3-Span Precast Prestressed Box Beam Bridge [PPBBB] Example

This example illustrates the design of a three span precast prestressed box beam bridge. The bridge has spans of 85.25, 86.50 and 85.25 feet resulting in equal lengths of the modified AASHTO BII-48 box beam in all spans. The bridge has zero skew. Standard ADOT 32-inch f-shape barriers will be used resulting in a bridge configuration of 1'-5" barrier, 12'-0" outside shoulder, one 12'-0" lane, a 4'-0" inside shoulder and a 1'-5" barrier. The overall out-to-out width of the bridge is 30'-10". A plan view and typical section of the bridge are shown in Figures 1 and 2.

The following legend is used for the references shown in the left-hand column:

[2.2.2]	LRFD Specification Article Number
[2.2.2-1]	LRFD Specification Table or Equation Number
[C2.2.2]	LRFD Specification Commentary
[A2.2.2]	LRFD Specification Appendix
[BDG]	ADOT LRFD Bridge Design Guideline

Bridge Geometry

Span lengths	85.25, 86.50, 85.25 ft
Bridge width	30.83 ft
Roadway width	28.00 ft
Superstructure depth	3.17 ft
Web spacing	4.00 ft
Web thickness	5.00 in
Cast-in-place deck thickness	5.00 in
Top slab thickness	5.50 in
Bottom slab thickness	6.00 in
Deck overhang	1.42 ft

[Table 2.5.2.6.3-1]

Minimum Requirements

The minimum span to depth ratio for a simple span adjacent box beam bridge is 0.030 resulting in a minimum depth of (0.030)(84) = 2.52 feet.
A nominal 5 inch concrete slab will be cast compositely with the 5.50 inch top slab of the precast member resulting in a 10.50 inch composite deck.

Concrete Deck Slab Minimum Requirements

Slab thickness	10.50 in
Top concrete cover	2.50 in
Bottom concrete cover	1.00 in
Wearing surface	0.50 in

Span Length

The bridge is composed of equal length box beams separated by 12 inches. The centerline of bearing is 9 inches from the end of the beam. The resulting span length of each box beam is 86.50 – 1.00 – (2)(0.75) = 84.00 feet.

