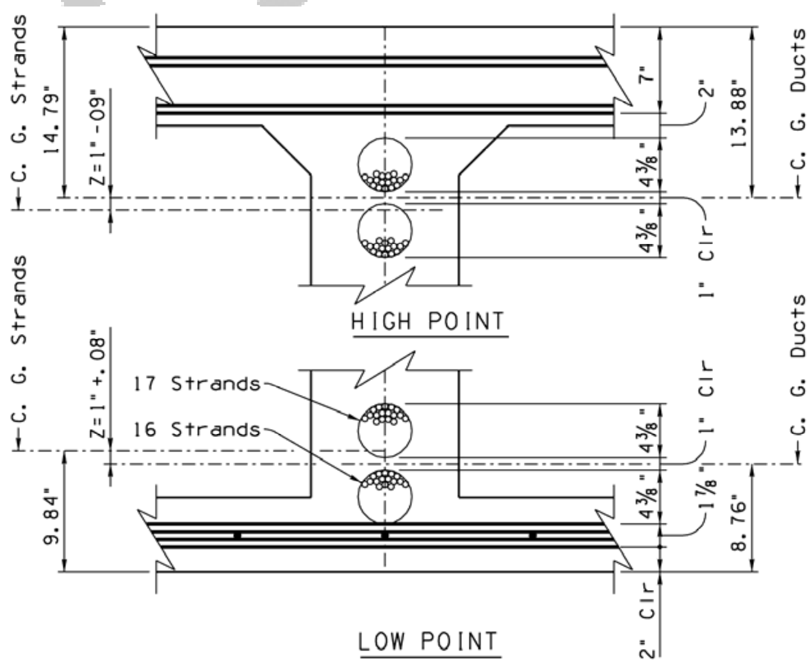


Figure 13 – Strand CG

Step 3 – Calculate Friction Losses

[5.9.5]
[BDG]

Total losses in prestress are due to friction loss, anchor set loss, elastic shortening, shrinkage, creep and relaxation. When the strands are pulled through the ducts, friction losses occur. Some loss is due to a uniform friction along the length of the path and some is due to angle changes in the cable path. Figure 14 is a diagram showing a summary of the friction losses, anchor set losses, elastic shortening losses and time dependent losses. Friction coefficients for rigid galvanized metal ducts are as follows:

k = 0.0002
μ = 0.25

For an unsymmetric two span bridge the cable should be jacked from the long end. The increased friction losses in the shorter span should not shift the critical location to the short span but the stresses in the short span should also be checked. The bridge will be jacked from the Abutment 2 end only.

The angle change for various segments (See Figure 12) is shown below:

$$\alpha_1 = \frac{2\cdot(33-16)\div 12}{47.20} = 0.060028 \text{ radians}$$

$$\alpha_2 = \alpha_3 = \frac{2\cdot(45.1667-16)\div 12}{59.00} = 0.082392 \text{ radians}$$

$$\alpha_4 = \alpha_5 = \frac{2\cdot(44.50-12)\div 12}{65.00} = 0.083333 \text{ radians}$$

$$\alpha_6 = \frac{2\cdot(33-12)\div 12}{52.00} = 0.067308 \text{ radians}$$

Computer software calculated the friction losses at the tenth points along the span. The friction loss calculations will be shown for the critical locations along the span.

[5.9.5.2.2b-1]

$$\Delta f_{pF} = f_{pj}\left(1-e^{-(Kx+\mu\alpha)}\right)$$

<u>0.6 Span 2</u>

kx + μα = (0.0002)(52) + (0.25)[0.067308] = 0.027227

$$\Delta f_{pF} = f_{pj}\left(1-e^{-(0.027227)}\right) = 0.0269 f_{pj}$$

<u>End Seat Loss (92.09 ft) (See Step 4)</u>

$kx + \mu\alpha = (0.0002)(92.09) + (0.25)[(92.09\text{-}52)/65(0.083333)+0.067308]$
$= 0.048094$

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(0.048094)}\right) = 0.0470 f_{pj}$$

<u>Span 2</u>

$kx + \mu\alpha = (0.0002)(130) + (0.25)[2(0.083333)+0.067308] = 0.084494$

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(0.084494)}\right) = 0.0810 f_{pj}$$

<u>0.4 Span 1</u>

$kx + \mu\alpha = (0.0002)(130+11.80+59.00) +$
$(0.25)[2(0.082392)+2(0.083333)+0.067308] = 0.139850$

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(0.139850)}\right) = 0.1305 f_{pj}$$

<u>Non-jacking End</u>

$kx + \mu\alpha = (0.0002)(118+130) +$
$(0.25)[0.060028+2(0.082392)+2(0.083333)+0.067308]$
$= 0.164297$

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(0.164297)}\right) = 0.1515 f_{pj}$$

A summary of friction losses at critical locations is shown in Figure 14.

Step 4 – Calculate Anchor Set Losses

The anchor set losses can be determined by the simplified method assuming straight line friction and anchor set losses. However, for this structure computer software was used that determined the losses using more sophisticated methods accounting for the curved loss path. The structure will be jacked from the Abutment 2 end only with an anchor set of 0.375 inches. A summary of the anchor set losses follows:

Anchor Set Length = 92.09 feet
Anchor Set Loss = 19.522 ksi

Jacking End: $\Delta f_{pA} = \dfrac{19.522}{0.77 \cdot (270)} = 0.0939 f_{pj}$

0.6 Span 2: $\Delta f_{pA} = 0.0939 - 2(0.0269) = 0.0401\ f_{pj}$

The initial force coefficients including both friction and anchor set losses at critical locations are as follows:

Jacking End	$FC_i = 1.000 - 0.0939 = 0.9061\ f_{pj}$
0.6 Span 2	$FC_i = 1.000 - 0.0269 - 0.0401 = 0.9330\ f_{pj}$
End Seating	$FC_i = 1.000 - 0.0470 = 0.9530\ f_{pj}$
Pier	$FC_i = 1.000 - 0.0810 = 0.9190\ f_{pj}$
0.4 Span 1	$FC_i = 1.000 - 0.1305 = 0.8695\ f_{pj}$
Non-Jacking End	$FC_i = 1.000 - 0.1515 = 0.8485\ f_{pj}$

A summary of the friction and anchor set losses is shown in Figure 14 below:

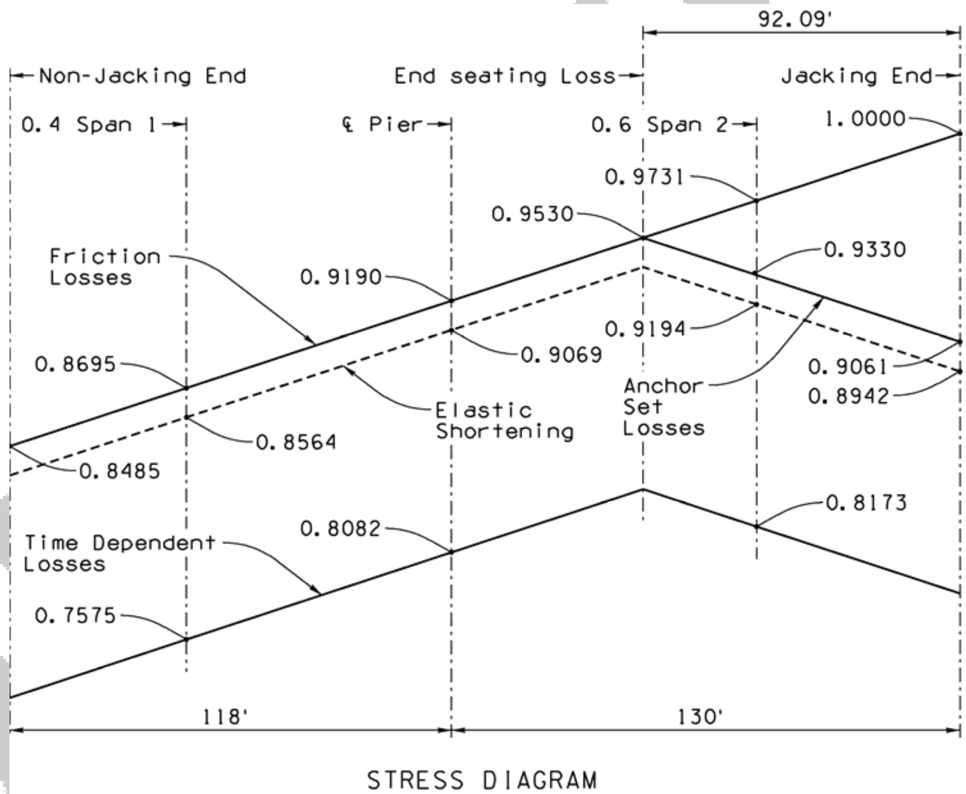


Figure 14 – Stress Diagram

Step 5 – Calculate Prestress Losses

Elastic Shortening [C 5.9.5.2.3b-1]

Elastic shortening losses require that the number of tendons in the bridge be known. The estimate of 12 tendons (2 tendons per web) will be used to calculate the elastic shortening losses. Elastic shortening losses can be calculated directly with a rather lengthy equation in lieu of a trial and error method. The equation for calculation of elastic shortening in the LRFD Commentary [C5.9.5.2.3b-1] is incorrect. The equation also does not directly consider the effect of the secondary prestress moment. The correct formula is given below:

$$\Delta f_{pES} = \frac{A_{ps}(FC_i) f_{pj}\left(I + Ae_m^{\ 2}\right) - e_m\left(M_g + M_{p/s}\right)A}{A_{ps}\left(I + Ae_m^{\ 2}\right) + \dfrac{A \cdot I \cdot E_{ci}}{E_p} \cdot \dfrac{2N}{(N-1)}}$$

This equation can be modified by dividing both the numerator and denominator by A and substituting r^2 for the ratio I / A. This version of the equation produces more manageable numbers.

$$\Delta f_{pES} = \frac{(FC_i) f_{pj} A_{ps}\left(r^2 + e_m^{\ 2}\right) - e_m\left(M_g + M_{p/s}\right)}{A_{ps}\left(r^2 + e_m^{\ 2}\right) + \dfrac{E_{ci} I}{E_p} \cdot \dfrac{2N}{N-1}}$$

$r^2 = I / A$ = 6,596,207 / 10,741 = 614.11 in^2

The problem of determination of elastic shortening losses is more difficult than first viewed. The secondary moment is a function of the stress in the tendons along the entire cable path. The secondary moment could be calculated considering the elastic shortening losses along the span but this refinement is not considered necessary considering the accuracy of the loss calculations. The secondary moment considering final losses will be used to determine the elastic shortening loss. The losses will be calculated for the three critical locations.

Prestress Loss
0.6 Span 2

0.6 Span 2

e = 36.63 – 12.00 = 24.63 in

$$A_{ps}\left(r^2 + e_m{}^2\right) = (43.400)\cdot\left(614.11 + (24.63)^2\right) = 52{,}980$$

$$\frac{I\cdot E_{ci}}{E_p}\cdot\frac{2\cdot N}{(N-1)} = \frac{(6{,}596{,}207)\cdot(3405)}{28500}\cdot\frac{2\cdot(12)}{(12-1)} = 1{,}719{,}432$$

Elastic Shortening

$$\Delta f_{pES} = \frac{(52{,}980)\cdot(0.9330)\cdot(0.77\cdot 270) - (24.63)\cdot(15{,}044 + 2817)\cdot(12)}{52{,}980 + 1{,}719{,}432}$$

Δf_{pES} = 2.82 ksi

$$FC_{ES} = 0.9330 - \frac{2.82}{(0.77)\cdot(270)} = 0.9194$$

Calculate f_{cgp} and verify the elastic shortening by substituting into [Eqn. 5.9.5.2.3b-1].

$$f_{cgp} = 43.400\cdot\left[(0.9330)\cdot(0.77)\cdot(270) - 2.82\right]\cdot\left(\frac{1}{10741} + \frac{(24.63)^2}{6{,}596{,}207}\right) - \frac{(15{,}044 + 2817)\cdot 12\cdot(24.63)}{6{,}596{,}207} = 0.735\ \text{ksi}$$

[5.9.5.2.3b-1]

$$\Delta f_{pES} = \frac{N-1}{2N}\cdot\frac{E_p}{E_{ci}}\cdot f_{cgp} = \left[\frac{12-1}{(2)\cdot(12)}\right]\cdot\left[\frac{28500}{3405}\right]\cdot(0.735) = 2.82\ \text{ksi}\ \ \text{OK}$$

Time-Dependent Losses
[5.9.5]
[BDG]

The method of loss calculation contained in the 2006 LRFD Specification shall not be used for post-tensioned box girder bridges. AASHTO LRFD Specifications Third Edition, 2004 shall be used. as specified in the Bridge Design Guidelines.

Shrinkage
[5.9.5.]
[BDG]

For Arizona, most locations have an average relative humidity of approximately 40%. The equation for shrinkage losses follows:

$\Delta f_{pSR} = (13.5 - 0.123\ H) = 13.5 - (0.123)(40) = 8.58$ ksi

Creep
[5.9.5]
[BDG]

The equation for creep follows:

$\Delta f_{pCR} = 12.0\ f_{cgp} - 7.0\ \Delta f_{cdp}$

where f_{cgp} has been previously calculated in the determination of elastic shortening losses and Δf_{cdp} equals the change in concrete stress due to externally applied dead loads excluding self weight.

$\Delta f_{cgp} = (1340 + 1308)(12)(24.63) / (6{,}596{,}207) = 0.119$ ksi

$\Delta f_{pCR} = 12.0(0.735) - 7.0(0.119) = 7.99$ ksi

Relaxation
[5.9.5]
[BDG]

For low relaxation strands, the relaxation in the prestressing strands equals 30% of the equation shown below:

$\Delta f_{pR2} = 20.0 - 0.3\ \Delta f_{pF} - 0.4\ \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$

where Δf_{pF} = the friction loss below 0.70 f_{pu} at the point under consideration.

At 0.6 Span 2 the friction stress is $0.9330 f_{pj}$ or $(0.9330)(0.77) f_{pu}$ = $0.7184\ f_{pu}$. Since this value is greater than 0.70, $\Delta f_{pF} = 0$.

$\Delta f_{pR2} = 0.3[20.0 - 0.3(0) - 0.4(2.82) - 0.2(8.58 + 7.99)] = 4.67$ ksi

Total Losses

The final total loss excluding friction and anchor set loss is:

Final Loss = 2.82 + 8.58 + 7.99 + 4.67 = 24.06 ksi

$$FC_f = 0.9330 - \frac{24.06}{(0.77)\cdot(270)} = 0.8173$$

Refer to Figure 14 for an overall view of losses.

Prestress Loss 0.4 Span 1

0.4 Span 1

e = 36.63 – 16.00 = 20.63 in

$$A_{ps}\left(r^2 + e_m{}^2\right) = (43.400)\cdot\left(614.11 + (20.63)^2\right) = 45{,}123$$

$$\frac{I \cdot E_{ci}}{E_p}\cdot\frac{2\cdot N}{(N-1)} = \frac{(6{,}596{,}207)\cdot(3405)}{28500}\cdot\frac{2\cdot(12)}{(12-1)} = 1{,}719{,}432$$

Elastic Shortening

$$\Delta f_{pES} = \frac{(45{,}123)\cdot(0.8695)\cdot(0.77\cdot 270) - (20.63)\cdot(10{,}990 + 2497)\cdot(12)}{45{,}123 + 1{,}719{,}432}$$

Δf_{pES} = 2.73 ksi

$$FC_{ES} = 0.8695 - \frac{2.73}{(0.77)\cdot(270)} = 0.8564$$

Calculate f_{cgp} and verify the elastic shortening by substituting into [Eqn. 5.9.5.2.3b-1].

$$f_{cgp} = (43.400)\cdot[(0.8695)\cdot(0.77)\cdot(270) - 2.73]\cdot\left(\frac{1}{10741} + \frac{(20.63)^2}{6{,}596{,}207}\right)$$

$$-\frac{(10990 + 2497)\cdot 12\cdot(20.63)}{6{,}596{,}207} = 0.712 \text{ ksi}$$

[Equation5.9.5.2.3b-1]

$$\Delta f_{pES} = \frac{N-1}{2N}\cdot\frac{E_p}{E_{ci}}\cdot f_{cgp} = \frac{12-1}{2\cdot 12}\cdot\frac{28500}{3405}\cdot 0.712 = 2.73 \text{ ksi OK}$$

Shrinkage

The equation for shrinkage losses follows:

Δf_{pSR} = (13.5 – 0.123 H) = 13.5 - (0.123)(40) = 8.58 ksi

Creep

The equation for creep follows:

$\Delta f_{pCR} = 12.0\ f_{cgp} - 7.0\ \Delta f_{cdp}$

where f_{cgp} has been previously calculated in the determination of elastic shortening losses and Δf_{cdp} equals the change in concrete stress due to externally applied dead loads excluding self weight.

Δf_{cgp} = (975 + 951)(12)(20.63) / (6,596,207) = 0.072 ksi

Δf_{pCR} = 12.0(0.712) – 7.0(0.072) = 8.04 ksi

Relaxation

For low relaxation strands, the relaxation in the prestressing strands equals 30% of the equation shown below:

$\Delta f_{pR2} = 20.0 - 0.3\ \Delta f_{pF} - 0.4\ \Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$

where Δf_{pF} = the friction loss below 0.70 f_{pu} at the point under consideration.

At 0.4 Span 1 the friction stress is 0.8695f_{pj} or (0.8695)(0.77)f_{pu} = 0.6695 f_{pu}. Since this value is less than 0.70, Δf_{pF} = (0.70 - 0.6695)(270) = 8.24 ksi.

Δf_{pR2} = 0.3[20.0 –0.3(8.24) – 0.4(2.73) – 0.2(8.58 + 8.04)]
= 3.93 ksi

The final loss excluding friction and anchor set loss is:

Final Loss = 2.73 + 8.58 + 8.04 + 3.93 = 23.28 ksi

$$FC_f = 0.8695 - \frac{23.28}{(0.77)\cdot(270)} = 0.7575$$

Refer to Figure 14 for an overall view of losses.