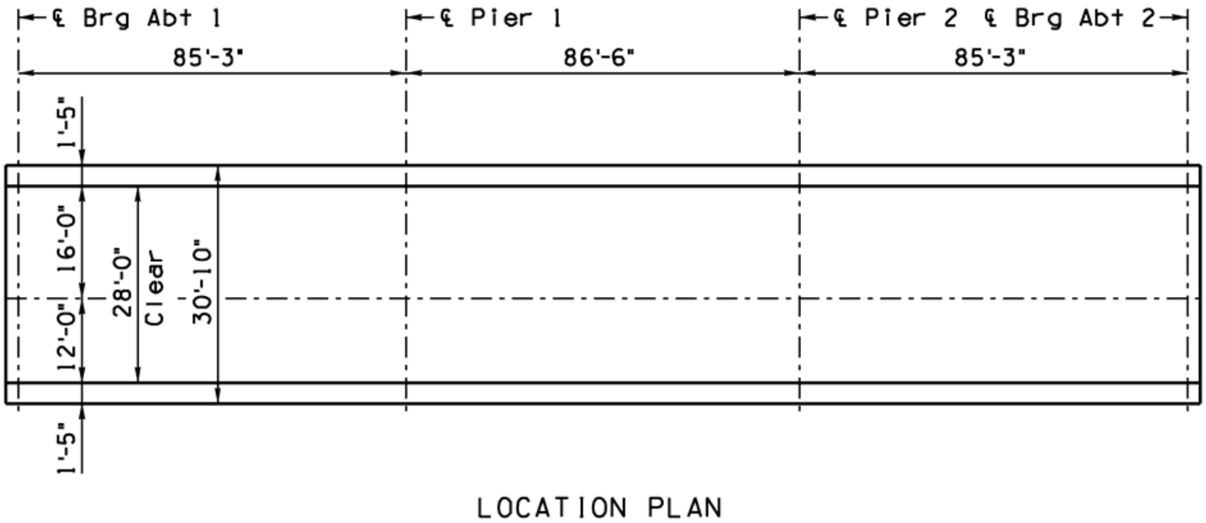


Figure 1

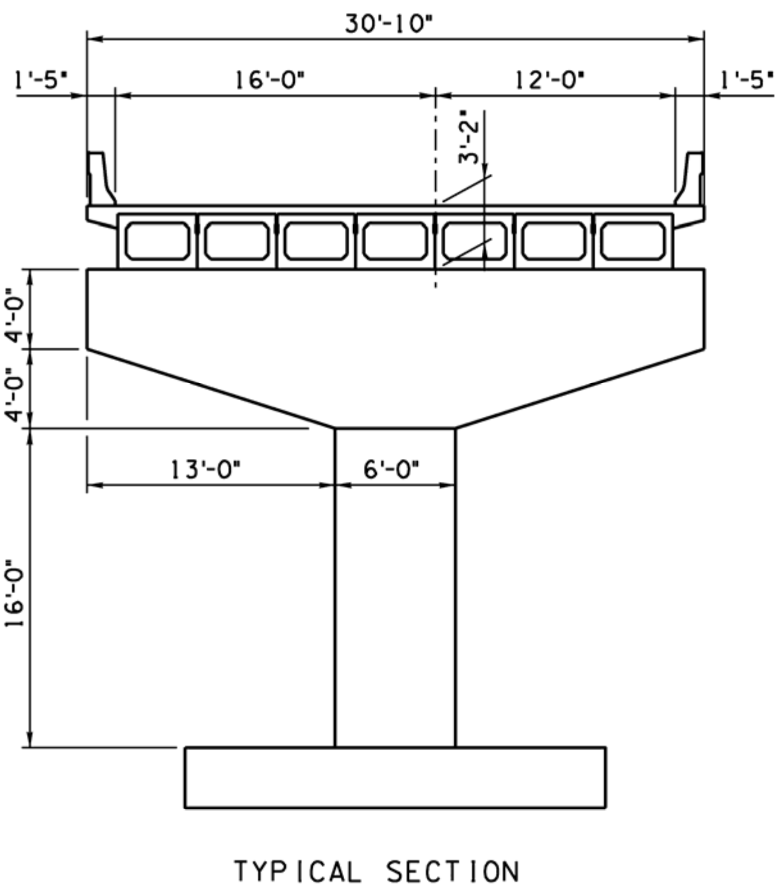


Figure 2

Material Properties

Reinforcing Steel

[5.4.3.1] Yield Strength $f_y = 60$ ksi

[5.4.3.2] Modulus of Elasticity $E_s = 29{,}000$ ksi

Prestressing Strand

[Table 5.4.4.1-1] Low relaxation prestressing strands

[5.4.4.2]

½" diameter strand	A_{ps}	$= 0.153\ in^2$
Tensile Strength	f_{pu}	= 270 ksi
Yield Strength	f_{py}	= 243 ksi
Modulus Elasticity	E_p	= 28500 ksi

Concrete

The final and release concrete strengths are specified below:

<u>Precast Box Beam</u>	<u>Deck</u>	<u>Pier & Footing</u>
$f'_c = 5.0$ ksi	$f'_c = 4.5$ ksi	$f'_c = 3.5$ ksi
$f'_{ci} = 4.4$ ksi		

[Table 3.5.1-1] Unit weight for normal weight concrete is listed below:

[C 3.5.1]

Unit weight for computing E_c = 0.145 kcf
Unit weight for DL calculation = 0.150 kcf

[C5.4.2.4] The modulus of elasticity for normal weight concrete where the unit weight is 0.145 kcf may be taken as shown below:

Precast Box Beam:

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{5.0} = 4070\ ksi$$

$$E_{ci} = 1820\sqrt{f'_{ci}} = 1820\sqrt{4.4} = 3818\ ksi$$

Deck Slab:

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{4.5} = 3861\ ksi$$

Pier and Footing:

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{3.5} = 3405\ ksi$$

[5.7.1]

The modular ratio of reinforcing to concrete should be rounded to the nearest whole number. An exception is made for prestressed members where the modular ratio is rounded to two places in this example.

Precast Box Beam

$n = \frac{28{,}500}{3818} = 7.46$ Use n = 7.46 for Prestressing in Beam at Transfer

$n = \frac{28{,}500}{4070} = 7.00$ Use n = 7.00 for Prestressing in Beam at Service

$n = \frac{29{,}000}{4070} = 7.13$ Use n = 7 for Reinforcing in Beam

Deck Slab:

$n = \frac{29{,}000}{3861} = 7.51$ Use n = 8 for Deck

Pier and Footing:

$n = \frac{29{,}000}{3405} = 8.52$ Use n = 9 for Pier and Footing

[5.7.2.2]

β_1 = The ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone stress block.

Precast Box Beam

$$\beta_1 = 0.85 - 0.05 \cdot \left[\frac{f'_c - 4.0}{1.0}\right] = 0.85 - 0.05 \cdot \left[\frac{5.0 - 4.0}{1.0}\right] = 0.800$$

Deck Slab

$$\beta_1 = 0.85 - 0.05 \cdot \left[\frac{f'_c - 4.0}{1.0}\right] = 0.85 - 0.05 \cdot \left[\frac{4.5 - 4.0}{1.0}\right] = 0.825$$

Pier and Footing

$\beta_1 = 0.85$

Modulus of Rupture [5.4.2.6]

The modulus of rupture for normal weight concrete has two values. When used to calculate service level cracking, as specified in Article 5.7.3.4 for side reinforcing or in Article 5.7.3.6.2 for determination of deflections, the following equation should be used:

Service Level Cracking

$$f_r = 0.24\sqrt{f'_c}$$

For superstructure calculations:

Deck: $f_r = 0.24\sqrt{4.5} = 0.509\ ksi$

Box Beam: $f_r = 0.24\sqrt{5.0} = 0.537\ ksi$

For substructure calculations:

$$f_r = 0.24\sqrt{3.5} = 0.449\ ksi$$

When the modulus of rupture is used to calculate the cracking moment of a member for determination of the minimum reinforcing requirement as specified in Article 5.7.3.3.2, the following equation should be used:

Minimum Reinforcing

$$f_r = 0.37\sqrt{f'_c}$$

For superstructure calculations:

Deck: $f_r = 0.37\sqrt{4.5} = 0.785\ ksi$

Box Beam: $f_r = 0.37\sqrt{5.0} = 0.827\ ksi$

For substructure calculations:

$$f_r = 0.37\sqrt{3.5} = 0.692\ ksi$$

Limit States [1.3.2]

In the LRFD Specification, the general equation for design is shown below:

$$\sum \eta_i \gamma_i Q_i \leq \varphi R_n = R_r$$

For loads for which a maximum value of γ_i is appropriate:

$$\eta_i = \eta_D \eta_R \eta_I \geq 0.95$$

For loads for which a minimum value of γ_i is appropriate:

$$\eta_i = \frac{1}{\eta_D \eta_R \eta_I} \leq 1.0$$

[1.3.3]

Ductility

For strength limit state for conventional design and details complying with the LRFD Specifications and for all other limit states:

[3.4.1] [BDG]

$$\eta_D = 1.00$$

[1.3.4]

Redundancy

For the strength limit state for conventional levels of redundancy and for all other limit states:

$$\eta_R = 1.0$$

[1.3.5]

Operational Importance

For the strength limit state for typical bridges and for all other limit states:

$$\eta_I = 1.0$$

[3.4.1] [BDG]

For an ordinary structure with conventional design and details and conventional levels of ductility, redundancy, and operational importance, it can be seen that η_i = 1.0 for all cases. Since multiplying by 1.0 will not change any answers, the load modifier η_i has not been included in this example.

For actual designs, the importance factor may be a value other than one. The importance factor should be selected in accordance with the ADOT LRFD Bridge Practice Guidelines.

DECK DESIGN

[BDG]

As bridges age, decks are one of the first element to show signs of wear and tear. As such ADOT has modified some LRFD deck design criteria to reflect past performance of decks in Arizona. Section 9 of the Bridge Design Guidelines provides a thorough background and guidance on deck design.

Effective Length
[9.7.2.3]

ADOT Bridge Practice Guidelines specify that deck design be based on the effective length rather than the centerline-to-centerline distance specified in the LRFD Specification. The effective length for monolithic cast-in-place concrete is the clear distance between supports. For this example with a centerline-to-centerline web spacing of 42.50 inches and web width of 5 inches, the effective length is 37.50 inches or 3.13 feet. The resulting minimum deck slab thickness per ADOT guidelines is 8.00 inches.

Method of Analysis
[9.6.1]
[BDG]

In-depth rigorous analysis for deck design is not warranted for ordinary bridges. The empirical design method specified in [9.7.2] is not allowed by ADOT Bridge Group. Therefore the approximate elastic methods specified in [4.6.2.1] will be used. Dead load analysis will be based on a strip analysis using the simplified moment equation of [$w S^2 / 10$] where "S" is the effective length.

Live Loads
[A4.1]

The unfactored live loads found in Appendix A4.1 will be used. Multiple presence and dynamic load allowance are included in the chart. Since ADOT bases deck design on the effective length, the chart should be entered under S equal to the effective length of 3.13 feet rather than the centerline-to-centerline distance of 3.54 feet. Since the effective length is used the correction for negative moment from centerline of the web to the design section should be zero. Entering the chart under the minimum span of 4'-0" yields the following live load moments:

Pos M = 4.68 ft-k/ft
Neg M = -2.68 ft-k/ft (0 inches from centerline)

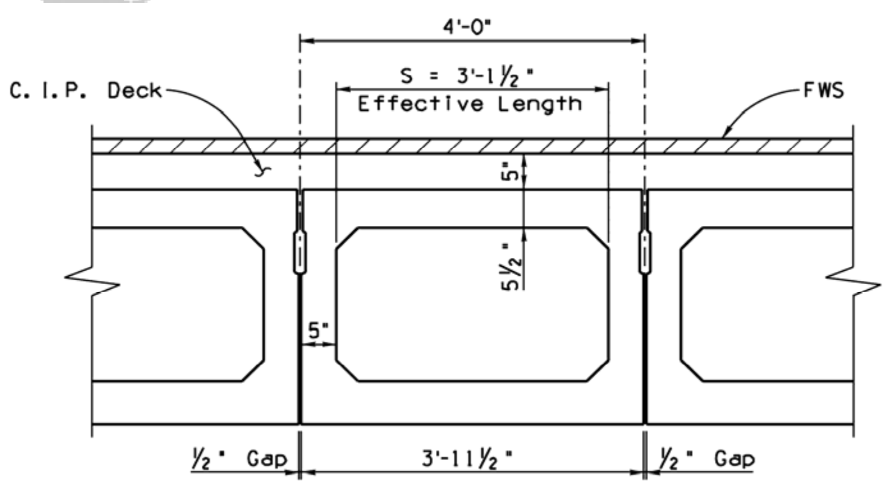


Figure 3

Positive Moment Design

A summary of positive moments follows:

DC Loads (Non-Composite)

Deck	$0.150(5.50/12)(3.13)^2 \div 10$	= 0.07
CIP Deck	$0.150(5.00/12)(3.13)^2 \div 10$	= 0.06
Build-up	$0.150(1.00/12)(3.13)^2 \div 10$	= 0.01
	DC	= 0.14 ft-k

DW Loads (Composite)

FWS $0.025(3.13)^2 \div 10$ = 0.02 ft-k

Vehicle (Composite)

LL + IM = 4.68 ft-k

Service I Limit State [9.5.4] [BDG]

Deck design is normally controlled by the service limit state. The working stress in the deck is calculated by the standard methods used in the past. However, the loads must be separated into those acting compositely and those acting non-compositely. For this check Service I moments should be used.

$$M_s = 1.0 \cdot (M_{DC} + M_{DW}) + 1.0 \cdot (M_{LL+IM})$$

Non-Composite

$M_s = 1.0(0.14) = 0.14$ ft-k

Composite

$M_s = 1.0(0.02) + 1.0(4.68) = 4.70$ ft-k

Try #5 reinforcing bars

Non-Comp $d_s = 5.50 – 1$ clr $–0.625/2 = 4.19$ in

Comp $d_s = 10.50 – 1$ clr $– 0.625/2 - 0.5$ ws $= 8.69$ in

Since the majority of the load is composite, determine approximate area reinforcing based on composite section properties as follows:

$$A_s \approx \frac{M_s}{f_s j d_s} = \frac{(0.14 + 4.70) \cdot (12)}{(24.0) \cdot (0.9) \cdot (8.69)} = 0.309 \text{ in}^2$$

Try #5 @ 12 inches

$A_s = 0.31$ in^2

Allowable Stress [9.5.2] [BDG]

The allowable stress for a deck under service loads is not limited by the LRFD Specifications. The 2006 Interim Revisions replaced the direct stress check with a maximum spacing requirement to control cracking. However, the maximum allowable stress for transverse reinforcing in a deck is limited to 24 ksi per the LRFD Bridge Design Guidelines.