Prestress Loss Pier Span 2

Pier Span 2

e = 51.00 - 36.63 = 14.37 in

Elastic Shortening

$$A_{ps}\left(r^2 + e_m{}^2\right) = (43.400)\cdot\left(614.11 + (14.37)^2\right) = 35{,}614$$

$$\frac{I \cdot E_{ci}}{E_p}\cdot\frac{2\cdot N}{(N-1)} = \frac{(6{,}596{,}207)\cdot(3405)}{28500}\cdot\frac{2\cdot(12)}{(12-1)} = 1{,}719{,}432$$

$$\Delta f_{pES} = \frac{(35{,}614)\cdot(0.9190)\cdot(0.77\cdot 270) - (14.37)\cdot(20{,}909 - 7041)\cdot(12)}{35{,}614 + 1{,}719{,}432}$$

Δf_{pES} = 2.51 ksi

$$FC_{ES} = 0.9190 - \frac{2.51}{(0.77)\cdot(270)} = 0.9069$$

Calculate f_{cgp} and verify the elastic shortening by substituting into [Eqn. 5.9.5.2.3b-1].

$$f_{cgp} = (43.400)\cdot[(0.9190)\cdot(0.77)\cdot(270) - 2.51]\cdot\left(\frac{1}{10741} + \frac{(14.37)^2}{6{,}596{,}207}\right) - \frac{(20{,}909 - 7041)\cdot 12\cdot(14.37)}{6{,}596{,}207} = 0.655 \text{ ksi}$$

$$\Delta f_{pES} = \frac{N-1}{2N}\cdot\frac{E_p}{E_{ci}}\cdot f_{cgp} = \frac{12-1}{2\cdot 12}\cdot\frac{28500}{3405}\cdot 0.655 = 2.51 \text{ ksi OK}$$

Shrinkage

The equation for shrinkage losses follows:

Δf_{pSR} = (13.5 – 0.123 H) = 13.5 - (0.123)(40) = 8.58 ksi

Creep

The equation for creep follows:

Δf_{pCR} = 12.0 f_{cgp} – 7.0 Δf_{cdp}

where f_{cgp} has been previously calculated in the determination of elastic shortening losses and Δf_{cdp} equals the change in concrete stress due to externally applied dead loads excluding self weight.

Δf_{cgp} = (1867 + 1822)(12)(14.37) / (6,596,207) = 0.096 ksi

$\Delta f_{pCR} = 12.0(0.655) - 7.0(0.096) = 7.19$ ksi

Relaxation

For low relaxation strands, the relaxation in the prestressing strands equals 30% of the equation shown below:

$\Delta f_{pR2} = 20.0 - 0.3\,\Delta f_{pF} - 0.4\,\Delta f_{pES} - 0.2(\Delta f_{pSR} + \Delta f_{pCR})$

where Δf_{pF} = the friction loss below 0.70 f_{pu} at the point under consideration.

At the pier the friction stress is $0.9190 f_{pj}$ or $(0.9190)(0.77) f_{pu} = 0.7076\, f_{pu}$. Since this value is greater than 0.70, $\Delta f_{pF} = 0$ ksi.

$\Delta f_{pR2} = 0.3[20.0 - 0.3(0) - 0.4(2.51) - 0.2(8.58 + 7.19)] = 4.75$ ksi

The final loss excluding friction and anchor set loss is:

Final Loss = 2.51 + 8.58 + 7.19 + 4.75 = 23.03 ksi

$$FC_f = 0.9190 - \frac{23.03}{(0.77)\cdot(270)} = 0.8082$$

Refer to Figure 14 for an overall view of losses.

Step 6 – Check Allowable Stress in Strands

Prestressing Strand Stress [5.9.3] [BDG] [Table 5.9.3-1]

There are four limits for stress in prestressing strands. The first allowable limit is prior to seating. Bridge Group has modified the LRFD allowable of 0.90 f_{py} = (0.90)(0.90) f_{pu} = 0.81f_{pu} to a maximum of 0.78 f_{pu}.

(1) f_{pj} = 0.77 f_{pu}, < 0.78 f_{pu} OK.

The second stress limit is 0.70 f_{pu} at anchorages immediately after anchor set. At this time friction losses and anchor set losses have occurred. This criteria will usually limit the allowable jacking stress.

At jacking end:

(2) Strand stress = 0.9061f_{pj} = (0.9061)(0.77)f_{pu} = 0.698 f_{pu} < 0.70 f_{pu}

At non-jacking end:

(2) Strand stress = 0.8485f_{pj} = (0.8485)(0.77)f_{pu} = 0.653 f_{pu} < 0.70 f_{pu}

The third stress limit to be checked occurs at the end of the seating loss zone immediately after anchor set.

(3) Strand stress = 0.9530f_{pj} = (0.9530)(0.77)f_{pu} = 0.734 f_{pu} < 0.74 f_{pu}

The fourth stress limit is a service limit state after all losses. The maximum prestress strand stress occurs at the end of the seating loss zone. The maximum composite dead load and live load plus dynamic load allowance occurs at the 0.6 Span 2. Since this criteria rarely controls, adding the two maximum values, even though they do not occur at the same place and using the losses at 0.6 Span 2, is a reasonable and conservative simplification.

f_{pe} = [0.9530 - (0.9330 - 0.8173)](0.77)f_{pu} = 0.6447 f_{pu} after all losses

At service limit state added dead load and live load plus dynamic allowance stresses are added to the strand stress since the strands are bonded through the grouting process.

$$f_{service} = \frac{(1340 + 1308 + 9559) \cdot 12 \cdot (24.63)}{6{,}596{,}207} \cdot \frac{28500}{3861} = 4.037 \text{ ksi}$$

Strand stress = 0.6447 f_{pu} +(4.037) / (270)f_{pu} = 0.660 f_{pu}

(4) Strand stress = 0.660 f_{pu} < 0.80 f_{py} = 0.80(0.90) f_{pu} = 0.720 f_{pu}

Since the four criteria for stress in the strand are met, the jacking coefficient of 0.77 is satisfactory.

Step 7 – Verify Initial Concrete Strength

Once the amount of prestressing steel is determined from tension criteria, the resulting concrete stress and required concrete strength can be determined. Service I limit state is used to determine the initial concrete compressive stress. The concrete stress in compression before time dependent losses is limited to:

[5.9.4.1.1]

$$Allowable\ Compression = 0.60 \cdot f'_{ci} = 0.60 \cdot (3.5) = 2.100\ ksi$$

The basic equation for stress in concrete follows:

$$f = \frac{P_j FC_{ES}}{A} + \frac{(P_j FC_{ES} e + M_{\sec})y}{I} + \frac{\sum(\gamma M)y}{I}$$

<u>0.6 Span 2</u>
Bottom fiber

$$f_b = \frac{(9023)\cdot(0.9194)}{10741} + \frac{[(9023)\cdot(0.9194)\cdot(24.63)-(2817)\cdot(12)]\cdot(36.63)}{6{,}596{,}207}$$
$$-\frac{(15{,}044)\cdot 12\cdot(36.63)}{6{,}596{,}207} = 0.772 + 0.947 - 1.003 = 0.716\ ksi \le 2.100\ ksi$$

Top fiber

$$f_t = \frac{(9023)\cdot(0.9194)}{10741} - \frac{[(9023)\cdot(0.9194)\cdot(24.63)-(2817)\cdot(12)]\cdot(28.87)}{6{,}596{,}207}$$
$$+\frac{(15{,}044)\cdot 12\cdot(28.87)}{6{,}596{,}207} = 0.772 - 0.746 + 0.790 = 0.816\ ksi \le 2.100\ \text{ksi}$$

<u>0.4 Span 1</u>
Bottom Fiber

$$f_b = \frac{(9023)\cdot(0.8564)}{10741} + \frac{[(9023)\cdot(0.8564)\cdot(20.63)-(2497)\cdot(12)]\cdot(36.63)}{6{,}596{,}207}$$
$$-\frac{(10{,}990)\cdot 12\cdot(36.63)}{6{,}596{,}207} = 0.719 + 0.719 - 0.732 = 0.706\ ksi \le 2.100\ ksi$$

Top fiber

$$f_t = \frac{(9023)\cdot(0.8564)}{10741} - \frac{[(9023)\cdot(0.8564)\cdot(20.63)-(2497)\cdot(12)]\cdot(28.87)}{6{,}596{,}207}$$

$$+\frac{(10{,}990)\cdot 12\cdot(28.87)}{6{,}596{,}207} = 0.719 - 0.567 + 0.577 = 0.729\ ksi \le 2.100\ \text{ksi}$$

Pier Span 2

Bottom Fiber

$$f_b = \frac{(9023)\cdot(0.9069)}{10741} - \frac{[(9023)\cdot(0.9069)\cdot(14.37)+(7041)\cdot(12)]\cdot(36.63)}{6{,}596{,}207}$$
$$+\frac{(20{,}909)\cdot 12\cdot(36.63)}{6{,}596{,}207} = 0.762 - 1.122 + 1.393 = 1.033\ ksi \le 2.100\ ksi$$

Top fiber

$$f_t = \frac{(9023)\cdot(0.9069)}{10741} + \frac{[(9023)\cdot(0.9069)\cdot(14.37)+(7041)\cdot(12)]\cdot(28.87)}{6{,}596{,}207}$$
$$-\frac{(20{,}909)\cdot 12\cdot(28.87)}{6{,}596{,}207} = 0.762 + 0.884 - 1.098 = 0.548\ ksi \le 2.100\ \text{ksi}$$

Jacking End

Independent analysis provides an elastic shortening loss at the support of 0.0119 P_j.

Bottom fiber

$$f_b = \frac{(9023)\cdot(0.8942)}{10741} + \frac{(9023)\cdot(0.8942)\cdot(3.63)\cdot(36.63)}{6{,}596{,}207} = 0.914\ ksi$$

The initial concrete stresses are less than the allowable compressive stress. Therefore f'_{ci} = 3.5 ksi is acceptable. The initial concrete stress must also be checked in the design of the anchor zone. This check may control the required initial strength.

Step 8 – Temporary Tension at Ends

[5.9.4.1.2] The ends of the structure should be checked to ensure that the end eccentricity has been limited so as to keep any tension within the allowable.

$$f_t = \frac{(9023)\cdot(0.8942)}{10741} - \frac{(9023)\cdot(0.8942)\cdot(3.63)\cdot(28.87)}{6{,}596{,}207} = 0.623\ ksi \ge 0$$

Since there is no tension at the ends, the criteria is met.

Step 9 – Determine Final Concrete Strength

[5.9.4.2.1] The required final concrete strength is determined after all prestress losses. Service I load combination is used.

0.6 Span 2

Case I – Permanent Loads plus Effective Prestress

Allowable Compression = 0.45 f'_c = (0.45)(4.5) = 2.025 ksi

$$f_t = \frac{(9023)\cdot(0.8173)}{10741} - \frac{[(9023)\cdot(0.8173)\cdot(24.63)-(2817)\cdot(12)]\cdot(28.87)}{6{,}596{,}207}$$
$$+\frac{(17{,}692)\cdot 12\cdot(28.87)}{6{,}596{,}207} = 0.687 - 0.647 + 0.929 = 0.969 \text{ ksi} < 2.025 \text{ ksi}$$

Case II – One-half the Case I loads plus LL + IM

Allowable Compression = 0.40 f'_c = (0.40)(4.5) = 1.800 ksi

$$f_t = \frac{1}{2}\cdot[0.969] + \frac{(9559)\cdot 12\cdot(28.87)}{6{,}596{,}207}$$

f_t = 0.987 ksi < 1.800 ksi Allowable OK

Case III – Effective Prestress, Permanent Loads and Transient Loads

[5.7.4.7.2c-1]

Allowable Compression = 0.60 φ_w f'_c = 0.60(1.00)(4.5) = 2.700 ksi

The reduction factor φ_w shall be taken equal to 1.0 when the wall slenderness ratio λ_w is not greater than 15. The critical ratio involves the bottom slab.

[5.7.4.7.1]

$$\lambda_w = \frac{X_u}{t} = \frac{(7.75-1.00)}{0.500} = 13.5 \le 15$$

Since the ratio is less than the allowable, the equivalent rectangular stress block can be used.

$$f_t = \frac{(9023)\cdot(0.8173)}{10741} - \frac{[(9023)\cdot(0.8173)\cdot(24.63)-(2817)\cdot(12)]\cdot(28.87)}{6{,}596{,}207}$$
$$+\frac{(27{,}251)\cdot 12\cdot(28.87)}{6{,}596{,}207} = 0.687 - 0.647 + 1.431 = 1.471\, ksi \le 2.700 \text{ ksi}$$

0.4 Span 1

Case I – Permanent Loads plus Effective Prestress

Allowable Compression = 0.45 f'_c = (0.45)(4.5) = 2.025 ksi

$$f_t = \frac{(9023)\cdot(0.7575)}{10741} - \frac{[(9023)\cdot(0.7575)\cdot(20.63)-(2497)\cdot(12)]\cdot(28.87)}{6{,}596{,}207}$$

$$+\frac{(12{,}916)\cdot 12\cdot(28.87)}{6{,}596{,}207} = 0.636 - 0.486 + 0.678 = 0.828 \text{ ksi} < 2.025 \text{ ksi}$$

Case II – One-half the Case I loads plus LL + IM

Allowable Compression = 0.40 f'_c = (0.40)(4.5) = 1.800 ksi

$$f_t = \frac{1}{2}\cdot[0.828] + \frac{(8673)\cdot 12\cdot(28.87)}{6{,}596{,}207}$$

f_t = 0.870 ksi < 1.800 ksi Allowable OK

Case III – Effective Prestress, Permanent Loads and Transient Loads

Allowable Compression = 0.60 φ_w f'_c = 0.60(1.00)(4.5) = 2.700 ksi

$$f_t = \frac{(9023)\cdot(0.7575)}{10741} - \frac{[(9023)\cdot(0.7575)\cdot(20.63)-(2497)\cdot(12)]\cdot(28.87)}{6{,}596{,}207}$$

$$+\frac{(21{,}589)\cdot 12\cdot(28.87)}{6{,}596{,}207} = 0.636 - 0.486 + 1.134 = 1.284\ ksi \le 2.700 \text{ ksi}$$

Pier Span 2

Case I – Permanent Loads plus Effective Prestress

Allowable Compression = 0.45 f'_c = (0.45)(4.5) = 2.025 ksi

$$f_b = \frac{(9023)\cdot(0.8082)}{10741} - \frac{[(9023)\cdot(0.8082)\cdot(14.37)+(7041)\cdot(12)]\cdot(36.63)}{6{,}596{,}207}$$

$$+\frac{(24{,}598)\cdot 12\cdot(36.63)}{6{,}596{,}207} = 0.679 - 1.051 + 1.639 = 1.267 \text{ ksi} < 2.025 \text{ ksi}$$

Case II – One-half the Case I loads plus LL + IM

Allowable Compression = 0.40 f'_c = (0.40)(4.5) = 1.800 ksi

$$f_b = \frac{1}{2}\cdot[1.267] + \frac{(8750)\cdot 12\cdot(36.63)}{6{,}596{,}207}$$

f_b = 1.217 ksi < 1.800 ksi Allowable OK

Case III – Effective Prestress, Permanent Loads and Transient Loads

Allowable Compression = 0.60 φ_w f'_c = 0.60(1.00)(4.5) = 2.700 ksi

$$f_b = \frac{(9023)\cdot(0.8082)}{10741} - \frac{[(9023)\cdot(0.8082)\cdot(14.37)+(7041)\cdot(12)]\cdot(36.63)}{6{,}596{,}207}$$

$$+\frac{(33{,}348)\cdot 12\cdot(36.63)}{6{,}596{,}207} = 0.679 - 1.051 + 2.222 = 1.850\ ksi \le 2.700 \text{ ksi}$$

Step 10 – Determine Final Concrete Tension

Determination of the tension in the concrete is a Service III Limit State. The allowable tension after all losses is limited to

[BPG]

$$Allowable\ Tension = 0.0948\sqrt{f'_c} = 0.0948\sqrt{4.5} = 0.201\ ksi$$

The basic equation for stress in concrete follows:

$$f = \frac{P_j FC_f}{A} + \frac{\left(P_j FC_f e + M_{P/S}\right)y}{I} + \frac{\sum(\gamma M)y}{I}$$

<u>Bottom fiber at 0.6 Span 2</u>

$$f_b = \frac{(9023)\cdot(0.8173)}{10741} + \frac{\left[(9023)\cdot(0.8173)\cdot(24.63) - (2817)\cdot(12)\right]\cdot(36.63)}{6{,}596{,}207}$$

$$-\frac{(25{,}339)\cdot 12\cdot(36.63)}{6{,}596{,}207} = 0.687 + 0.821 - 1.689 = -0.181\ ksi \geq -0.201\ ksi$$

<u>Bottom fiber at 0.4 Span 1</u>

$$f_b = \frac{(9023)\cdot(0.7575)}{10741} + \frac{\left[(9023)\cdot(0.7575)\cdot(20.63) - (2497)\cdot(12)\right]\cdot(36.63)}{6{,}596{,}207}$$

$$-\frac{(19{,}854)\cdot 12\cdot(36.63)}{6{,}596{,}207} = 0.636 + 0.617 - 1.323 = -0.070\ ksi \geq -0.201\ ksi$$

<u>Top fiber at Pier Span 2</u>

$$f_t = \frac{(9023)\cdot(0.8082)}{10741} + \frac{\left[(9023)\cdot(0.8082)\cdot(14.37) + (7041)\cdot(12)\right]\cdot(28.87)}{6{,}596{,}207}$$

$$-\frac{(31{,}598)\cdot 12\cdot(28.87)}{6{,}596{,}207} = 0.679 + 0.828 - 1.660 = -0.153\ ksi \geq -0.201\ ksi$$

[BPG]

The member must also be checked to ensure that there is no tension under dead load and effective prestress.

Bottom fiber at 0.6 Span 2

$$f_b = 0.687 + 0.821 - \frac{(17{,}692)\cdot(12)\cdot(36.63)}{6{,}596{,}207} = 0.329\ ksi \geq 0$$

Bottom fiber at 0.4 Span 1

$$f_b = 0.636 + 0.617 - \frac{(12{,}916)\cdot(12)\cdot(36.63)}{6{,}596{,}207} = 0.392\ ksi \geq 0$$

Top fiber at Pier Span 2

$$f_t = 0.679 + 0.828 - \frac{(24{,}598)\cdot(12)\cdot(28.87)}{6{,}596{,}207} = 0.215\ ksi \geq 0$$

Since both the above criteria are met, the superstructure has adequate prestress reinforcement for serviceability.

Fatigue Limit State [5.5.3.1]

Fatigue of the reinforcement need not be checked for fully prestressed components designed to have extreme fiber tensile stress due to Service III Limit State within the tensile stress limit specified in Table 5.9.4.2.2-1.

Flexural Resistance [5.7.3]

Step 11 – Flexural Resistance

The flexural resistance of the structure must exceed the factored loads. Strength I Limit State should be compared to the resistance. The Strength I Limit State includes the secondary moment from prestressing.