Non-Composite

Determine non-composite stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.31}{(12)\cdot(4.19)} = 0.006165$$

np = 7(0.006165) = 0.04316

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.04316)+(0.04316)^2} - 0.04316 = 0.254$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.254}{3} = 0.915$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(0.14)\cdot(12)}{(0.31)\cdot(0.915)\cdot(4.19)} = 1.41\ \text{ksi}$$

Composite

Determine composite stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.31}{(12)\cdot(8.69)} = 0.002973$$

Use the cast-in-place deck concrete properties for positive moment compressive analysis.

np = 8(0.002973) = 0.02378

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.02378)+(0.02378)^2} - 0.02378 = 0.196$$

kd = (0.196)(8.69) = 1.70 in < CIP deck slab thickness = 4.50 in

$$j = 1 - \frac{k}{3} = 1 - \frac{0.196}{3} = 0.935$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(4.70)\cdot(12)}{(0.31)\cdot(0.935)\cdot(8.69)} = 22.39\ \text{ksi}$$

The applied stress is the sum of the non-composite and composite stresses equal to 1.41 + 22.39 = 23.80 ksi.

Allowable Stress

Since the applied stress is less than 24 ksi, the LRFD Bridge Practice Guideline service limit state requirement is satisfied.

Control of Cracking [5.7.3.4]

For all concrete components in which the tension in the cross-section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in Equation 5.7.3.4-1 shall be satisfied.

$$f_{sa} = 0.80f_r = 0.80(0.24\sqrt{f'_c}) = 0.80(0.537) = 0.430 \text{ ksi}$$

$$S_{cr} = (12.00)(5.50)^2 \div 6 = 60.5 \text{ in}^3 \text{ (non-composite)}$$

$$S_{cr} = (12.00)(10.00)^2 \div 6 = 200 \text{ in}^3 \text{ (composite)}$$

$$f_{cr} = \frac{M_s}{S_{cr}} = \frac{(0.14)\cdot(12)}{60.5} + \frac{(4.70)\cdot(12)}{200} = 0.310\ ksi < f_{sa} = 0.430 \text{ ksi}$$

Since the service limit state stress is less than the allowable, the control of cracking requirement is met.

Strength I Limit State [Table 3.4.1-1]

Factored moment for Strength I is as follows:

$$M_u = \gamma_{DC}(M_{DC}) + \gamma_{DW}(M_{DW}) + 1.75(M_{LL+IM})$$

$$M_u = 1.25\cdot(0.14) + 1.50\cdot(0.02) + 1.75\cdot(4.68) = 8.40 \text{ ft-k}$$

Flexural Resistance [5.7.3] [5.7.3.2.2-1]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right)$$

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.310)\cdot(60)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(12)} = 0.491 \text{ in}$$

[5.7.3.2.3]

$$a = \beta_1 c = (0.825)(0.491) = 0.41 \text{ in}$$

The tensile strain must be calculated as follows:

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{8.69}{0.491} - 1 \right) = 0.050$$

[5.5.4.2.1]

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 0.90$.

$$M_r = (0.90) \cdot (0.310) \cdot (60) \cdot \left(8.69 - \frac{0.41}{2} \right) \div 12 = 11.84 \text{ ft-k}$$

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the strength limit state is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated this limit. Below a net tensile strain in the extreme tension steel of 0.005, the factored resistance is reduced as the tension reinforcement quantity increases. This reduction compensates for the decreasing ductility with increasing overstrength.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u. Since the deck is composed of two different strengths of concrete, the cast-in-place deck will be transformed into an equivalent width of the higher strength concrete of the box beam.

The width of the cast-in-place deck will be modified based on the ratio of modulus of elasticity of the deck to the beam. The ½ inch wearing surface will not be subtracted since the thicker slab creates a higher cracking moment.

$$n = \frac{3861}{4070} = 0.949$$

n	W	H	A	y	Ay	Io	A(y-yb)^2
0.949	12.00	5.00	56.94	8.00	456	119	450
1.000	12.00	5.50	66.00	2.75	182	166	393
			122.94		638	285	843

$y_b = 638 / 122.94 = 5.190$ in
$I = 285 + 843 = 1128$ in^4
$S_c = I / y_b = 1128 / 5.190 = 217$ in^3

$$1.2M_{cr} = 1.2 f_r S_c = (1.2)\cdot(0.827)\cdot(217) \div 12 = 17.95 \text{ ft-k}$$

$$1.33\ M_u = (1.33)(8.40) = 11.17 \text{ ft-k} < M_r = 11.84 \text{ ft-k}$$

$\therefore$ The minimum reinforcement limit is satisfied.

Fatigue Limit State [9.5.3] & [5.5.3.1]

Fatigue need not be investigated for concrete decks.

The deck is adequately reinforced for positive moment using #5 @ 12" in the bottom of the box beam top slab.

Distribution Reinforcement [9.7.3.2]

Reinforcement shall be placed in the secondary direction in the bottom of slabs as a percentage of the primary reinforcement for positive moments as follows:

$$\frac{220}{\sqrt{S}} = \frac{220}{\sqrt{3.13}} = 124 \ \textit{percent} < 67 \text{ percent}$$

Use 67% Maximum.

$$A_s = 0.67(0.31) = 0.208 \text{ in}^2$$

Use #5 @ 12" $\Rightarrow A_s = 0.31 \text{ in}^2$

Skewed Decks [9.7.1.3] [BDG]

For bridges with skews less than 20 degrees, the ADOT LRFD Bridge Design Guidelines specifies that the primary reinforcement shall be placed parallel to skew. For zero degree skew in this example, transverse deck reinforcement shall be placed normal to the webs.

Negative Moment Design

A summary of negative moments follows:

DC Loads (Non-Composite)

Deck	$0.150(5.50/12)(3.13)^2 \div 10$	= -0.07
CIP Deck	$0.150(5.00/12)(3.13)^2 \div 10$	= -0.06
Build-up	$0.150(1.00/12)(3.13)^2 \div 10$	= -0.01
	DC	= -0.14 ft-k

DW Loads (Composite)

FWS	$0.025(3.13)^2 \div 10$	= -0.02 ft-k

Vehicle (Composite)

LL + IM		= -2.68 ft-k

Service I Limit State
[9.5.2]
[BDG]
[Table 3.4.1-1]

Deck design is normally controlled by the service limit state. The working stress in the deck is calculated by the standard methods used in the past. For this check Service I moments should be used.

$$M_s = 1.0(M_{DC} + M_{DW}) + 1.0(M_{LL+IM})$$

Non-Composite

$$M_s = 1.0 \cdot (0.14) = 0.14 \text{ ft-k}$$

Composite

M_s = 1.0(0.02) + 1.0(2.68) = 2.70 ft-k

Try #5 reinforcing bars

Non-Comp d_s = 5.50 – 1.0 clear – 0.625 / 2 = 4.19 inches
Composite d_s = 10.50 – 2.50 clear – 0.625 / 2 = 7.69 inches

Determine approximate area reinforcing as follows:

$$A_s \approx \frac{M_s}{f_s j d_s} = \frac{(2.70)\cdot(12)}{(24.0)\cdot(0.9)\cdot(7.69)} = 0.195 \text{ in}^2$$

Try #5 @ 12 inches

$A_s = 0.31 \text{ in}^2$

Determine stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.31}{(12)\cdot(7.69)} = 0.003359$$

Use the box beam concrete properties for negative moment compressive analysis.

np = 7(0.003359) = 0.02352

$$k = \sqrt{2np + np^2} - np = \sqrt{2 \cdot (0.02352) + (0.02352)^2} - 0.02352 = 0.195$$

kd = (0.195)(7.69) = 1.50 in < Box beam top slab = 5.50 in

$$j = 1 - \frac{k}{3} = 1 - \frac{0.195}{3} = 0.935$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(2.70)\cdot(12)}{(0.31)\cdot(0.935)\cdot(7.69)} = 14.54\ ksi \leq 24.0\,\text{ksi}$$

Allowable Stress

Since the applied stress is less than 24 ksi, the LRFD Bridge Design Guideline service limit state requirement is satisfied.

Control of Cracking [5.7.3.4]

The deck must be checked for control of cracking. For all concrete components in which the tension in the cross section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in Equation 5.7.3.4-1 shall be satisfied. For negative moments the cast-in-place deck concrete strength will be used for tensile checks.

$f_{sa} = 0.80f_r = 0.80(0.24\sqrt{f'_c}) = 0.80(0.509) = 0.407$ ksi

$S_{cr} = (12.00)(10.00)^2 \div 6 = 200$ in^3

$$f_{cr} = \frac{M_s}{S_b} = \frac{(2.70)\cdot(12)}{200} = 0.162\ ksi < f_{sa} = 0.407\text{ ksi}$$

Since the service limit state stress is less than the allowable, the control of cracking requirement is satisfied.

Strength I Limit State [3.4.1]

$$M_u = \gamma_{DC}(M_{DC}) + \gamma_{DW}(M_{DW}) + 1.75(M_{LL+IM})$$

$$M_u = 1.25 \cdot (0.14) + 1.50 \cdot (0.02) + 1.75 \cdot (2.68) = 4.90 \text{ ft-k}$$

The flexural resistance of a reinforced concrete rectangular section is:

Flexural Resistance [5.7.3]

$$M_r = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right)$$

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.310) \cdot (60)}{(0.85) \cdot (5.0) \cdot (0.800) \cdot (12)} = 0.456 \text{ in}$$

[5.7.3.1.1-4]

$a = \beta_1 c = (0.800)(0.456) = 0.36$ inches

[5.7.3.2.3]

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{7.69}{0.456} - 1 \right) = 0.048$$

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 0.90$.

[5.5.4.2.1]

$$M_r = (0.90) \cdot (0.310) \cdot (60) \cdot \left(7.69 - \frac{0.36}{2} \right) \div 12 = 10.48 \text{ ft-k}$$

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the strength limit state is satisfied.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u. The critical cracking load for negative moment will be caused by ignoring the 0.5 inch wearing surface and considering the full depth of the section.

$S_c = 12.0(10.50)^2 \div 6 = 220.5 \text{ in}^3$

$1.2 M_{cr} = 1.2 f_r S_c = (1.2) \cdot (0.785) \cdot (220.5) \div 12 = 17.31 \text{ ft-k}$

Factored moment for Strength I is as follows:

$1.33\ M_u = 1.33(4.90) = 6.52 \text{ ft-k} < M_r = 10.48 \text{ ft-k}$

$\therefore$ The minimum reinforcement limit is satisfied.

Fatigue Limit State [9.5.3] & [5.5.3.1]

Fatigue need not be investigated for concrete deck slabs in multigirder applications.

The cast-in-place deck is adequately reinforced for negative moment using #5 @ 12 inches.

There will also be stresses in the top slab of the box beam caused by its self-weight, the cast-in-place slab and build-up. By inspection these moments are very low and using #5 @ 12 inches for the top of the box beam slab is adequate.

Shear [C4.6.2.1.6]

Past practice has been not to check shear in typical slabs. For a standard concrete deck shear need not be investigated.