$M_r = \varphi M_n < \sum\gamma M$

<u>0.6 Span 2</u>

STRENGTH I: $\sum\gamma M$ = 41,987 ft-k

$A_{ps} = (0.217)(200) = 43.400 \text{ in}^2$

[5.7.3.1.1-1]

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

[5.7.3.1.1-2]

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 2\left(1.04 - \frac{243}{270}\right) = 0.28$$

For a rectangular section without mild reinforcing steel:

d_p = 65.50 – 12.00 = 53.50 inches

[5.7.3.1.1-4]

$$c = \frac{A_{ps} f_{pu}}{0.85 f'_c \beta_1 b + kA_{ps}\dfrac{f_{pu}}{d_p}}$$

$$c = \frac{(43.400)\cdot(270)}{0.85\cdot(4.5)\cdot(0.825)\cdot(542) + 0.28\cdot(43.400)\cdot\dfrac{270}{53.50}} = 6.61 < t_{slab} = 7.50''$$

Since the stress block depth is less than the slab, the section is treated as a rectangular section:

[5.7.3.2.3]

$$a = c\beta_1 = (6.61)\cdot(0.825) = 5.45\ in$$

$$f_{ps} = (270)\cdot\left(1 - (0.28)\cdot\frac{6.61}{53.50}\right) = 260.66\ ksi$$

[5.7.3.2.2-1]

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

$$M_n = (43.400)\cdot(260.66)\cdot\left(53.50 - \frac{5.45}{2}\right) \div 12 = 47{,}867 \text{ ft-k}$$

Determine the tensile strain as follows:

$$\varepsilon_T = 0.003\left(\frac{d_T}{c} - 1\right) = 0.003\cdot\left(\frac{53.50}{6.61} - 1\right) = 0.021$$

Since $\varepsilon_T > 0.005$, the member is tension controlled.

[5.5.4.2]
[BDG]

The resistance factor φ = 0.95 for flexure of cast-in-place prestressed concrete.

φM_n = (0.95)(47,867) = 45,474 ft-k > 41,987 ft-k

∴ Section is adequate for flexural strength at 0.6 Span 2.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated the maximum reinforcing requirement replacing it with the strain limitations associated with the phi factors.

Minimum Reinforcing [5.7.3.3.2]

There is also a minimum amount of reinforcement that must be provided in a section. The amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

1.2 M_{cr}

1.33 $\sum\gamma M$ = 1.33 M_u

The cracking moment is determined on the basis of elastic stress distribution and the modulus of rupture of the concrete.

[5.7.3.3.2-1]

$$M_{cr} = S_c\left(f_r + f_{cpe}\right) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c f_r$$

Since the structure is designed for the monolithic section to resist all loads, S_{nc} should be substituted for S_c resulting in the second term equaling zero.

$$S_c = S_{nc} = \frac{I}{y_b} = \frac{6{,}596{,}207}{36.63} = 180{,}077 \text{ in}^3$$

Since the effect of the secondary moment is included in the applied loads, it is not repeated in the stress equation.

$$f_{cpe} = \frac{P_j FC_f}{A} + \frac{P_j FC_f e_m y_b}{I}$$

$$f_{cpe} = \frac{(9023)\cdot(0.8173)}{10{,}741} + \frac{(9023)\cdot(0.8173)\cdot(24.63)\cdot(36.63)}{6{,}596{,}207} = 1.695\,ksi$$

1.2 M_{cr} = 1.2(180,077)(0.785 + 1.695) / 12 = 44,659 ft-k

Since 1.2 M_{cr} = 44,659 < φM_n = 45,474, the minimum reinforcing limit is satisfied.

<u>0.4 Span 1</u>

Flexural Resistance

STRENGTH I: $\sum\gamma M$ = 34,058 ft-k

A_{ps} = (0.153)(282) = 43.400 in^2

[5.7.3.1-1]

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

For a rectangular section without mild reinforcing steel:

d_p = 65.50 – 16.00 = 49.50 inches

[5.7.3.1.1-4]

$$c = \frac{A_{ps} f_{pu}}{0.85 f'_c \beta_1 b + k A_{ps}\frac{f_{pu}}{d_p}}$$

$$c = \frac{(43.400)\cdot(270)}{0.85\cdot(4.5)\cdot(0.825)\cdot(542) + 0.28\cdot(43.400)\cdot\frac{270}{49.50}} = 6.60 < t_{slab} = 7.50"$$

Since the stress block depth is less than the slab, the section is treated as a rectangular section:

$$a = c\beta_1 = (6.60)\cdot(0.825) = 5.44\,in$$

$$f_{ps} = (270)\cdot\left(1 - (0.28)\cdot\frac{6.60}{49.50}\right) = 259.92\,ksi$$

[5.7.3.2.2-1]

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

$$M_n = (43.400)\cdot(259.92)\cdot\left(49.50 - \frac{5.44}{2}\right)\div 12 = 43{,}975 \text{ ft-k}$$

Determine the tensile strain as follows:

$$\varepsilon_T = 0.003\left(\frac{d_T}{c} - 1\right) = 0.003\cdot\left(\frac{49.50}{6.60} - 1\right) = 0.020$$

Since $\varepsilon_T > 0.005$, the member is tension controlled.

[BDG]

The resistance factor φ = 0.95 for flexure of cast-in-place prestressed concrete.

φM_n = (0.95)(43,975) = 41,776 ft-k > 34,058 ft-k

∴ Section is adequate for flexural strength at 0.4 Span 1.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated the maximum reinforcing requirement replacing it with the strain limitations associated with the phi factors.

Minimum Reinforcing [5.7.3.3.2]

There is also a minimum amount of reinforcement that must be provided in a section. The cracking moment is determined on the basis of elastic stress distribution and the modulus of rupture of the concrete.

[5.7.3.3.2-1]

$$M_{cr} = S_c\left(f_r + f_{cpe}\right) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \ge S_c f_r$$

$$S_c = S_{nc} = \frac{I}{y_b} = \frac{6{,}596{,}207}{36.63} = 180{,}077 \text{ in}^3$$

$$f_{cpe} = \frac{P_j FC_f}{A} + \frac{P_j FC_f e_m y_b}{I}$$

$$f_{cpe} = \frac{(9023)\cdot(0.7575)}{10{,}741} + \frac{(9023)\cdot(0.7575)\cdot(20.63)\cdot(36.63)}{6{,}596{,}207} = 1.419\, ksi$$

1.2 M_{cr} = 1.2(180,077)(0.785 + 1.419)÷12 = 39,689 ft-k < 41,776 ft-k

∴The minimum reinforcing limit is satisfied.

Flexural Resistance
[5.7.3]

Flexural Resistance

The flexural resistance of the structure must exceed the factored loads. Strength I Limit State should be compared to the resistance.

Pier Span 2

STRENGTH I: $\sum\gamma M$ = -39,475 ft-k

A_{ps} = (0.217)(200) = 43.400 in^2

[5.7.3.1.1-1]

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

For a rectangular section without mild reinforcing steel:

d_p = 66.00 – 15.00 = 51.00 inches

[5.7.3.1.1-4]

$$c = \frac{A_{ps} f_{pu}}{0.85 f'_c \beta_1 b + kA_{ps}\frac{f_{pu}}{d_p}}$$

$$c = \frac{(43.400)\cdot(270)}{0.85\cdot(4.5)\cdot(0.825)\cdot(436) + 0.28\cdot(43.400)\cdot\frac{270}{51.00}} = 8.14 > t_{slab} = 6.00"$$

Since the stress block is deeper than the slab, the section must be treated as a T-section:

[5.7.3.1.1-3]

$$c = \frac{A_{ps} f_{pu} - 0.85 f'_c (b - b_w)h_f}{0.85 f'_c \beta_1 b_w + kA_{ps}\frac{f_{pu}}{d_p}}$$

$$c = \frac{(43.400)\cdot(270) - 0.85\cdot(4.5)\cdot(436 - 73.83)\cdot(6.0)}{0.85\cdot(4.5)\cdot(0.825)\cdot(73.83) + (0.28)\cdot(43.400)\frac{270}{51.00}} = 11.46 \text{ in}$$

[5.7.3.2.3]

$$a = c\beta_1 = (11.46)\cdot(0.825) = 9.45 \text{ in}$$

$$f_{ps} = (270)\cdot\left(1 - (0.28)\cdot\frac{11.46}{51.00}\right) = 253.01 \text{ ksi}$$

[5.7.3.2.2-1]

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85 f'_c (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

$$M_n = (43.400)\cdot(253.01)\cdot\left(51.00 - \frac{9.45}{2}\right)$$
$$+ 0.85\cdot(4.5)\cdot(436 - 73.83)\cdot(6.00)\cdot\left(\frac{9.45}{2} - \frac{6.00}{2}\right) = 522{,}467 \text{ in-k}$$

Determine the tensile strain as follows:

$$\varepsilon_T = 0.003\left(\frac{d_T}{c} - 1\right) = 0.003\cdot\left(\frac{51.00}{11.46} - 1\right) = 0.010$$

Since $\varepsilon_T > 0.005$, the member is tension controlled.

[BDG]

The resistance factor φ = 0.95 for flexure of cast-in-place prestressed concrete.

$\varphi M_n = (0.95)(522{,}467) / 12 = 41{,}362$ ft-k $> M_u = 39{,}475$ ft-k

∴ Section is adequately reinforced for flexure.

Minimum Reinforcing [5.7.3.3.2]

There is also a minimum amount of reinforcement that must be provided in a section.

[5.7.3.3.2-1]

The cracking moment is determined on the basis of elastic stress distribution and the modulus of rupture of the concrete.

$$M_{cr} = S_c\left(f_r + f_{cpe}\right) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c f_r$$

Since the structure is designed for the monolithic section to resist all loads, S_{nc} should be substituted for S_c resulting in the second term equaling zero.

$$S_c = S_{nc} = \frac{I}{y_b} = \frac{6{,}596{,}207}{28.87} = 228{,}480 \text{ in}^3$$

$$f_{cpe} = \frac{P_j FC_f}{A} + \frac{P_j FC_f e_m y_b}{I}$$

$$f_{cpe} = \frac{(9023)\cdot(0.8082)}{10{,}741} + \frac{(9023)\cdot(0.8082)\cdot(14.37)\cdot(28.87)}{6{,}596{,}207} = 1.138\ ksi$$

1.2 M_{cr} = (1.2)(228,480)(0.785 + 1.138) ÷ 12 = 43,937 ft-k

1.33M_u = 1.33(39,475) = 52,502 ft-k

Since neither criteria is satisfied, add mild steel over pier for additional resistance.

d_s = 66.00 – 2.50 cl – 0.625 – 0.625 / 2 = 62.56 in

Try #5 @ 12 inches

A_s = (0.31)(45) = 13.95 in^2

[5.7.3.1.1-3]

$$c = \frac{A_{ps} f_{pu} + A_s f_y - 0.85 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w + k A_{ps} \frac{f_{pu}}{d_p}}$$

$$c = \frac{(43.400)\cdot(270) + (13.95)\cdot(60) - (0.85)\cdot(4.5)\cdot(362.17)\cdot(6.0)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(73.83) + (0.28)\cdot(43.400)\cdot\frac{270}{51.00}} = 14.27 \text{ in}$$

a = (0.825)(14.27) = 11.77 in

$$f_{ps} = (270)\cdot\left(1 - 0.28\cdot\frac{14.27}{51.00}\right) = 248.85 \text{ ksi}$$

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y \left(d_s - \frac{a}{2}\right) + 0.85 f'_c (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

$$M_n = (43.400)\cdot(248.85)\cdot\left(51.00 - \frac{11.77}{2}\right) + (13.95)\cdot(60)\cdot\left(62.56 - \frac{11.77}{2}\right) + 0.85\cdot(4.5)\cdot(362.17)\cdot(6.00)\cdot\left(\frac{11.77}{2} - \frac{6.00}{2}\right) = 558{,}663 \text{ in-k}$$

Determine the tensile strain as follows:

$$\varepsilon_T = 0.003\left(\frac{d_T}{c} - 1\right) = 0.003 \cdot \left(\frac{62.56}{14.27} - 1\right) = 0.010$$

Since $\varepsilon_T > 0.005$, the member is tension controlled.

[BDG]

The resistance factor φ = 0.95 for flexure of cast-in-place prestressed concrete.

φM_n = (0.95)(558,663) / 12 = 46,555 ft-k > $1.2M_{cr}$ = 43,937 ft-k

∴ The minimum reinforcing criteria is satisfied.

Shear
[5.8]

The LRFD method of shear design is a complete change from the methods specified in the Standard Specifications and that used by ADOT. For this example an in-depth shear design will be performed at the critical location in Span 2 near the pier.

Critical Section
[5.8.3.2]

d_v
[5.8.2.9]

The critical location is located a distance d_v from the face of the support. This creates a problem in that d_v is largest of three values, two of which are a function of distance from the pier. To eliminate the iterative process in determining the critical shear location, a simplification is required. It is recommended that the equation, $d_v = 0.72$ h be used to determine the critical shear location. Since d_v is the larger of the three values determined in Step 3, using $d_v = 0.72h = (0.72)(65.5) / 12 = 3.93$ feet will be conservative. Since the pier diaphragm is 6 feet wide, the critical shear is located 3.00 + 3.93 = 6.93 feet from the centerline of the pier.

Step 1 – Determine Shear

The shears were determined from a computer program as follows:

	2.0	2.1	Units
Super	958.2	804.7	k
Barriers	86.1	72.1	k
DC	1044.3	876.8	k
DW (FWS)	84.1	70.4	k
LL+IM Vehicle	143.9	130.4	k
Secondary M	-54.2	-54.2	k

Super $V_{Crit} = 958.2 - (958.2 - 804.7)(6.93) / 13.0 = 876.4$ kips
Barrier $V_{Crit} = 86.1 - (86.1 - 72.1)(6.93) / 13.0 = 78.6$ kips
DC $V_{Crit} = 876.4 + 78.6 = 955.0$ kips

DW $V_{Crit} = 84.1 - (84.1 - 70.4)(6.93) / 13.0 = 76.8$ kips
Vehicle $V_{Crit} = 143.9 - (143.9 - 130.4)(6.93) / 13.0 = 136.7$ kips

Live Load Distribution
[4.6.2.2.1]

The live load distribution factor for shear will be determined based on the provisions for a whole width design. The distribution of live load per lane for shear for an interior beam in Span 2 for one design lane loaded is:

[Table 4.6.2.2.3a-1]

$$LL\ Distribution = \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1} = \left(\frac{7.75}{9.5}\right)^{0.6}\left(\frac{65.5}{12.0\cdot(130)}\right)^{0.1} = 0.645$$

The distribution for two or more design lanes loaded is:

$$LL\ Distribution = \left(\frac{S}{7.3}\right)^{0.9}\left(\frac{d}{12.0L}\right)^{0.1} = \left(\frac{7.75}{7.3}\right)^{0.9}\left(\frac{65.5}{12.0\cdot(130)}\right)^{0.1} = 0.769$$

For a whole width bridge, LL Distribution = (0.769)(6 webs) = 4.614

Skew Effect
[Table 4.6.2.2.3c-1]

For skewed bridges, the shear shall be adjusted to account for the effects of the skew. For a right angle bridge the correction factor equals one.

LL+IM V_{Crit} = (136.7)(4.614)(1.00) = 630.7 kips
$V_u = \sum\gamma_i V_i$ = 1.25(955.0) + 1.50(76.8) + 1.75(630.7) + 1.00(–54.2) = 2358 kips

Step 2 – Determine Analysis Model

Sectional Model
[5.8.3]

The sectional model of analysis is appropriate for the design of typical bridge webs where the assumptions of traditional beam theory are valid. Where the distance from the point of zero shear to the face of the support is greater than 2d the sectional model may be used. Otherwise, the strut-and-tie model should be used.

From Computer output:
Point of Zero Shear to Face of Support = 79.70 - 3.00 = 76.70 ft
2d = 2(5.50) = 11.00 ft < 76.70 ft

$\therefore$ Sectional model may be used.

[5.8.2.9]

Step 3 – Shear Depth, d_v

The shear depth is the maximum of the following criteria:

1) $d_v = 0.9\ d_e$ where $d_e = \dfrac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = d_p$ when $A_s = 0$

From Figure 12:

$$d_p = 51.00 - (51.00 - 44.50)\cdot\left(\frac{6.93}{13.00}\right)^2 = 49.15\ in$$

$$d_v = 0.9 d_p = 0.9(49.15) = 44.24\ \text{in}$$

2) 0.72h = 0.72(65.50) = 47.16 in

3) $d_v = \dfrac{M_n}{A_s f_y + A_{ps} f_{pu}}$

For a T-Section at the critical location:

[5.7.3.1.1-3]

$$c = \frac{(43.400)\cdot(270) - 0.85\cdot(4.5)\cdot(436-73.83)\cdot(6.0)}{0.85\cdot(4.5)\cdot(0.825)\cdot(73.83) + (0.28)\cdot(43.400)\dfrac{270}{49.15}} = 11.36 \text{ in}$$

[5.7.3.2.3]

a = $c\beta_1$ = (11.36)(0.825) = 9.38 in

[5.7.3.1.1-1]

$$f_{ps} = (270)\cdot\left[1 - (0.28)\cdot\frac{11.36}{49.15}\right] = 252.53 \text{ ksi}$$

[5.7.3.2.2-1]

$$M_n = (43.400)\cdot(252.53)\cdot\left(49.15 - \frac{9.38}{2}\right)$$

$$+\,0.85\cdot(4.5)\cdot(436-73.83)\cdot(6.00)\cdot\left(\frac{9.38}{2} - \frac{6.00}{2}\right) = 501{,}320 \text{ in-k}$$

M_n = (501,320) / 12 = 41,777 ft-k

$$d_v = \frac{(41{,}777)\cdot(12)}{0 + (43.400)\cdot(252.53)} = 45.74 \text{ in}$$

Based on the above, the shear depth, d_v, equals 47.16 inches.

Step 4 – Calculate, V_p

Due to the cable curvature, some of the prestress force is in the upward vertical direction and directly resists the applied shear. See Figure 12 for the angle of the cable path and Figure 14 for the stress diagram as shown below:

$$\alpha = \frac{2(51.00 - 44.50)}{12(13.00)}\frac{6.93}{13.00} = 0.04442 \text{ rads}$$

FC_f = (0.9530 - 0.9190)(6.93) / 37.91 + 0.8082 = 0.8144

V_p = (9023)(0.8144)(0.04442) = 326 kips

Step 5 – Check Shear Width, b_v

[5.8.2.9]

The LRFD Specification requires that web width be adjusted for the presence of voided or grouted ducts. For ungrouted ducts, 50% of the width should be subtracted from the gross width and for grouted ducts, 25% should be subtracted. When the structure is first prestressed, the ducts are ungrouted. For this condition of dead load and prestressing, the shear should be checked with the 50% reduction for ducts. For the final condition, the ducts are grouted and only the 25% reduction is required.