OVERHANG DESIGN [A13.4.1]

Design Case 1

The overhang shall be designed for the three design cases described below:

Design Case 1: Transverse forces specified in [A13.2]
Extreme Event Limit State

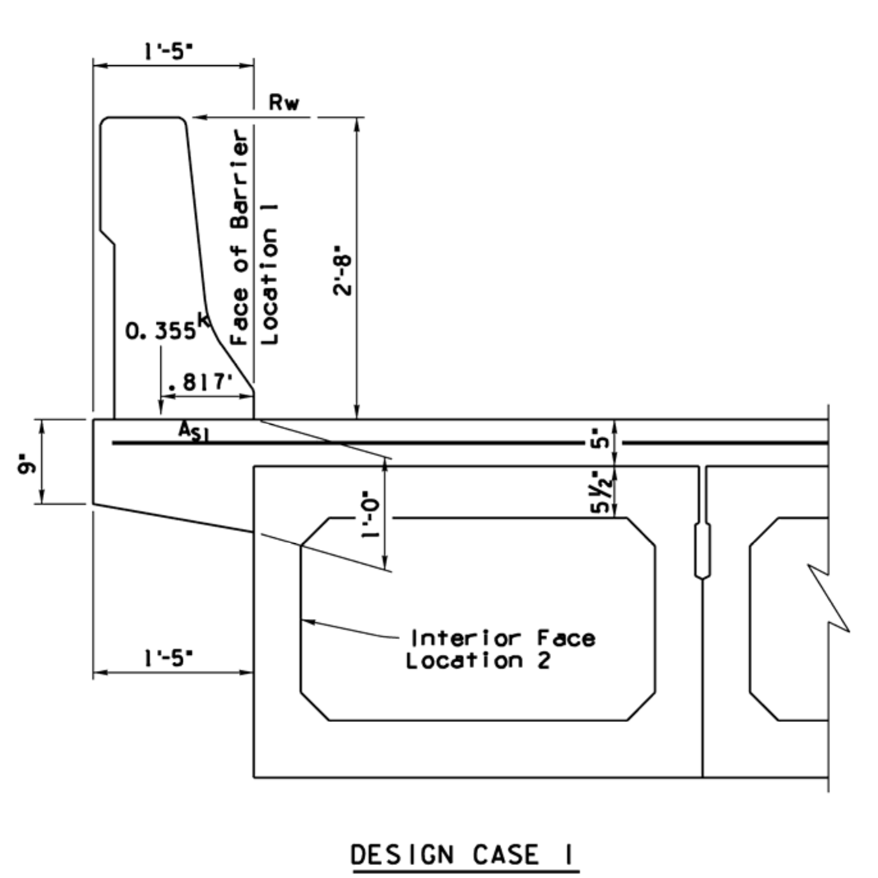


Figure 4

The deck overhang must be designed to resist the forces from a railing collision using the forces given in Section 13, Appendix A. A TL-4 railing is generally acceptable for the majority of applications on major roadways and freeways. A TL-4 rail will be used. A summary of the design forces is shown below:

[A13.2-1]

Design Forces		Units
F_t, Transverse	54.0	kips
F_l, Longitudinal	18.0	kips
F_v, Vertical Down	18.0	kips
L_t and L_l	3.5	feet
L_v	18.0	feet
H_e Minimum	32.0	inch

Rail Design

The philosophy behind the overhang analysis is that the deck should be stronger than the barrier. This ensures that any damage will be done to the barrier which is easier to repair and that the assumptions made in the barrier analysis are valid. ADOT generally avoids attaching the barrier directly to the precast member since a damaged deck will be easier and less expensive to repair than a damaged precast member. The forces in the barrier must be known to analyze the deck.

R_w = total transverse resistance of the railing.
L_c = critical length of yield line failure. See Figures 5 and 6.

<u>For impacts within a wall segment:</u>

[A13.3.1-1]

$$R_w = \left(\frac{2}{2L_c - L_t}\right)\left(8M_b + 8M_w + \frac{M_c L_c^{\,2}}{H}\right)$$

[A13.3.1-2]

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_w)}{M_c}}$$

The railing used on the bridge is the 32 inch f-shape concrete barrier as shown on ADOT SD 1.01. From previous analysis of the barrier the following values have been obtained:

M_b = 0.00 ft-k
M_c = 6.17 ft-k
M_w = 28.66 ft-k

$$L_c = \frac{3.50}{2} + \sqrt{\left(\frac{3.50}{2}\right)^2 + \frac{8\cdot(2.67)\cdot(0+28.66)}{6.17}} = 11.86\,\text{ft}$$

$$R_w = \left(\frac{2}{2\cdot(11.86) - 3.50}\right)\cdot\left(8\cdot(0) + 8\cdot(28.66) + \frac{(6.17)\cdot(11.86)^2}{2.67}\right) = 54.83\,\text{k}$$

Since the railing resistance to a transverse load, R_w = 54.83 kips, is greater than the applied load, F_t = 54.00 kips, the rail is adequately designed.

Barrier Connection To Deck

The strength of the attachment of the barrier to the deck must also be checked. The deck will only see the lesser of the strength of the barrier or the strength of the connection. For the 32 inch barrier, #4 at 16 inches connects the barrier to the deck.

$A_s = (0.20)(12) / (16) = 0.150 \text{ in}^2$

$d_s = 14.75 – 1 \tfrac{1}{2} \text{ clear} – 0.50 / 2 = 13.00 \text{ inches}$

Flexure

For a reinforcing bar not parallel to the compression face only the parallel component is considered. The #4 reinforcing is oriented at an angle of 26 degrees.

$$c = \frac{A_s f_y \cos\theta}{0.85 f'_c \, \beta_1 b} = \frac{(0.150)\cdot(60)\cos(26)}{(0.85)\cdot(4.0)\cdot(0.85)\cdot(12)} = 0.234 \text{ in}$$

$a = \beta_1 c = (0.85)(0.234) = 0.20 \text{ inches}$

$$M_n = A_s f_y \cos(\theta)\left(d_s - \frac{a}{2}\right)$$

$$M_n = (0.150)\cdot(60)\cdot\cos(26)\cdot\left(13.00 - \frac{0.20}{2}\right) \div 12 = 8.70 \text{ ft-k}$$

$\varphi M_n = (1.00)(8.70) = 8.70 \text{ ft-k}$

$\varphi P_u = (8.70)(12) \div (32) = \underline{3.261 \text{ k/ft}}$

Shear

The barrier to deck interface must also resist the factored collision load. The normal method of determining the strength is to use a shear friction analysis. However, in this case with the sloping reinforcing, the horizontal component of reinforcing force will also directly resist the horizontal force.

$R_n = A_s f_y \sin\theta = (0.150)(60)\sin(26) = 3.945 \text{ k/ft}$

The strength of the connection is limited by the lesser of the shear or flexural strength. In this case, the resistance of the connection is 3.261 k/ft.

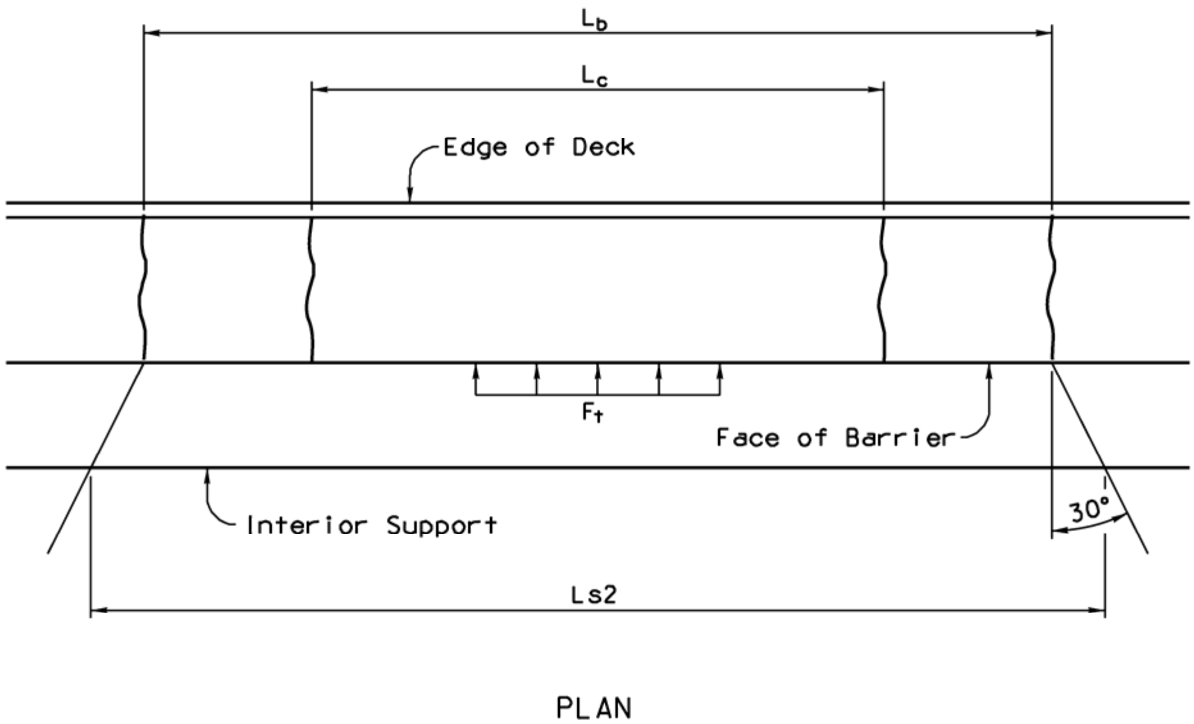


Figure 5

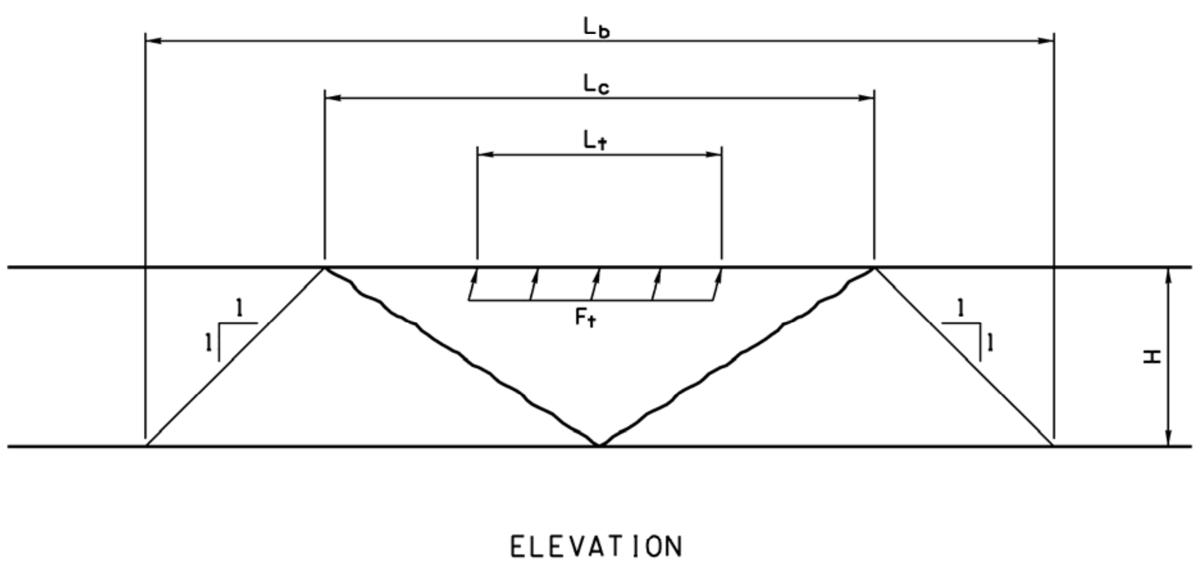


Figure 6

Face of Barrier
Location 1
Figure 4

The design horizontal force in the barrier is distributed over the length L_b equal to L_c plus twice the height of the barrier. See Figures 5 and 6.