For ungrouted ducts under DC dead load of superstructure and diaphragm:

[5.8.3.3-2]

$V_u <= \varphi V_n = \varphi(0.25f'_c b_v d_v + V_p)$

$$\text{Required } b_v = \frac{\frac{V_u}{\varphi} - V_p}{0.25 d_v f'_c} = \frac{\frac{1.25\cdot(876.4)}{0.9} - 326}{(0.25)\cdot(47.16)\cdot(4.5)} = 16.80 \text{ inches}$$

Available b_v = 73.83 - 0.50(4.375)(6 webs) = 60.71 inches, ok

For grouted ducts under full load:

$$\text{Required } b_v = \frac{\frac{2358}{0.9} - 326}{(0.25)\cdot(47.16)\cdot(4.5)} = 43.24 \text{ inches}$$

Available b_v = 73.83 – 0.25(4.375)(6 webs) = 67.27 inches, ok

Use b_v = 67.27 inches for all future calculations involving shear.

Step 6 – Evaluate Shear Stress

[5.8.2.9-1]

$$v_u = \frac{V_u - \varphi V_p}{\varphi b_v d_v} = \frac{2358 - 0.90\cdot(326)}{0.90\cdot(67.27)\cdot(47.16)} = 0.723 \text{ ksi}$$

$$\frac{v_u}{f'_c} = \frac{0.723}{4.5} = 0.161$$

Step 7 – Estimate Crack Angle θ

The LRFD method of shear design involves several cycles of iteration. The first step is to estimate a value of θ, the angle of inclination of diagonal compressive stress. Since the formula is not very sensitive to this estimate assume that θ = 26.5 degrees. This simplifies the equation somewhat by setting the coefficient 0.5 cotθ = 1.0.

Step 8 – Calculate strain, ε_x

[5.8.3.4.2]

There are two formulae for the calculation of strain for sections containing at least the minimum amount of transverse reinforcing. The first formula is used for positive values of strain indicating tensile stresses, while the second formula is used for negative values of strain indicating compressive stresses.

Formula for ε_x for positive values:

[5.8.3.4.2-1]

$$\varepsilon_x = \left[\frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}}{2(E_s A_s + E_p A_{ps})} \right]$$

Formula for e_x for negative values:

[5.8.3.4.2-3]

$$\varepsilon_x = \left[\frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}}{2(E_c A_c + E_s A_s + E_p A_{ps})} \right]$$

where:

A_c = area of concrete on the flexural tension side of the member.
A_c = 10741 - (436)(6) - 2(0.5)(6)(2.4) – (26.75)(73.83) = 6136 in^2

A_{ps} = area of prestressing steel on the flexural tension side of the member.
A_{ps} = 43.400 in^2 since all the strands are located above mid-depth.

A_s = area of nonprestressed steel on flexural tension side of the member.
A_s = 0.

f_{po} = a parameter taken as the modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete. For the usual levels of prestressing, a value of 0.7 f_{pu} will be appropriate.
f_{po} = 0.70(270) = 189 ksi

N_u = factored axial force taken as positive if tensile.
N_u = 0 kips

V_u = factored shear force.
V_u = 2358 kips

M_u = factored moment but not to be taken less than $V_u d_v$.

The moment at the critical shear location is required. The moment at the pier centerline is used for interpolation and not the reduced moment that was used in the calculation of stresses and flexural capacity. A summary of moments follows:

	2.0	2.1	Units
Super	-23,554	-12,095	ft-k
Barriers	-4158	-2125	ft-k
DC	-27,712	-14,220	ft-k
DW (FWS)	-2736	-1656	ft-k
LL+IM	-8750	-5299	ft-k
Secondary M	7041	6337	ft-k

Super $M_{Crit} = -23554 + (-12095 + 23554)(6.93) / 13.0 = -17{,}445$ ft-k
Barrier $M_{Crit} = -4158 + (-2125 + 4158)(6.93) / 13.0 = -3074$ ft-k
DC $M_{Crit} = -17{,}445 - 3074 = -20{,}519$ ft-k

DW $M_{Crit} = -2736 + (-1656 + 2736)(6.93) / 13.0 = -2160$ ft-k
LL+IM $M_{Crit} = -8750 + (-5299 + 8750)(6.93) / 13.0 = -6910$ ft-k
Secondary $M_{Crit} = 7041 + (6337 - 7041)(6.93) / 13.0 = 6666$ ft-k

$M_{Crit} = 1.25(-20{,}519) + 1.50(-2160) + 1.75(-6910) + 1.00(6666) = -34{,}315$ ft-k

$M_u = 34{,}315 \text{ ft-k} > V_u d_v = (2358)(47.16) / 12 = 9267$ ft-k

Determine strain as follows:

$$\varepsilon_x = \left[\frac{\dfrac{|34315 \cdot 12|}{47.16} + 0 + 1.0 \cdot |2358 - 326| - (43.400) \cdot (189)}{2(29000 \cdot 0 + 28500 \cdot 43.400)}\right]$$

$\varepsilon_x = 0.00104 = 1.04 \times 10^{-3}$

Since the value is positive the first formula must be used.

[5.8.3.4.2-1]

Now go into [Table 5.8.3.4.2-1] to read the values for θ and β. From the previously calculated value of $v_u/f'_c = 0.161$, enter the ≤ 0.175 row and the ≤ 1.00 column even though the value is larger than one. The estimate for values is shown below:

θ = 36.8 degrees
β = 1.96

With the new value of θ, the strain must be recalculated.

$$\varepsilon_x = \left[\frac{\frac{|34315 \cdot 12|}{47.16} + 0 + 0.5 \cdot |2358 - 326| \cot(36.8) - 43.400 \cdot (189)}{2{,}473{,}800} \right]$$

$\varepsilon_x = 0.00076 = 0.76 \times 10^{-3}$

With this new estimate for strain, reenter the table and determine new values for θ and β. Since our new values are the same as assumed, our iterative portion of the design is complete.

Step 9 - Calculate Concrete Shear Strength, V_c

The nominal shear resistance from concrete, V_c, is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316 \beta \sqrt{f'_c} b_v d_v$$

$$V_c = 0.0316 \cdot (1.96) \cdot \sqrt{4.5} \cdot (67.27) \cdot (47.16) = 417 \text{ kips}$$

Step 10 - Determine Required Vertical Reinforcement, V_s

$V_n = V_c + V_s + V_p$

[5.8.3.3-2]

$V_n = 0.25f'_c b_v d_v + V_p$

[5.8.3.3-1]

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s} = \frac{A_v f_y d_v \cot\theta}{s} \; where\ \alpha = 90°$$

[5.8.3.3-4]

$$V_u \le V_R = \phi V_n = \phi(V_c + V_s + V_p)$$

$$V_s = \frac{V_u}{\phi} - V_c - V_p = \frac{A_v f_y d_v \cot\theta}{s}$$

$$s = \frac{A_v f_y d_v \cot\theta}{\frac{V_u}{\phi} - V_c - V_p} = \frac{(3.72) \cdot (60) \cdot (47.16) \cdot \cot(36.8)}{\frac{2358}{0.90} - 417 - 326} = 7.5 \text{ in}$$

Use #5 stirrups at 7 inch spacing

$$V_s = \frac{(3.72)\cdot(60)\cdot(47.16)\cot(36.8)}{7} = 2010 \text{ kips}$$

The shear strength is the lesser of:

V_n = [417 + 2010 + 326] = 2753 kips

V_n = [0.25(4.5)(67.27)(47.16) + 326] = 3895 kips

φV_n = (0.90)(2753) = 2478 k > V_u = 2358 k

Step 11 - Longitudinal Reinforcement

[5.8.3.5]

In addition to vertical reinforcement, shear requires a minimum amount of longitudinal reinforcement. The requirement for longitudinal reinforcement follows:

$$A_s f_y + A_{ps} f_{ps} \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

Considering only the prestressing steel yields the following:

$$(43.400)\cdot(252.53) \geq \frac{|34315\cdot 12|}{(47.16)\cdot(0.95)} + \left(\left|\frac{2358}{0.90} - 326\right| - 0.5\cdot(2010.)\right)\cot(36.8)$$

10,960 kips > 10,914 kips

$\therefore$ The prestressing strands are adequate for longitudinal reinforcement without additional mild reinforcing. Note that the Specification states that the area of longitudinal reinforcement need not be greater than the area required to resist the maximum moment alone.

A summary of critical shear design values at the tenth points of Span 2 is shown below:

	2.1	2.2	2.3	2.4	2.5
Vu	2200	1852	1497	1139	779
Vp	612	493	372	248	123
Vc	578	636	668	551	498
Reqd Spa	18.0	25.8	40.8	33.8	61.4
Max Spa	12.0	24.0	24.0	24.0	24.0
Spa Used	12.0	24.0	24.0	24.0	24.0
Vs	2050	1881	999	1059	745
φVn	2916	2709	1835	1671	1229

	2.6	2.7	2.8	2.9	d crit
Vu	431	776	1145	1530	1772
Vp	0.0	122	242	360	426
Vc	511	531	551	771	1269
Reqp Spa	100.0	79.3	43.8	47.2	99.9
Max Spa	24.0	24.0	24.0	24.0	24.0
Spa Used	24.0	24.0	24.0	12.0	12.0
Vs	854	920	1168	2238	2285
φVu	1228	1416	1764	3369	3980

[BDG]

Note that while the required spacing is low at the abutment, the stirrups are spaced at 12 inches. ADOT Guidelines require the 12 inch maximum spacing within 20 feet of the abutment diaphragms to allow for flaring of the tendons without the need to change the stirrup spacing.

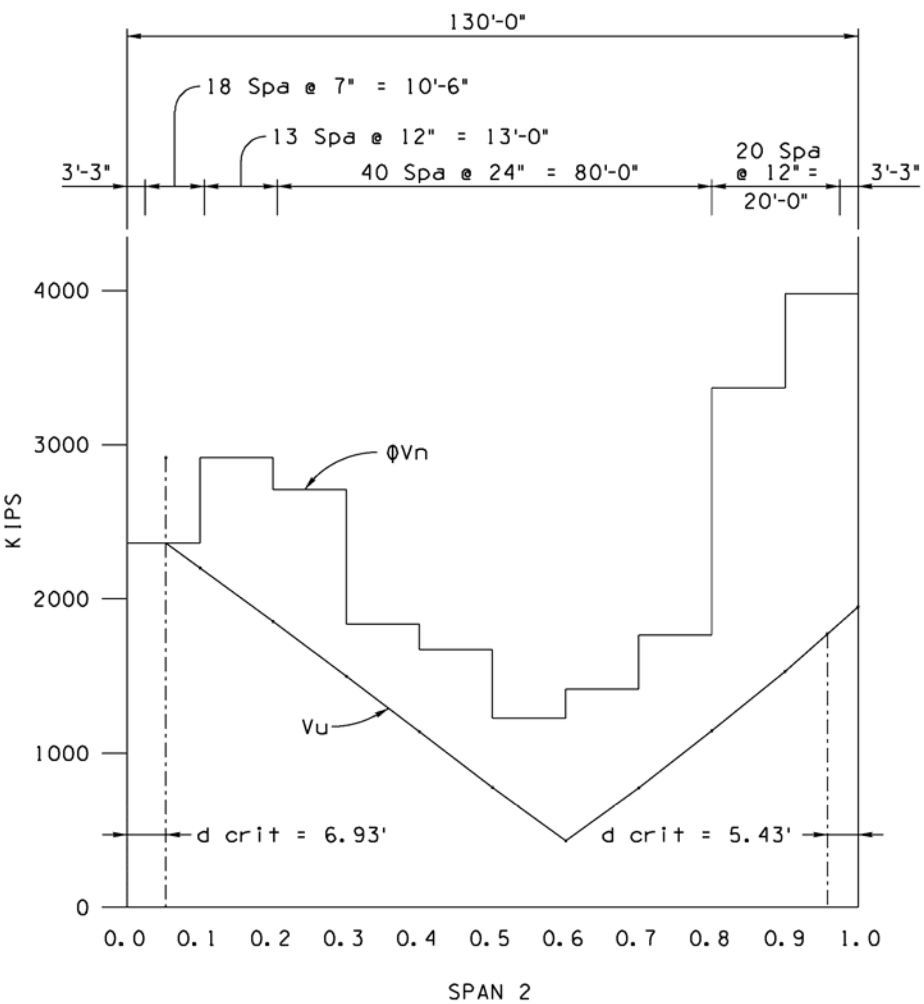


Figure 15

Interface Shear Transfer [5.8.4]

For cast-in-place box girder bridges, the deck is cast separately from the bottom slab and webs. Thus the shear transfer across this surface would appear to require investigation. In the past this was sometimes performed but was rarely a controlling criteria. The current specifications would appear to require analysis for interface shear across the horizontal joint at the top of the web since the concrete is poured across the joint at different times.

However, the method contained in the specification is more appropriate for precast girders with concrete decks poured after the member is erected. For a post-tensioned box girder bridge, the deck is poured prior to the prestressing. Once stressed the member acts as a unit with the vertical reinforcing providing adequate strength for horizontal shear. Application of the current specification to this problem resulted in a requirement for wider webs and additional vertical reinforcing.

In 2006 the Specification added a diagram and discussion concerning web-flange interfaces. This has traditionally not been a problem with the usual configuration of cast-in-place post-tensioned concrete box girder bridges used in Arizona. For single cell boxes or those with widely spaced webs the shear transfer mechanism should be investigated.

Based on the above discussion interface shear need not be checked for typical a cast-in-place post-tensioned concrete box girder bridge.

Post-Tensioned Anchor Zone [5.10.9] [5.10.9.3.1]

The design of anchor zone involves strength limit states involving factored jacking forces. Three design methods are provided in the LRFD Specification: strut-and-tie, refined elastic stress and approximate methods. The refined elastic stress method is very involved and not deemed appropriate for ordinary bridges. The approximate methods do not adequately consider the I-shape nature of the box and therefore may provide inaccurate answers. The strut-and-tie method will be used for analysis purposes.

Step 1 – Define Geometry

The first step in the analysis process is to define the geometry of the anchor zone. Figure 16 below shows a plan view of the end diaphragm while Figure 17 shows an elevation view.

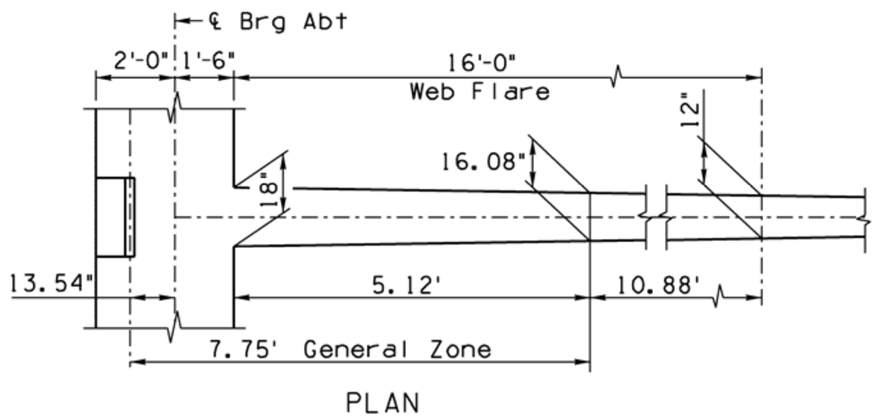


Figure 16 – Plan View Abutment Diaphragm

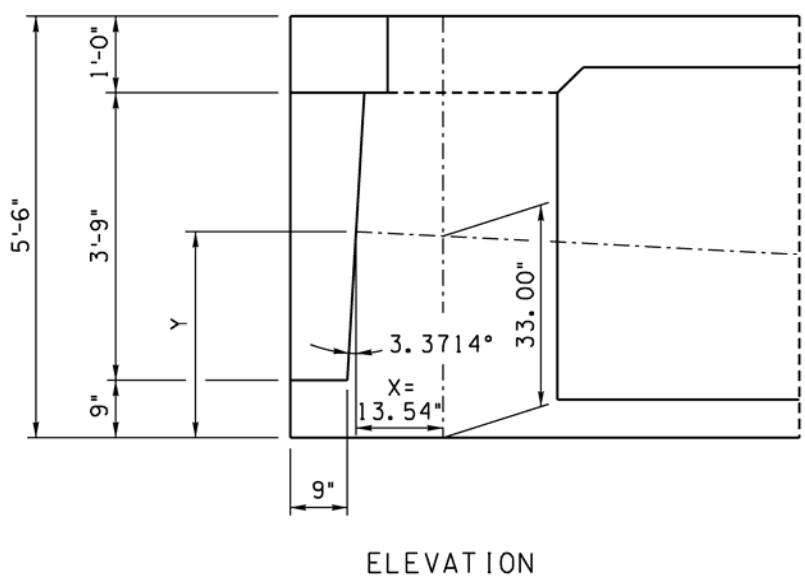


Figure 17 – Elevation View Abutment Diaphragm

The anchor zone design is based on the location of the actual anchorage devices. At the back face of the blockout, two equations exist for the location of the center of gravity of the strands:

$Y = 33.00 + X\tan\alpha$
$X = 24.00 – 9.00 – (Y - 9.00)\tan\alpha$

Solving for X yields: $x = \dfrac{15.00 - 24.00\tan(3.3714)}{1 + \tan^2(3.3714)} = 13.54 \text{ in}$

where α = 3.3714 degrees (Step 6).

Step 2 – Determine Anchorage Zone

The anchorage zone is geometrically defined as the volume of concrete through which the concentrated prestressing force at the anchorage device spreads to a more linear stress distribution across the entire cross-section at some distance from the anchorage device. Within this zone, the assumption that plane sections remain plane is not valid, requiring a different method of analysis. The anchorage zone may be taken as the maximum depth or width of the section but not larger than the longitudinal extent of the anchorage zone.

Step 3 – Determine Local Zone

[5.10.9.2.3]

The local zone is the rectangular prism of the concrete surrounding and immediately ahead of the anchorage device and any integral confining reinforcement. The local zone is the region of high compressive stresses immediately ahead of the anchorage device.

[5.10.9.7.1]

When the manufacturer has not provided a minimum edge distance as assumed for this problem, the transverse dimension in each direction shall be taken as the greater of:

1. The bearing plate size plus twice the minimum cover.
2. The outer dimension of any required confining reinforcement plus the required concrete cover.

[5.12.3-1]

Based on the flexural design, either 16 or 17 strands are required per duct based on usage of 0.6" diameter strands. From post-tensioning literature, the spirals for systems with 19-0.6" strands are 15 inches long with a 14.5 inch outside diameter. Adding two inch clearance to each side yields a local zone of 18.5 inches diameter. This produces an equivalent square of 16.40 inches.

The length of the local zone shall not be taken to be less than:

1. The maximum width of the local zone = 16.40"
2. The length of the anchorage device confining reinforcement = 15.00"

The length of the local zone shall not be taken as greater than 1.5 times the width of the local zone = 1.5(16.40) = 24.60". The length of the local zone should be greater than 16.40 inches and less than 24.60 inches. For this problem a length of 16.40 inches will be used.

Step 4 – Determine General Zone

[5.10.9.2.2]

The general zone extent is the same as the anchorage zone. The general zone is the region subjected to tensile stresses due to spreading of the tendon force into the structure and includes the local zone.

The minimum general zone length is the maximum of the width (7.75 feet) or depth (5.50 feet). The maximum general zone length equals 1.5 times this value. Use a general zone length of 7.75 feet.

Step 5 – Determine Section Properties

The section properties are required at the end of the anchorage zone to allow for the determination of the stresses. At this location the web is flared requiring that the dimension between the anchorages and the centerline bearing be known. Based on the previous calculations the anchorages can be assumed to be 13.54 inches (1.13 feet) behind the centerline. Based on an anchorage zone length of 7.75 feet, the width of the flared web can be determined at the end of the anchorage zone.

web = 12.00 + (18 - 12)(1.13 + 1.50 + 16.00 – 7.75)/(16) = 16.08 inches
$\sum$web = 16.08[2/cos(21.80) + 4] = 98.96 inches

For anchor zone design the ½ inch wearing surface has not been subtracted. The calculations for the section at the end of the anchor zone are not shown. A summary of the section properties follows:

Area	12,349	in^2
Inertia	7,160,048	in^4
y_b	36.831	in
y_t	29.169	in

Step 6 – Determine External Loads

[3.4.3.2]

For post-tensioning, a load factor of 1.2 is used. This is applied to the maximum stress in the strand that can be interpreted to be the jacking stress. The total jacking force is as follows:

$$P_u = (1.2)\cdot(0.77)\cdot(270)\cdot(200)\cdot(0.217) = 10{,}827 \text{ kips}$$

While the cable path follows a parabolic shape, in reality near the anchorage device, the path will be straight. The anchorage device and trumpets are straight and must be installed as such. The tangent segment length is assumed to be 14.13 feet for this problem placing the transition at a tenth point of the span. This will require that the tendon path be located on an angle from the horizontal as follows:

$$y_{b2.9} = 12 + [33.00 - 12.00] \cdot \left(\frac{39}{52}\right)^2 = 23.81 \text{ in}$$

$$\alpha = \tan^{-1}\left[\frac{33.00 - 23.81}{(12)\cdot(13.00)}\right] = 3.3714 \text{ degrees}$$

The total tendon force is divided into vertical and horizontal components as follows:

$$P_{uh} = (10827) \cdot \cos(3.3714) = 10808 \text{ kips}$$
$$P_{uv} = (10827) \cdot \sin(3.3714) = 637 \text{ kips}$$

This force is proportionally divided at the anchorage end based on the number of strands in each tendon.

<u>Top Tendon</u> (17 strands x 6 webs = 102 strands)
P_{uh} = (10808)(102) / (200) = 5512 kips
P_{uv} = (637)(102) / (200) = 325 kips

<u>Bottom Tendon</u> (16 strands x 4 webs plus 17 strands x 2 webs = 98 strands)
P_{uh} = (10808)(98) / (200) = 5296 kips
P_{uv} = (637)(98) / (200) = 312 kips

Step 7 – General Zone Stress Distribution

The stress at the end of the anchor zone is determined by classical methods. The stress on each structural shape is calculated to determine the forces acting on the various shapes. The eccentricity at the end of the anchor zone follows:

$e_{genzone}$ = 36.831 - [33.00 - (33.00 - 23.81)(7.75 - 1.13)/13] = 8.511 inch

The stress (ksi) acting on each interface is determined as follows:

Top	[10808][1/12349 - (8.511)(29.169)/(7160048)] = 0 50047
Soffit	[10808][1/12349 - (8.511)(21.169)/(7160048)] = 0.60325
Overhang	[10808][1/12349 - (8.511)(20.169)/(7160048)] = 0.61610
Fillet	[10808][1/12349 - (8.511)(17.169)/(7160048)] = 0.65464
Top Bot	[10808][1/12349 + (8.511)(30.831)/(7160048)] = 1.27131
Bottom	[10808][1/12349 + (8.511)(36.831)/(7160048)] = 1.34839

Step 8 – Determine Forces at End of Anchorage Zone

The stresses calculated in Step 7 must now be applied to the various shapes of the cross section to determine the magnitude of the force acting on each area and the location of the center gravity of the load. See Figure 18 for an idealized typical section and the stresses at the various levels. These forces are combined into three groups: top slab, web and bottom slab with the top fillets included in the web force.

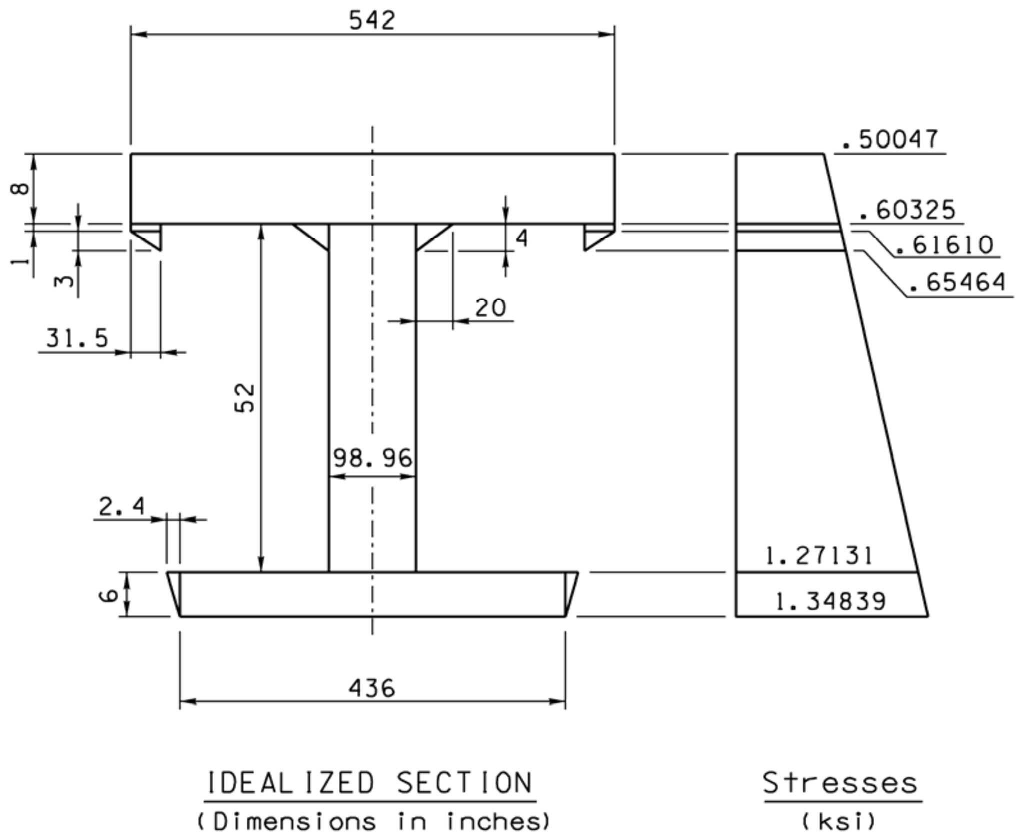


Figure 18

Determine the forces and center gravity resulting from the stress distribution acting on the smaller member shapes.

Top Slab

Force 1 = [0.50047](8.00)(542.00) = 2170.04 k

Force 2 = [0.60325 – 0.50047](8.00)(542.00) / 2= 222.83 k

CG = 66.00 - [(2170.04)(8.00/2) + (222.83)(8.00)(2/3)] / 2392.87
= 61.8758 in

Overhang

Force 3 = [0.60325](1.00)(63.00) = 38.00 k

Force 4 = [0.61610 – 0.60325](1.00)(63.00) / 2 = 0.40 k

c.g. = [38.00(1.00/2) + 0.40(1.00)(2/3)] / 38.40 = 0.5017 in

Exterior Fillets

Force 5 = [0.61610](3.00)(63.00) / 2 = 58.22 k

Force 6 = [0.65464 – 0.61610](3.00)(63.00) / 6 = 1.21 k

c.g. = [58.22(3.00/3) + 1.21(3.00/2)] / 59.43 = 1.0102 in

Interior Fillets

Force 7 = [0.60325](4.00)(40.00) / 2 = 48.26 k

Force 8 = [0.65464 – 0.60325](4.00)(40.00) / 6 = 1.37 k

c.g. = [48.26(4.00/3) + 1.37(4.00/2)] / 49.63 = 1.3517 in

Web

Force 9 = [0.60325](52.00)(98.96) = 3104.28 k

Force 10= [1.27131 – 0.60325](52.00)(98.96)/2 = 1718.89 k

c.g. = [3104.28(52.00/2) + 1718.89(52.00)(2/3)] / 4823.17 = 29.0886 in

Combination of Overhang, Fillets and Web

Force = 38.40 + 59.43 + 49.63 + 4823.17 = 4970.63 k

c.g. = [38.40(0.5017) + 59.43(1.00+1.0102) + 49.63(1.3517)
+ 4823.17(29.0886)] / 4970.63 = 28.2671 in

CG = 66.00 - 8.00 – 28.2671 = 29.7329 in

Bottom Slab

Force 11 = [1.27131](6.00)(436.00) = 3325.75 k
Force 12 = [1.34839-1.27131](6.00)(436.00) / 2 = 100.82 k
Force 13 = [1.27131](6.00)(4.80) / 2 = 18.31 k
Force 14 = [1.34839-1.27131](6.00)(4.80) / 6 = 0.37 k

CG = [(3325.75)(6.00/2) + (100.82)(6.00/3) + (18.31)(6.00)(2/3)
+ (0.37)(6.00/2)] / (3445.25) = 2.9761 in

The sum of the forces from all the members is 2392.87 + 4970.63 + 3445.25 = 10,808.75 kips compared to the 10,808 kips horizontally applied load. ok

Step 9 – Create Strut-and-Tie Model

Using the calculated center gravity as the y-coordinate, the strut-and-tie model can be created. Joints 1, 2 and 3 are located at the end of the Anchorage Zone.

Joint 5
y-Coord = 29.7329 + (93.000 - 33.000)(637) / 4971 = 37.4215

Due to the unequal number of strands in each tendon calculate the c.g. of the tendons spaced 18 inches apart at the anchorage
c.g. bottom = (18.00)(102) / (200) = 9.18 inches
c.g. top = 18.00 – 9.18 = 8.82 inches

Joint 7
y-Coord = 33.00 + (13.54)tan(3.3714) + (8.82)cos(3.3714) = 42.6024
x-Coord = (8.82)sin(3.3714) = 0.5187

Joint 8
y-Coord = 33.00 + (13.54)tan(3.3714) – (9.18)cos(3.3714) = 24.6335
x-Coord =-(9.18)sin(3.3714) = -0.5399

A summary of coordinates and applied forces follows:

Joint	x-Coord	y-Coord	Fx	Fy
1	93.0000	61.8758	-2393	
2	93.0000	29.7329	-4971	637
3	93.0000	2.9761	-3445	
4	33.0000	61.8758		
5	33.0000	37.4215		
6	33.0000	2.9761		
7	0.5187	42.6024	5512	-325
8	-0.5399	24.6335	5296	-312

A diagram showing the strut-and-tie model with the externally applied forces is shown in Figure 19.

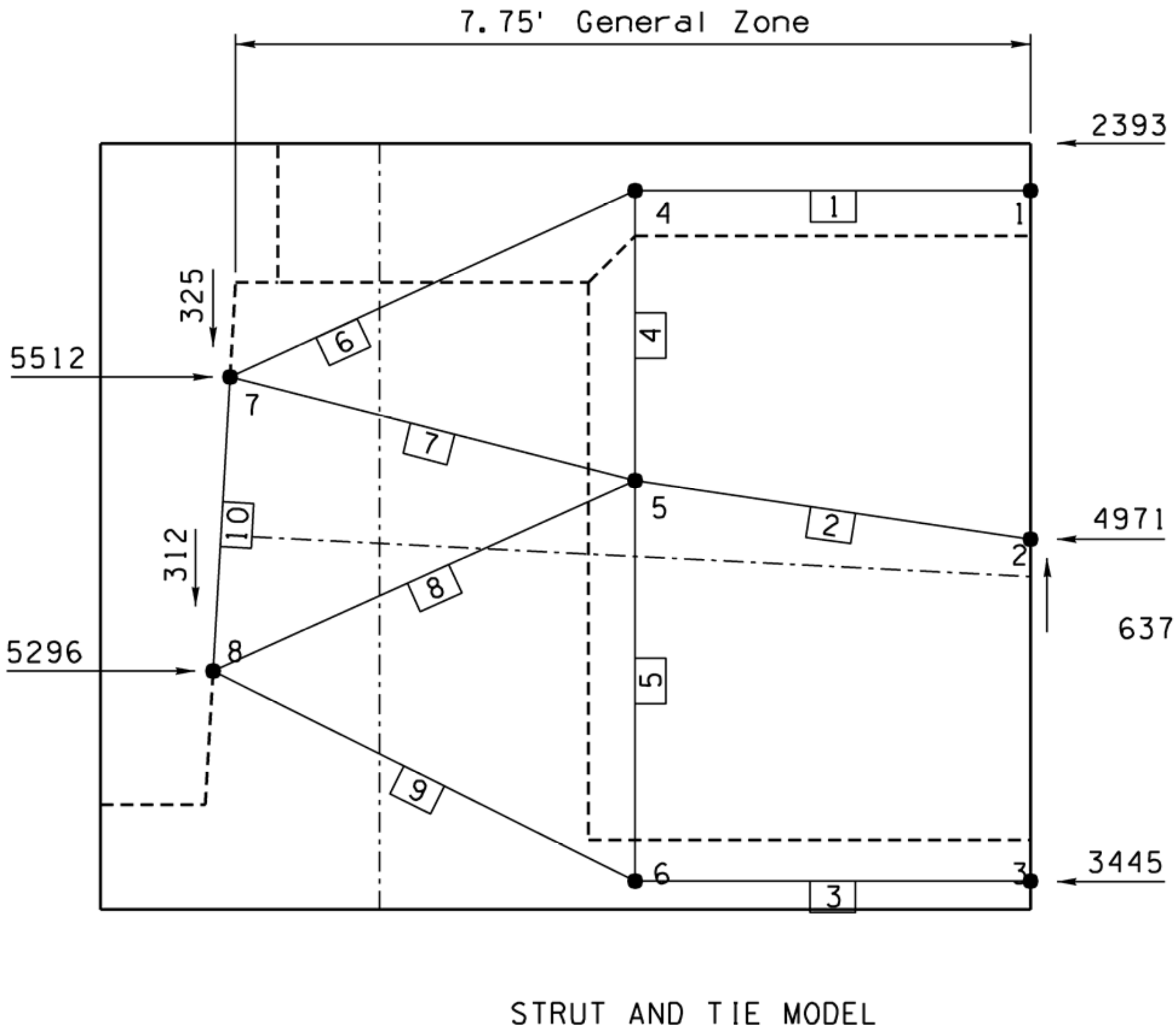


Figure 19 Strut-and-Tie Model

Step 10 – Solve for Member Forces

Member 1 and Member 3 carry the applied forces at the end of the anchorage zone. Member 2 has both a vertical and a horizontal component. The member forces are shown below:

F1 = 2393 kips
F2 = $[(4971)^2 + (637)^2]^{1/2}$ = 5012 kips
F3 = 3445 kips

The remainder of the member forces must be calculated by equating the sum of the forces at a node equal to zero in both the vertical and horizontal directions.

Node 4

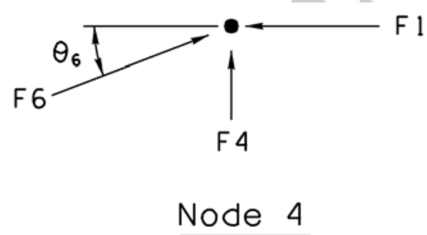


Figure 20

$\theta_6 = \tan^{-1}[(61.8758 - 42.6024) / (33.0000 - 0.5187)] = 30.6836$ degrees

$F6 = F1 / \cos\theta_6 = 2393 / \cos(30.6836) = 2783$ kips

$F4 = -F6 \sin\theta_6 = -2783 \sin(30.6836) = -1420$ kips

Node 6

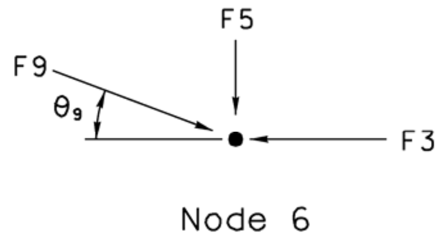


Figure 21

$\theta_9 = \tan^{-1}[(24.6335 - 2.9761) / (33.0000 + 0.5399)] = 32.8512$ degrees

$F9 = F3 / \cos\theta_9 = 3445 / \cos(32.8512) = 4101$ kips

$F5 = -F9 \sin\theta_9 = -4101 \sin(32.8512) = -2225$ kips

Node 7

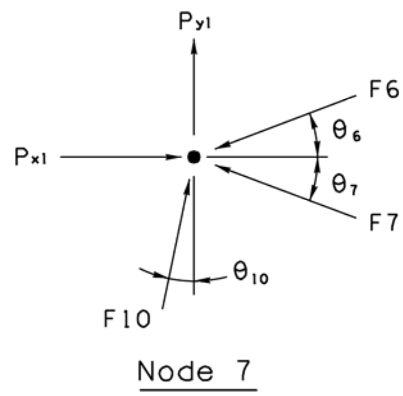


Figure 22

P_{x1} = 5512 k
P_{y1} = -325 k
F6 = 2783 k

θ_6 = 30.6836 degrees
$\theta_7 = \tan^{-1}[(42.6024 - 37.4215) / (33.0000 - 0.5187)]$ = 9.0626 degrees
θ_{10} = 3.3714 degrees

Sum Forces in x-direction

$P_{x1} - F6 \cos\theta_6 - F7 \cos\theta_7 + F10 \sin\theta_{10} = 0$

Sum Forces in y-direction

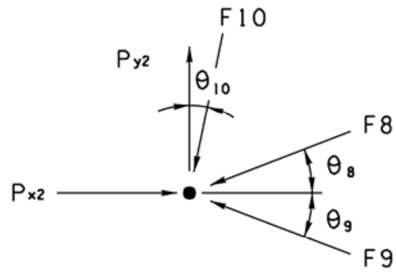
$P_{y1} - F6 \sin\theta_6 + F7 \sin\theta_7 + F10 \cos\theta_{10} = 0$

Solve the second equation for F10 and substitute into the first equation solving for F7:

$$F7 = \frac{P_{x1} - F6(\cos\theta_6 - \sin\theta_6 \tan\theta_{10}) - P_{y1} \tan\theta_{10}}{\cos\theta_7 + \sin\theta_7 \tan\theta_{10}} = 3232 \text{ k}$$

$$F10 = \frac{-P_{y1} + F6 \sin\theta_6 - F7 \sin\theta_7}{\cos\theta_{10}} = 1238 \text{ k}$$

Node 8



Node 8
Figure 23

P_{x2} = 5296 k
P_{y2} = -312 k
F9 = 4101 k

$\theta_8 = \tan^{-1}[(37.4215 - 24.6335) / (33.0000 + 0.5399)] = 20.8707$ degrees
θ_9 = 32.8512 degrees
θ_{10} = 3.3714 degrees

Sum Forces in x-direction

$P_{x2} - F8 \cos\theta_8 - F9 \cos\theta_9 - F10 \sin\theta_{10} = 0$

Sum Forces in y-direction

$P_{y2} - F8 \sin\theta_8 + F9 \sin\theta_9 - F10 \cos\theta_{10} = 0$

Solve for F10 in the second equation and substitute into the first equation to solve for F8:

$$F8 = \frac{P_{x2} - P_{y2} \tan\theta_{10} - F9(\cos\theta_9 + \sin\theta_9 \tan\theta_{10})}{\cos\theta_8 - \sin\theta_8 \tan\theta_{10}} = 1903 \text{ k}$$

$$F10 = \frac{P_{y2} - F8\sin\theta_8 + F9\sin\theta_9}{\cos\theta_{10}} = 1237 \text{ k}$$

Note that the above calculated value of F10 = 1237 k is approximately equal to the previously calculated value of F10 = 1238 k.

Node 5

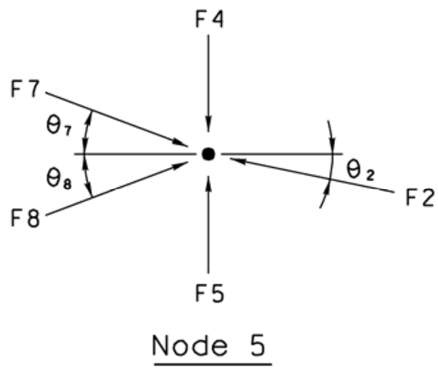


Figure 24

F2 = 5012 k
F4 = -1420 k
F5 = -2225 k
F7 = 3232 k
F8 = 1903 k

$\theta_2 = \tan^{-1}[(37.4215 - 29.7329) / (93.0000 - 33.0000)] = 7.3023$ degrees
$\theta_7 = 9.0626$ degrees
$\theta_8 = 20.8707$ degrees

Sum Forces in x-direction for static check

$$\sum F_x = F7 \cos\theta_7 + F8 \cos\theta_8 - F2 \cos\theta_2 = 0$$

$$\sum F_x = (3232)\cos(9.0626) + (1903)\cos(20.8707) - (5012)\cos(7.3023) = -1.56 \text{ k} \approx 0$$

Sum Forces in y-direction for statics check

$$\sum F_y = -F7 \sin\theta_7 + F8 \sin\theta_8 + F2 \sin\theta_2 - F4 + F5 = 0$$

$$\sum F_y = -(3232)\sin(9.0626) + (1903)\sin(20.8707) + (5012)\sin(7.3023) - (-1420) + (-2225) = 0.93 \text{ k} \approx 0$$

Since the static check produces very good results, the model is acceptable.

The greatest discrepancy is with the horizontal forces. However, due to rounding the applied loads on the left side equal 5512 + 5296 = 10,808 kips while the applied loads on the right side equal 2393 + 4971 + 3445 = 10,809 kips accounting for the majority of the error.

Step 11 – Web Bursting Design

Determine the maximum vertical tensile force in the web. Member 5 has the largest tensile force of –2225 kips. Divide this force by the number of webs to obtain a force of –370.83 kips per web. For tension in steel in anchorage zones use φ = 1.0. Determine the required area of reinforcement.

[5.5.4.2.1]

$$A_s = \frac{F_{\max}}{\phi f_y} = \frac{370.83}{(1.00)\cdot(60)} = 6.18 \text{ in}^2$$

Try 7 - #6 stirrups. $A_s = (0.44)(2)(7) = 6.16 \text{ in}^2$. Center these stirrups about the tie (Member 4 and 5) in the strut-and-tie model. The tie is located 33 inches from the anchorage or 33.00 – 13.54 = 19.46 inches from the centerline of bearing. Space the bursting stirrups at 5 inch spacing about the tie. This results in the first stirrup being 19.46 – 3(5.00) = 4.46 inches from the centerline of bearing or about 13.54 + 4.46 – 16.40 = 1.60 inches from the end of the local zone.

See Figure 32 for reinforcement placement.

Step 12 – Spalling Reinforcing

[5.10.9.3.2]

For multiple anchorages with a center-to-center spacing of less than 0.4 times the depth of the section, the spalling force shall not be taken to be less than 2 percent of the total factored tendon force. Since the strut-and-tie analysis did not reveal any tension between anchorages and our spacing of 18 inches is less than 0.4(66) = 26.4 inches, use the 2 percent criteria.

Spalling Force = 0.02(10,827) / (6 webs) = 36.09 kips per web.

$$A_s = \frac{T}{\varphi f_y} = \frac{36.09}{(1.00)\cdot(60)} = 0.602 \text{ in}^2$$

Use 2 - #5 rebar per web for spalling, yielding an $A_s = 2(0.31) = 0.62 \text{ in}^2$.

See Figure 32 for reinforcement placement.

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Step 13 – Concrete Stresses

Concrete stresses in the local zone can be very high. The use of spiral reinforcement increases the allowable concrete stress in the local region with the designs verified by testing. The responsibility of this region is given to the post-tensioning device supplier.

However, at the local zone/general zone interface the concrete stresses must be checked. From the anchorage head to the interface the stresses spread on a 1:3 angle. For a local zone 16.40 x 16.40 inches, the width of the interface is 16.40 + 2(16.40)/3 = 27.33 inches. The height equals the spacing plus the spread = 18.00 + 27.33 = 45.33 inches. The force per web equals 10,827 / 6 = 1805 kips

f_{ci} = [1805] / [(27.33) (45.33)] = 1.457 ksi

[5.5.4.2.1]

φ = 0.80

Allowable stress = 0.7 φ f'_{ci} = 0.7 (0.80) (3.5) = 1.960 ksi

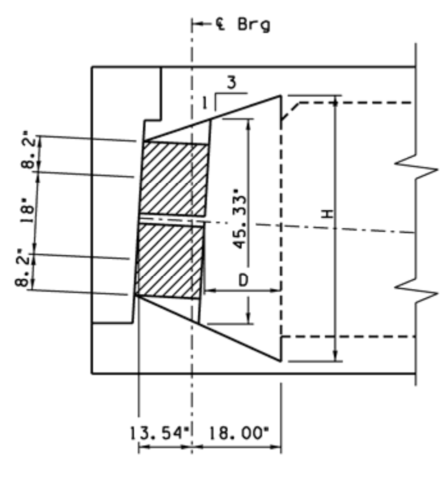
Discontinuity

The concrete must also be checked at the end of the diaphragm where the width of the spread is limited to the width of the web member. The distance between the local zone and the diaphragm equals:

D = 13.54 + 18.00 – 16.40 = 15.14 inches
H = [45.33 + 2(15.14) / 3] = 55.42 inches

f_{ci} = [1805] / [(18.00) (55.42)] = 1.809 ksi < 1.960 ksi

The limits on the initial concrete strength are satisfied.



LOCAL ZONE/GENERAL ZONE INTERFACE

Figure 25

Step 14 – Top Slab Analysis

The top slab must also disperse the concentrated forces from the webs to the entire width of the slab. A strut-and-tie model (Figure 27) was created with one set of nodes at the web top slab interface 33.00 inches from the anchors and the other set half the web spacing away. At the end of the general zone the stresses are uniformly distributed with nodes placed between the webs or the exterior web and the edge of deck. The force applied at each web equals the force in the top slab divided by the number of webs. The uniform load equals the top slab force divided by the web width.

Pweb = 2393 / 6 = 398.83 kips
Uniform = 2393 / 542.00 = 4.4151 kips/inch

Coordinate geometry of the top slab is shown below:

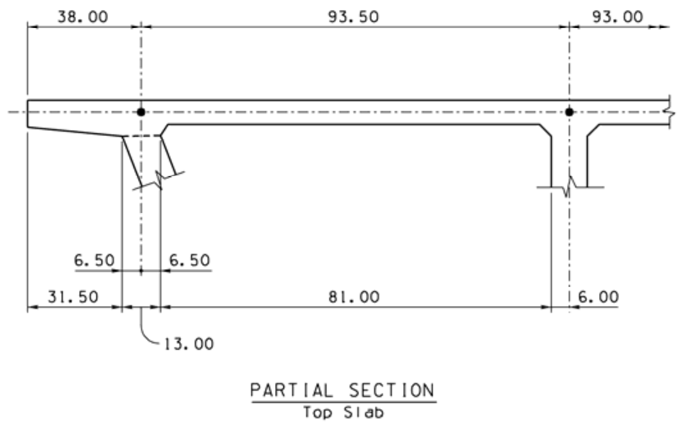
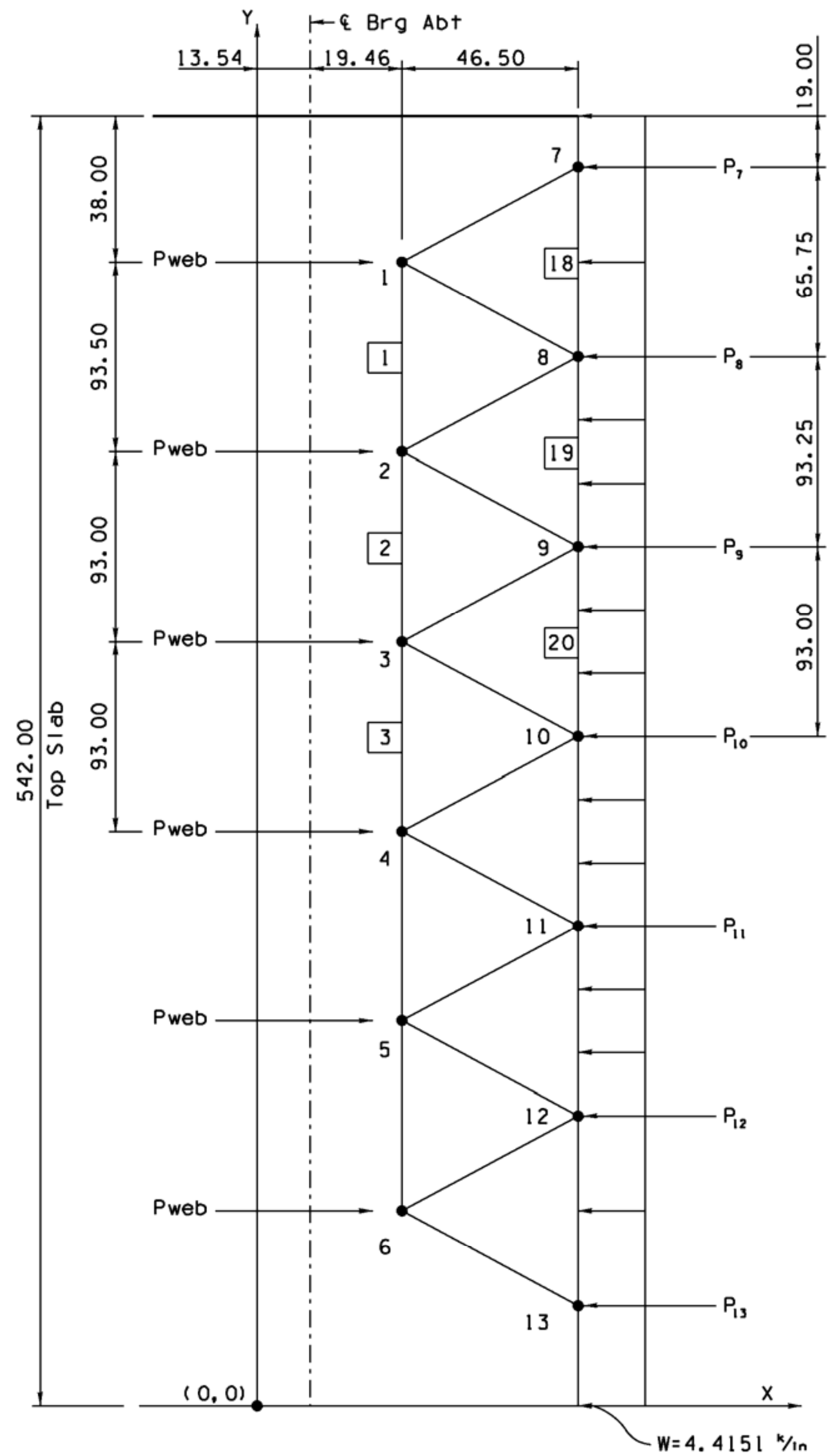


Figure 26

Joint coordinates and applied forces for the model are shown below:

Joint	x-Coord	y-Coord	Joint Force
1	33.00	504.00	398.83
2	33.00	410.50	398.83
3	33.00	317.50	398.83
4	33.00	224.50	398.83
5	33.00	131.50	398.83
6	33.00	38.00	398.83
7	79.50	523.00	-167.77
8	79.50	457.25	-412.81
9	79.50	364.00	-410.60
10	79.50	271.00	-410.60
11	79.50	178.00	-410.60
12	79.50	84.75	-412.81
13	79.50	19.00	-167.77



PLAN-TOP SLAB
Strut And Tie Model
Figure 27

The forces applied at the joints due to the uniformly distributed force in the top slab equals the uniform load multiplied by the contributing area as follows:

P7 = P13 = (4.4151)(38.00) = 167.77 k
P8 = P12 = (4.4151)(93.50) = 412.81 k
P9 = P10 = P11 = (4.4151)(93.00) = 410.60 k

The complete analysis of the strut-and-tie model is not shown here. The forces in each member can be determined by calculating the angles of the members and summing the forces in both the x and y directions at each node to determine the member forces. A simple method to obtain the tension tie forces is to cut a section through a joint and sum moments dividing by the distance between the nodes. This method will be demonstrated on the following pages.

<u>First tie</u>
Sum forces about Joint 8:

Sum M Jt8: P_{web} (Y1 – Y8) – P_7(Y7-Y8) + F_1d = 0
F_1 = [P_7(Y7 – Y8) – P_{web}(Y1 – Y8)] / d

F1 = [(167.77)(523.00 - 457.25) - (398.83)(504.00 - 457.25)] / 46.50
= -163.75 k

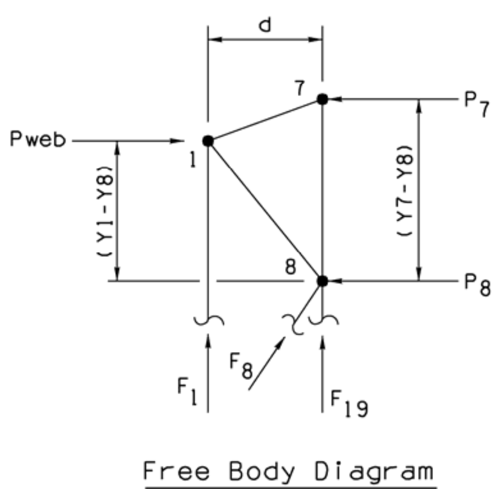


Figure 28

Sum forces about Joint 9:
F2 = [(167.77)(523.00 - 364.00) + (412.81)(457.25 - 364.00) – (398.83)(504.00 - 364.00) – (398.83)(410.50 - 364.00)] / 46.50
= -198.10 k

Sum forces about Joint 10:
F3 = [(167.77)(523.00 - 271.00) + (412.81)(457.25 - 271.00) + (410.60)(364.00 - 271.00) – (398.83)(504.00 - 271.00) – (398.83)(410.50 - 271.00) – (398.83)(317.50 - 271.00)] / 46.50
= -209.89 k

Second Tie

Sum forces about Joint 1:

Sum M Jt 1: $P_7(Y7 - Y1) + F_{18}d = 0$

$F_{18} = -[P_7(Y7 - Y1)] / d$

F18 = [-(167.77)(523.00 – 504.00)] / 46.50 = -68.55 k

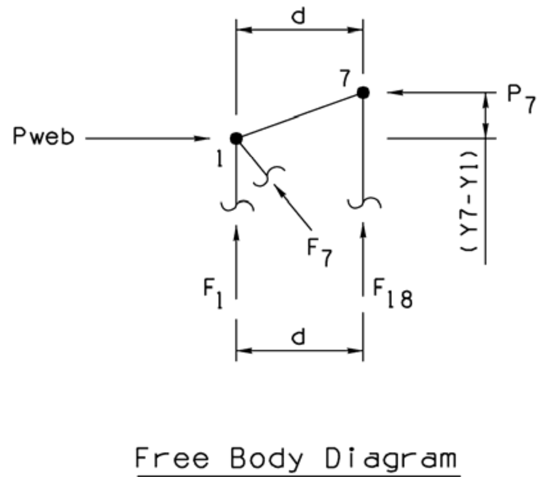


Figure 29

Sum forces about Joint 2:

F19 = [-(167.77)(523.00 – 410.50) - (412.81)(457.25 – 410.50) + (398.83)(504.00 – 410.50)] / 46.50 = -18.98 k

Sum forces about Joint 3:

F20 = [-(167.77)(523.00 – 317.50) - (412.81)(457.25 – 317.50) – (410.60)(364.00 – 317.50) + (398.83)(504.00 – 317.50) + (398.83)(410.50 – 317.50)] / 46.50 = 4.58 k

The first tie consists of forces F1, F2 and F3, while the second tie consists of forces F18, F19 and F20. Both ties have tension forces with the required tensile reinforcement as follows:

First tie $A_s = 209.89 / [(1.00)(60)] = 3.50 \text{ in}^2$

Use 6 - #7 at 7 inches ($A_s = 3.60 \text{ in}^2$)

Second tie $A_s = 68.55 / [(1.00)(60)] = 1.14 \text{ in}^2$

Use 4 - #5 at 7 inches ($A_s = 1.24 \text{ in}^2$)

See Figure 32 for reinforcing placement.

Step 15 – Bottom Slab Analysis

The bottom slab must also disperse the concentrated forces from the webs to the entire width of the slab. A strut-and-tie model (Figure 31) was created with one set of nodes at the web bottom slab interface 33 inches from the anchors and the other set half the web spacing (46.50 inches) away. At the end of the general zone the stresses can be uniformly distributed with nodes placed between the webs or between the exterior web and the edge of deck.

However, for the bottom slab with the sloping exterior web and no bottom cantilever, the assumption of a uniformly distributed stress in the bottom slab is not reasonable. A better assumption is that the two exterior webs will be distributed from the edge of the slab for a distance midway between the second and third webs. The force applied at each web equals the force in the bottom slab divided by the number of webs. The force applied at the other joints equals the uniform load multiplied by the appropriate distance. Due to the sloping face the bottom slab is assumed to be a rectangle with a width of 436.00 + 2.40 = 438.40 inches. See Figure 30.

Pweb = 3445 / 6 = 574.17 kips

Exterior Width = 1.20 + 78.50 + 93.00 / 2 = 126.20 inches
Exterior Uniform = (2)(574.17) / 126.20 = 9.0994 kips/inch

Interior Uniform = (574.17) / 93.00 = 6.1739 kips/inch

The forces applied to the joints due to the uniformly distributed force in the bottom slab equals the uniform load multiplied by the corresponding area as follows:

P7 = P11 = (9.0994)(79.70) = -725.22 k
P8 = P10 = (9.0994)(93.00 / 2) + (6.1739)(93.00 / 2) = -710.21 k
P9 = (6.1739)(93.00) = 574.17 k

Calculations for the y-coordinates for the two exterior webs are shown below:

y-Coordinate
Jt. 7: y = 542.00 – 53.00 + 1.20 – 79.70 / 2 = 450.35
Jt. 11: y = 53.00 – 1.20 + 79.70 / 2 = 91.65

To maintain equilibrium the first interior joint must be located at the center of gravity of the assumed load. Summing moments about Joint 3 yields:

H = [9.0994(46.50)(69.75) + 6.1739(46.50)(23.25)] / 710.21 = 50.95
Jt. 8: y = 317.50 + 50.95 = 368.45
Jt 10: y = 224.50 – 50.95 = 173.55

Using the wider web at the end of the general zone is conservative but helpful. The diagram used to determine the coordinates for the exterior webs is as follows:

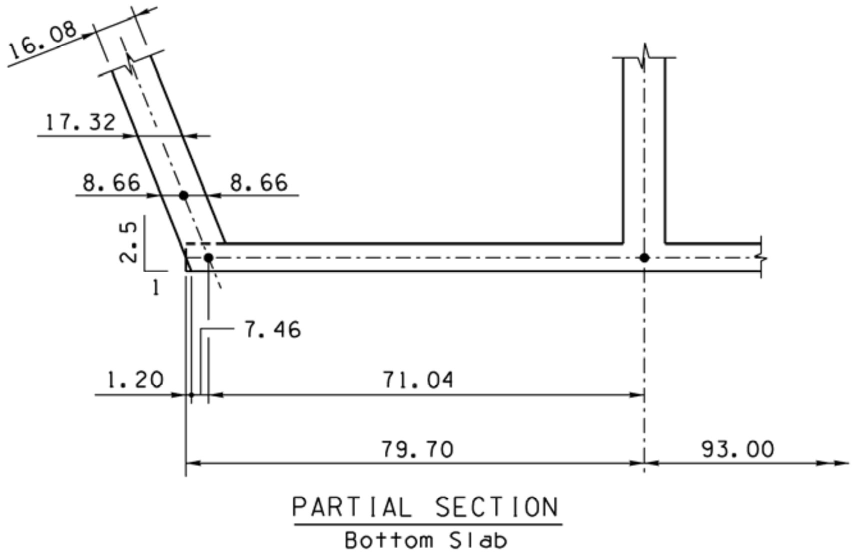


Figure 30

Joint coordinates and applied forces for the model are shown below:

Joint	x-Coord	y-Coord	Member Force
1	33.00	481.54	574.17
2	33.00	410.50	574.17
3	33.00	317.50	574.17
4	33.00	224.50	574.17
5	33.00	131.50	574.17
6	33.00	60.46	574.17
7	79.50	450.35	-725.22
8	79.50	368.45	-710.21
9	79.50	271.00	-574.17
10	79.50	173.55	-710.21
11	79.50	91.65	-725.22

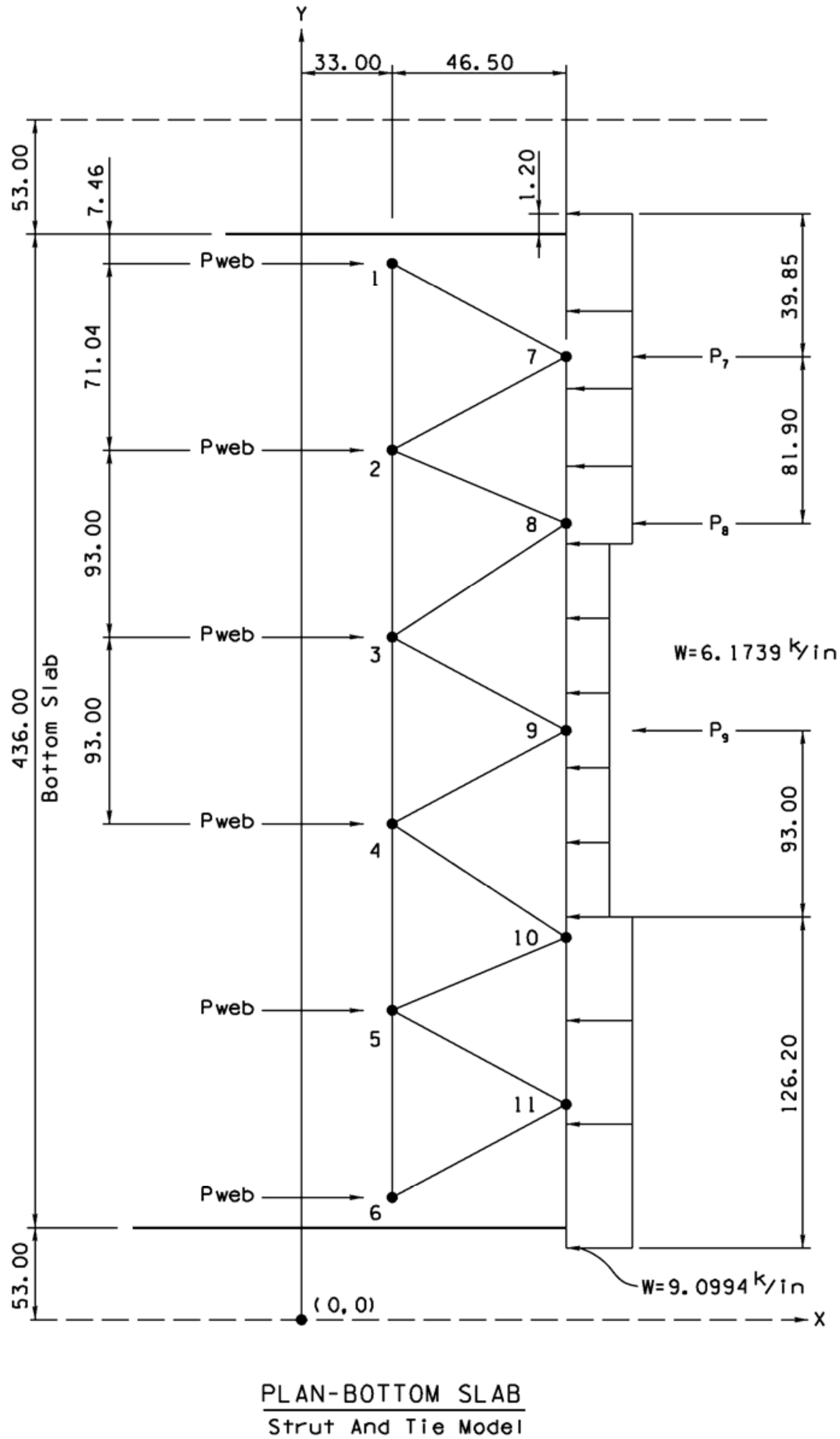

Y
33.00
46.50
53.00
7.46
1.20
Pweb
1
7
39.85
P7
71.04
Pweb
2
81.90
8
P8
93.00
Pweb
3
W=6.1739 k/in
9
P9
436.00
Bottom Slab
93.00
Pweb
4
93.00
10
Pweb
5
126.20
11
Pweb
6
53.00
(0,0)
W=9.0994 k/in
X
PLAN-BOTTOM SLAB
Strut And Tie Model

Figure 31

The complete analysis of the strut-and-tie model is not shown here. The results can be determined by calculating the angles of the members and summing the forces in both the x and y directions at each node to determine the member forces. A simple method to obtain the tension tie forces is to cut a section through a joint and sum moments dividing by the x-distance between the nodes. This method will be demonstrated below:

First tie

Sum forces about Joint 7:

F1 = [-(574.17)(481.54 - 450.35)] / 46.50 = -385.13 k

Sum forces about Joint 8:

F2 = [-(574.17)(481.54 - 368.45) - (574.17)(410.50 - 368.45) + (725.22)(450.35 - 368.45)] / 46.50 = -638.31 k

Sum forces about Joint 9:

F3 = [-(574.17)(481.54 - 271.00) - (574.17)(410.50 - 271.00) – (574.17)(317.50 - 271.00) + (725.22)(450.35 - 271.00) + (710.21)(368.45 - 271.00)] / 46.50 = -610.82 k

Second Tie

Sum forces about Joint 2:

F16 = [(574.17)(481.54 – 410.50) - (725.22)(450.35 - 410.50)] / 46.50 = 255.68 k

Sum forces about Joint 3

F17 = [(574.17)(481.54 – 317.50) + (574.17)(410.50 – 317.50) – (725.22)(450.35 – 317.50) - (710.21)(368.45 - 317.50)] / 46.50 = 323.74 k

Only the first tie has tension forces. Calculate the required tensile reinforcement for this tie:

A_s = 638.31 / [(1.00)(60)] = 10.64 in^2

Use 6 - #9 bundles at 8 inches (A_s = 12.00 in^2). Space the bars symmetrically about the center of the tie located 33 inches from the anchorage plates or 33.00 – 13.54 = 19.46 inches from the centerline of bearing of the abutment.

See Figure 32 for reinforcing placement.

A summary of reinforcing required for the anchor zone is shown below:

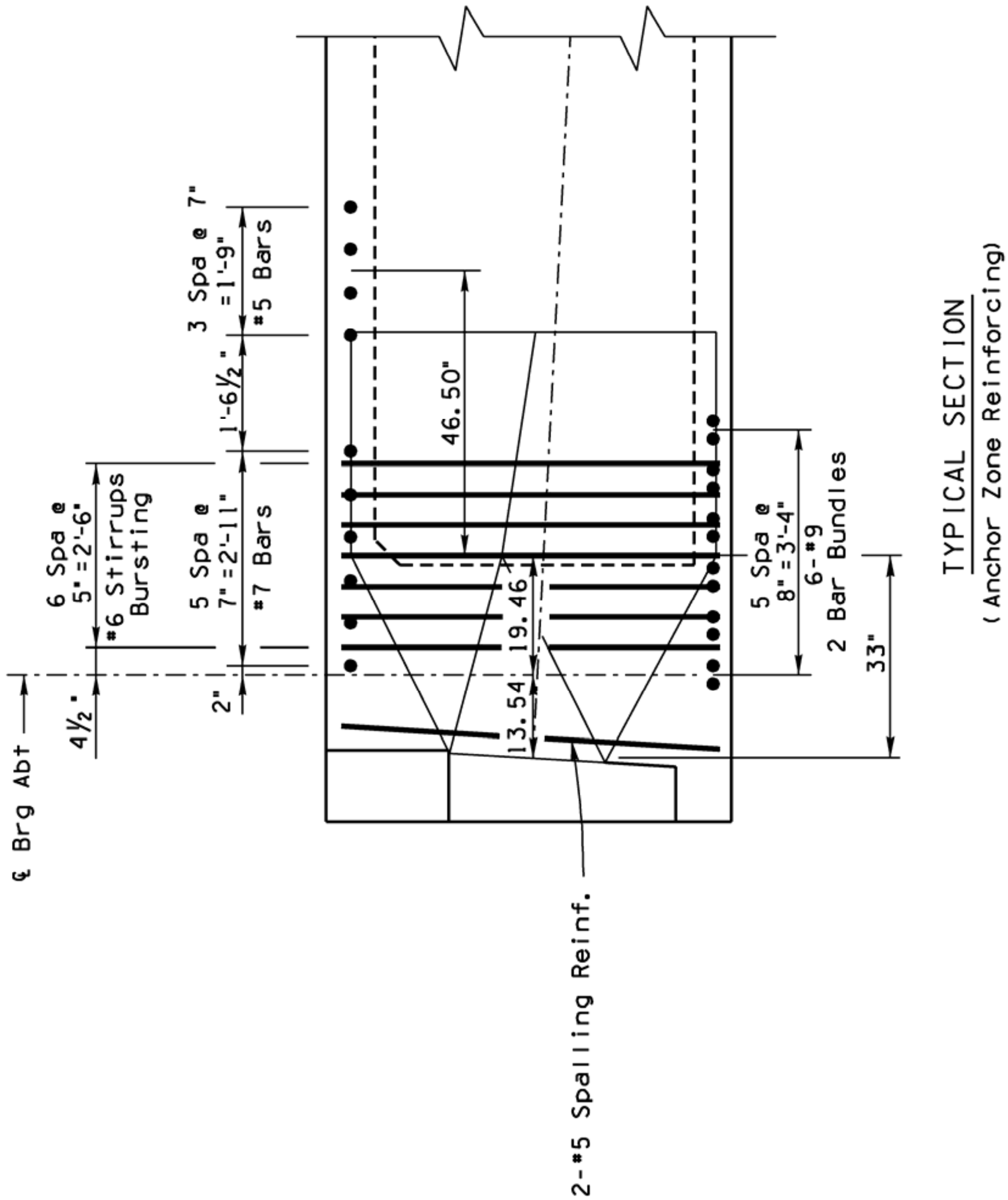


Figure 32

Deflections
[5.7.3.6]

Deflections must be calculated so the superstructure can be cambered to provide for a smooth riding surface.

Deflections can be calculated for dead load using the previously calculated modulus of elasticity based on the 28-day concrete strength, E_c = 3861 ksi.

[5.7.3.6.2]
[BDG]

The requirements for determination of moment of inertia and corresponding creep factor are the same as taken from the 17th Edition of the Standard Specifications for reinforced concrete. The LRFD Specification is based on the unified theory of concrete where the design requirements for reinforced and prestressed concrete have been combined under one section. However, the formula for determining an effective moment of inertia is not appropriate for fully prestressed members that never crack under full dead load and final prestress loss. Therefore, using gross section properties and a creep factor of 2 is recommended. The final deflection is then the instantaneous deflection multiplied by 3.

A summary of dead load and prestressed deflections shown in feet at the maximum points in each span follows. In an actual design the deflections would be determined at tenth points to provide a smooth camber.

	0.4 Span 1	0.6 Span 2
Superstructure	0.063	0.115
Barriers	0.006	0.010
Final Prestress	-0.053	-0.093

The resulting camber at the critical locations follows:

0.4 Span 1
Camber = (3.0)(0.063 + 0.006 – 0.053) = 0.048 feet

0.6 Span 2
Camber = (3.0)(0.115 + 0.010 – 0.093) = 0.096 feet

[5.7.3.6.2]
[BDG]

If the span arrangement had produced a less favorable ratio of short to long span, the shorter span deflection could have been negative or upwards. This would require that the bridge be constructed lower than the profile grade raising the issue of how much creep to apply. If the bridge is constructed low and the total creep does not occur, the bridge will always have the undesirable dip. To eliminate this condition raising the cable path in the short span should be investigated in an attempt to eliminate the upward growth. When negative camber results, the creep portion of the deflection should be ignored. For small values of upward deflection, the designer may desire to ignore the deflection altogether.

Appendix A
to
LRFD Example 2

Precise Overhang Design

A simplified method of determining the adequacy of an overhang subjected to both tension and flexure is included in the example. This appendix shows the more complex and precise method along with the assumptions made to derive the approximate simplified method.

The solution of the problem involves determining the resistance of the deck overhang to a combination of tension and flexure. Members subjected to eccentric tension loading, which induces both tensile and compressive stresses in the cross section, shall be proportioned in accordance with the provisions of Article 5.7.2.

Tension and Flexure [5.7.6.2]

[5.7.2]

Assumptions for a valid analysis for an extreme event limit state are contained in Article 5.7.2. Factored resistance of concrete components shall be based on the conditions of equilibrium and strain compatibility and the following:

Strain is directly proportional to the distance from the neutral axis.

For unconfined concrete, the maximum usable strain at the extreme concrete compressive fiber is not greater than 0.003.

The stress in the reinforcement is based on a stress-strain curve of the steel or on an approved mathematical representation.

Tensile strength of the concrete is ignored.

The concrete compressive stress-strain distribution is assumed to be a rectangular stress block in accordance with Article 5.7.2.2.

The development of the reinforcing is considered.

While the article specifies the use of the reduction factors in Article 5.5.4.2, that requirement only applies to a strength limit state analysis. For an extreme event limit state, the resistance factor shall be taken as 1.0.

[1.3.2.1]

The above assumptions as shown in Figures A-1, A-2 and A-3 were used in the development of the equations for resistance from tension and flexure that occur with a vehicular collision with a traffic railing.

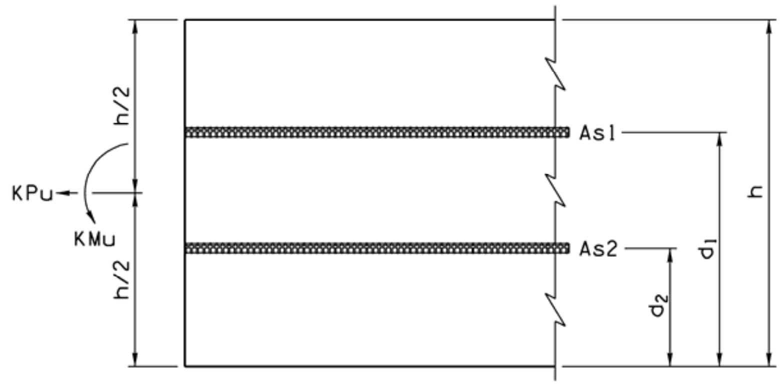


Figure A-1

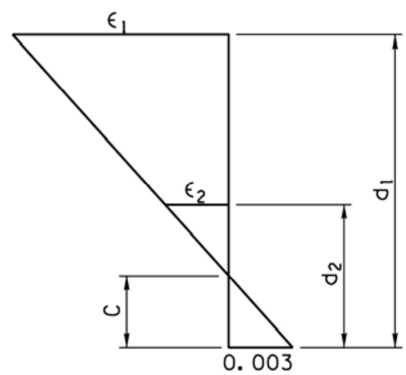


Figure A-2

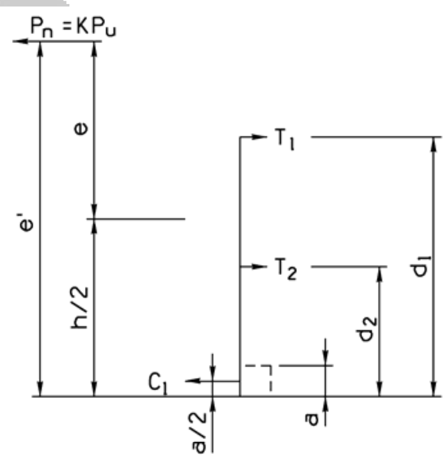


Figure A-3

The design of the deck overhang is complicated because both a bending moment and a tension force are applied. The problem can be solved using equilibrium and strain compatibility. The following trial and error approach may be used:

1. Assume a stress in the reinforcing
2. Determine force in reinforcing
3. Solve for k, the safety factor
4. Determine values for 'a' and 'c'
5. Determine corresponding strain
6. Determine stress in the reinforcing
7. Compare to assumed value and repeat if necessary

Face of Barrier
Location 1
Figure 4

The design horizontal force in the barrier is distributed over the length L_b equal to L_c plus twice the height of the barrier. See Figures 5 and 6.

L_b = 17.10 + 2(3.50) = 24.10 ft
P_u = 99.25 / 24.10 = 4.118 k/ft < 6.063 k/ft per connection strength

Dimensions

h = 9.00 + (3.00) (1.583) / (2.625) = 10.81 in
d_1 = 10.81 – 2.50 clr – 0.625 / 2 = 8.00 in
d_2 = 10.81 – 8.00 + 1.00 clr + 0.625 / 2 = 4.12 in

Moment at Face of Barrier

Deck = $0.150(9.00 / 12)(1.58)^2 \div 2$ = 0.14 ft-k
$0.150(1.81 / 12)(1.58)^2 \div 6$ = <u>0.01 ft-k</u>
= 0.15 ft-k

Barrier = 0.538(0.946) = 0.51 ft-k

Collision = 4.118[3.50 + (10.81/12) / 2] = 16.27 ft-k

[A13.4.1]

The load factor for dead load shall be taken as 1.0.

Extreme Event II
[3.4.1]

M_u = 1.00(0.15 + 0.51) + 1.00(16.27) = 16.93 ft-k

e = M_u / P_u = (16.93)(12) / (4.118) = 49.33 in

1. Assume Stress

Assume both layers of reinforcing yield and f_s = 60 ksi

2. Determine Forces

Determine resulting forces in the reinforcing:

T_1 = (0.676)(60)
= 40.56 k
T_2 = (0.531)(60)
= 31.86 k

3. Determine k
Safety Factor

Strength Equation

Solve the equations of equilibrium by summing the forces on the section and summing the moments about the soffit and setting them equal to zero. This yields the following two equations. See Figure A-3.

Sum forces in horizontal direction

Eqn 1: $-kP_u + T_1 + T_2 - C_1 = 0$ where $C_1 = 0.85f'_c ab$

Sum of moments

Eqn 2: $kP_u(e') - T_1(d_1) - T_2(d_2) + C_1(a/2) = 0$

Solving the above equations for k, the ratio of strength to applied force and moment, results in a quadratic equation with the following coefficients:

$$A = \frac{P_u^{\ 2}}{1.70 f'_c\, b}$$

$$B = P_u\left(e + \frac{h}{2} - \frac{T_1 + T_2}{0.85 f'_c\, b}\right)$$

$$C = -T_1 d_1 - T_2 d_2 + \frac{(T_1 + T_2)^2}{1.70 f'_c\, b}$$

Solution

Substituting in specific values yields:

$$A = \frac{(4.118)^2}{1.70 \cdot (4.5) \cdot (12)} = 0.184727$$

$$B = (4.118) \cdot \left(49.33 + \frac{10.81}{2} - \frac{(40.56 + 31.86)}{0.85 \cdot (4.5) \cdot (12)}\right) = 218.9014$$

$$C = -(40.56) \cdot (8.00) - (31.86) \cdot (4.12) + \frac{(40.56 + 31.86)^2}{1.70 \cdot (4.5) \cdot (12)} = -398.6119$$

Solution of the quadratic equation yields the value k, the safety factor.

$$k = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

$$k = \frac{-218.9014 + \sqrt{(218.9014)^2 - 4 \cdot (0.184727) \cdot (-398.6119)}}{2 \cdot (0.184727)} = 1.818$$

Since the value of k is greater than one the deck is adequately reinforced at this location.

4. Determine ‘a’ and ‘c’

Calculate the depth of the compression block from Eqn 1. See Figure A-3.

$$a = \frac{(T_1 + T_2 - kP_u)}{0.85 f'_c\, b}$$

$$a = \frac{(40.56 + 31.86 - (1.818) \cdot (4.118))}{0.85 \cdot (4.5) \cdot (12)} = 1.415 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{1.415}{0.825} = 1.715 \text{ in}$$

5. Strains
6. Stresses

Determine the resulting strain in the two layers of reinforcing. See Figure A-2.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

$\varepsilon_1 = 0.003(d_1 / c - 1) = 0.003(8.00 / 1.715 - 1) = 0.01099$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

$\varepsilon_2 = 0.003(d_2 / c - 1) = 0.003(4.12 / 1.715 - 1) = 0.00421$
Since $\varepsilon_2 > \varepsilon_y$ the bottom layer yields and $f_{s2} = 60$ ksi

7. Verify Assumption

Since both layers of reinforcing yield the assumptions made in the analysis are valid.

Maximum Strain

The LRFD Specification does not have an upper limit on the amount of strain in a reinforcing bar. ASTM does require that smaller diameter rebar have a minimum elongation at tensile strength of 8 percent. This appears to be a reasonable upper limit for an extreme event state where $\varphi = 1.00$. For this example the strain of 1.0 percent is well below this limit.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section. This unnecessary step is included for educational purposes.

$\varphi P_n = \varphi k P_u = \varphi[T_1 + T_2 – 0.85 f'_c ba]$
$= 1.0[40.56 + 31.86 - 0.85(4.5)(12.0)(1.415)] = 7.47$ k

Solve for equilibrium from Figure A-3 by substituting M_n for kP_ue and taking moments about the center of the compression block:

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) + T_2\left(d_2 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (40.56)\cdot\left(8.00 - \frac{1.415}{2}\right) + (31.86)\cdot\left(4.12 - \frac{1.415}{2}\right)$$

$$-(1.818)\cdot(4.118)\cdot\left(\frac{10.81}{2} - \frac{1.415}{2}\right) = 369.34 \text{ in-k}$$

φM_n = (1.00)(369.34) / 12 = 30.78 ft-k

The factor of safety for flexure is 30.78 / 16.93 = 1.818 approximately the same as for axial strength of 7.47 / 4.118 = 1.814. Thus demonstrating that the method provides a strength in tension and flexure with the same safety factor.

Simplified Method

A simplified method of analysis is also available. If only the top layer of reinforcing is considered in determining strength, the assumption can be made that the reinforcing will yield. By assuming the safety factor for axial tension is 1.0 the strength equation can be solved directly. This method will determine whether the section has adequate strength. However, the method does not consider the bottom layer of reinforcing, does not maintain the required constant eccentricity and does not determine the maximum strain.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{40.56 - 4.118}{(0.85)\cdot(4.5)\cdot(12)} = 0.79$ in

$$\varphi M_n = (1.00)\cdot\left[(40.56)\cdot\left(8.00 - \frac{0.79}{2}\right) - (4.118)\cdot\left(\frac{10.81}{2} - \frac{0.79}{2}\right)\right] \div 12$$

φM_n = 23.99 ft-k

Since $\varphi M_n = 23.99$ ft-k > $M_u = 16.93$ ft-k the overhang has adequate strength. Note that the resulting eccentricity equals (23.99)(12) ÷ 4.118 = 69.91 inches compared to the actual eccentricity of 49.33 inches that is fixed by the constant deck thickness, barrier height and dead load moment.

Independent analysis using the more complex method but considering only the top layer of reinforcing results in a flexural strength equal to 23.38 ft-k. Thus it would appear that the simplified analysis method produces a greater non-conservative result. However, the simplified method uses a safety factor of 1.0 for axial load leaving more resistance for flexure. As the applied load approaches the ultimate strength the two methods will converge to the same result.

[A13.4.1]

Extreme Event II
[3.4.1]

1. Assume Stress

2. Determine Forces

Exterior Support Location 2
Figure 4

The deck slab must also be evaluated at the exterior overhang support. At this location the design horizontal force is distributed over a length L_{s1} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the exterior support. See Figures 4, 5 and 6. Using a distribution of 30 degrees from the face of barrier to the exterior support results in the following:

L_{S1} = 17.10 + 2(3.50) + (2)tan(30)(1.04) = 25.30 ft
P_u = 99.25 / 25.30 = 3.923 k/ft

Dimensions

h = 12.00 in
d_1 = 12.00 – 2.50 clr – 0.625 / 2 = 9.19 in
d_2 = 4.00 + 1.00 clr + 0.625 / 2 = 5.31 in

Moment at Exterior Support

.
DC Loads

Deck = 0.150(9.00 / 12)$(2.63)^2$ / 2 = 0.39 ft-k
= 0.150(3.00 / 12)$(2.63)^2$ / 6 = 0.04 ft-k
Barrier = 0.538(0.946 + 1.042) = 1.07 ft-k

DC = 1.50 ft-k

DW Loads

FWS = 0.025$(1.04)^2$ / 2 = 0.01 ft-k

Collision = 3.923[3.50 + (12.00 / 12) / 2] = 15.69 ft-k

The load factor for dead load shall be taken as 1.0.

M_u = 1.00(1.50) + 1.00(0.01) + 1.00(15.69) = 17.20 ft-k

e = M_u / P_u = (17.20)(12) / (3.923) = 52.61 in

Assume both layers of reinforcing yield and f_s = 60 ksi

Determine resulting forces in the reinforcing:

T_1 = (0.676)(60) = 40.56 k
T_2 = (0.531)(60) = 31.86 k

$T_1 + T_2$ = 40.56 + 31.86 = 72.42 k

Solution

3. Determine k Safety Factor

4. Determine ‘a’ and ‘c’

$$k = \frac{-223.7374 + \sqrt{(223.7374)^2 - 4 \cdot (0.167646) \cdot (-484.7917)}}{2 \cdot (0.167646)} = 2.163$$

Since the value of k is greater than one, the deck is adequately reinforced at this location.

Calculate the depth of the compression block.

$$a = \frac{(T_1 + T_2 - kP_u)}{0.85 f'_c\, b}$$

5. Strains
6. Stress

$$a = \frac{(72.42 - (2.163) \cdot (3.923))}{0.85 \cdot (4.5) \cdot (12)} = 1.393 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{1.393}{0.825} = 1.688 \text{ in}$$

Determine the resulting strain in the two layers of reinforcing. See Figure A-2.

Using the previously derived equations for safety factor yields the following:

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

$\varepsilon_1 = 0.003(d_1 / c - 1) = 0.003(9.19 / 1.688 - 1) = 0.01333$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

$$A = \frac{(3.923)^2}{1.70 \cdot (4.5) \cdot (12)} = 0.167646$$

$\varepsilon_2 = 0.003(d_2 / c - 1) = 0.003(5.31 / 1.688 - 1) = 0.00644$
Since $\varepsilon_2 > \varepsilon_y$ the bottom layer yields and $f_{s2} = 60$ ksi

7. Verify Assumption

$$B = (3.923) \cdot \left(52.61 + \frac{12.00}{2} - \frac{(72.42)}{0.85 \cdot (4.5) \cdot (12)}\right) = 223.7374$$

Maximum Strain

Verify Results

$$C = -(40.56) \cdot (9.19) - (31.86) \cdot (5.31) + \frac{(72.42)^2}{1.70 \cdot (4.5) \cdot (12)} = -484.7917$$

Solution of the quadratic equation yields the value k, the safety factor.

$\varphi P_n = \varphi k P_u = \varphi[T_1 + T_2 - 0.85f'_c ba]$

$= 1.0[40.56 + 31.86 - 0.85(4.5)(12.0)(1.393)] = 8.48$ k

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) + T_2\left(d_2 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (40.56)\cdot\left(9.19 - \frac{1.393}{2}\right) + (31.86)\cdot\left(5.31 - \frac{1.393}{2}\right)$$

$$-(2.163)\cdot(3.923)\cdot\left(\frac{12.00}{2} - \frac{1.393}{2}\right) = 446.50 \text{ in-k}$$

$\varphi M_n = (1.00)(446.50) / 12 = 37.21$ ft-k

The factor of safety for flexure is 37.21 / 17.20 = 2.163 approximately the same as for axial strength of 8.48 / 3.923 = 2.162.

Simplified Method

A simplified method of analysis is available based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{40.56 - 3.923}{(0.85)\cdot(4.5)\cdot(12)} = 0.80$ in

Since both layers of reinforcing yield the assumptions made in the analysis are valid.

$$\varphi M_n = (1.00)\cdot\left[(40.56)\cdot\left(9.19 - \frac{0.80}{2}\right) - (3.923)\cdot\left(\frac{12.00}{2} - \frac{0.80}{2}\right)\right] \div 12$$

$\varphi M_n = 27.88$ ft-k $> M_u = 17.20$ ft-k

The maximum strain of 1.3 percent is less than the ADOT limit of 8 percent and is therefore satisfactory.

Interior Support
Location 3
Figure 4

Verify the results by calculating the tensile strength and flexural resistance of the section.

not be effective. Only the top layer will be considered. At this location the design horizontal force is distributed over a length L_{s2} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the interior support. See Figures 4, 5 and 6. Using a distribution of 30 degree from the face of the barrier to the interior support results in the following:

L_{S2} = 17.10 + 2(3.50) + (2)tan(30)(2.13) = 26.56 ft
P_u = 99.25 / 26.56 = 3.737 k/ft

Dimensions
h = 8.00 in
d_1 = 8.00 – 2.50 clr – 0.625 / 2 = 5.19 in

Moment at Interior Support
For dead loads use the maximum negative moments for the interior cells used in the interior deck analysis
.

[A13.4.1]
Extreme Event II
[3.4.1]

DC = 0.46 ft-k
DW = 0.11 ft-k
Collision = 3.737[3.50 + (8.00 / 12) / 2] = 14.33 ft-k

1. Assume Stress

The load factor for dead load shall be taken as 1.0.

2. Determine Force

M_u = 1.00(0.46) + 1.00(0.11) + 1.00(14.33) = 14.90 ft-k
e = M_u / P_u = (14.90)(12) / (3.737) = 47.85 in

Assume the top layer of reinforcing yields and f_s = 60 ksi

Determine resulting force in the reinforcing:

T_1 = (0.676)(60) = 40.56 k

Using the previously derived equations for safety factor yields the following:

$$A = \frac{(3.737)^2}{1.70 \cdot (4.5) \cdot (12)} = 0.152126$$

$$B = (3.737) \cdot \left(47.85 + \frac{8.00}{2} - \frac{(40.56)}{0.85 \cdot (4.5) \cdot (12)}\right) = 190.4612$$

$$C = -(40.56) \cdot (5.19) + \frac{(40.56)^2}{1.70 \cdot (4.5) \cdot (12)} = -192.5858$$

The deck slab must also be evaluated at the interior point of support. For this thinner slab the bottom reinforcing will be near the neutral axis and will

3. Determine k
Safety Factor

yields the value k, the safety factor.

$$k = \frac{-190.4612 + \sqrt{(190.4612)^2 - 4\cdot(0.152126)\cdot(-192.5858)}}{2\cdot(0.152126)} = 1.010$$

Since the value of k is greater than one, the deck is adequately reinforced at this location.

4. Determine 'a' and 'c'

Calculate the depth of the compression block.

$$a = \frac{(T_1 - kP_u)}{0.85 f'_c\, b}$$

$$a = \frac{(40.56 - (1.010)\cdot(3.737))}{0.85\cdot(4.5)\cdot(12)} = 0.801 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{0.801}{0.825} = 0.971 \text{ in}$$

5. Strain

Determine the resulting strain in the top layer of reinforcing. See Figure A-2.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

6. Stress

$\varepsilon_1 = 0.003(d_1 / c - 1) = 0.003(5.19 / 0.971 - 1) = 0.01304$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

7. Verify Assumption

Since the top layer of reinforcing yields the assumption made in the analysis is valid.

Maximum Strain

The maximum strain of 1.3 percent is less than the ADOT limit of 8 percent and is therefore satisfactory.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section.

$\varphi P_n = \varphi k P_u = \varphi[T_1 - 0.85 f'_c b a]$

$= 1.0[40.56 - 0.85(4.5)(12.0)(0.801)] = 3.79$ k

Solution of the quadratic equation

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (40.56)\cdot\left(5.19 - \frac{0.801}{2}\right) - (1.010)\cdot(3.737)\cdot\left(\frac{8.00}{2} - \frac{0.801}{2}\right)$$

M_n = 180.68 in-k

φM_n = (1.00)(180.68) / 12 = 15.06 ft-k

The factor of safety for flexure is 15.06 / 14.90 = 1.011 approximately the same as for axial strength of 3.79 / 3.737 = 1.014.

Simplified Method

A simplified method of analysis is available based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{40.56 - 3.731}{(0.85)\cdot(4.5)\cdot(12)} = 0.80\ \text{in}$

$$\varphi M_n = (1.00)\cdot\left[(40.56)\cdot\left(5.19 - \frac{0.80}{2}\right) - (3.737)\cdot\left(\frac{8.00}{2} - \frac{0.80}{2}\right)\right] \div 12$$

φM_n = 15.07 ft-k > M_u = 14.90 ft-k

It is interesting to note that for a safety factor approximately equal to one that the results of the precise and approximate methods are nearly the same.