$L_b = 11.86 + 2(2.67) = 17.20$ ft
$P_u = 54.83 / 17.20 = \underline{3.188 \text{ k/ft}} < 3.261$ k/ft connection strength.

Dimensions
h = 12.00 in
$d_1 = 12.00 - 2.50 \text{ clr} - 0.625 / 2 = 9.19$ in

Moment at Face of Barrier

Deck $= 0.150(9.00 / 12)(1.42)^2 \div 2 = 0.11$ ft-k
$0.150(3.00 / 12)(1.42)^2 \div 6 = \underline{0.01 \text{ ft-k}}$
$= 0.12$ ft-k

Barrier $= 0.355(0.817) = 0.29$ ft-k

Collision = 3.188[2.67 + (12.00/12) / 2] = 10.11 ft-k

[A13.4.1]
Extreme Event II
[3.4.1]

The load factor for dead load shall be taken as 1.0.

$M_u = 1.00(0.12 + 0.29) + 1.00(10.11) = 10.52$ ft-k

$e = M_u / P_u = (10.52)(12) / (3.188) = 39.60$ in

Determine resulting force in the reinforcing:

$T_1 = (0.310)(60) = 18.60$ k

Simplified Method

A simplified method of analysis is available. If only the top layer of reinforcing is considered in determining strength, the assumption can be made that the reinforcing will yield. By assuming the safety factor for axial tension is 1.0 the strength equation can be solved directly. This method will determine whether the section has adequate strength. However the method does not consider any bottom layer of reinforcing, does not maintain the required constant eccentricity and does not determine the maximum strain.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \frac{T_1 - P_u}{0.85 f'_c\, b} = \frac{18.60 - 3.188}{(0.85)\cdot(4.5)\cdot(12)} = 0.34$ in

$$\varphi M_n = (1.00)\cdot\left[(18.60)\cdot\left(9.19 - \frac{0.34}{2}\right) - (3.188)\cdot\left(\frac{12.00}{2} - \frac{0.34}{2}\right)\right] \div 12$$

φM_n = 12.43 ft-k

Since φ $M_n > M_u$ the overhang has adequate strength. Note that the resulting eccentricity equals (12.43)(12) ÷ 3.188 = 46.80 inches compared to the actual eccentricity of 39.60 inches that is fixed by the constant deck thickness, barrier height and dead load.

For an in-depth analysis of the overhang resisting the combination of tension and flexure using a stress-strain analysis refer to Appendix A.

Development Length [5.11.2]

The reinforcing must be properly developed from the barrier face towards the edge of deck. The available embedment length equals 17 inches minus 2 inches clear or 15 inches. For the #5 transverse reinforcing in the deck the required development length is as follows:

[5.11.2.1.1]

For No. 11 bar and smaller: $\frac{1.25 A_b f_y}{\sqrt{f'_c}} = \frac{(1.25)\cdot(0.31)\cdot(60)}{\sqrt{4.5}} = 10.96$ in

But not less than 0.4 d_b f_y = (0.4)(0.625)(60) = 15.00 in

Since the available length is equal to the required length, the reinforcing is adequately developed using straight bars.

If #6 transverse reinforcing was used as the top deck reinforcing, the development length would have been inadequate and the bars would require hooks unless modification factors reduced the demand to 15 inches of less. The reduction for excess reinforcing cannot be directly used since the analysis is based on a strain that ensures that the reinforcing yields. To use a reduced development length based on excess reinforcing, a stress-strain analysis must be performed that limits the strain in the reinforcing to a limit that produces less than the yield stress in the reinforcing. Consideration must also be given to the magnitude of the strain in the compressive zone in the concrete. For low levels of stress the analysis will default to a working stress limit in the concrete with the standard triangular stress block. Use of this method is complicated and is not recommended.

Interior Support Location 2 Figure 4

The deck slab must also be evaluated at the interior point of support. For this location only the top reinforcing in the cast-in-place slab will be considered. At this location the design horizontal force is distributed over a length L_{s2} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the interior support. See Figures 4, 5 and 6. Using a distribution of 30 degree from the face of the barrier to the interior support results in the following:

L_{S2} = 11.86 + 2(2.67) + (2)[tan(30)](0.42) = 17.68 ft
P_u = 54.83 / 17.68 = 3.101 k/ft

Dimensions

h = 10.50 in
d_1 = 10.50 – 2.50 clr – 0.625 / 2 = 7.69 in

Moment at Interior Support

For dead loads use the maximum negative moments for the interior cells used in the interior deck analysis.
.

DC = 0.14 ft-k
DW = 0.02 ft-k
Collision = 3.101[2.67 + (10.50 / 12) / 2] = 9.64 ft-k

The load factor for dead load shall be taken as 1.0.

[A13.4.1] Extreme Event II [3.4.1]

M_u = 1.00(0.14) + 1.00(0.02) + 1.00(9.64) = 9.80 ft-k
e = M_u / P_u = (9.80)(12) / (3.101) = 37.92 in

Determine resulting force in the reinforcing:

T_1 = (0.310)(60) = 18.60 k

Simplified Method

The simplified method of analysis is available based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{18.60 - 3.101}{(0.85)\cdot(5.0)\cdot(12)} = 0.30\ \text{in}$

$$\varphi M_n = (1.00)\cdot\left[(18.60)\cdot\left(7.69 - \frac{0.30}{2}\right) - (3.101)\cdot\left(\frac{10.50}{2} - \frac{0.30}{2}\right)\right] \div 12$$

$\varphi\, M_n$ = 10.37 ft-k

Design Case 1

Since the axial and flexural strength of the deck at the two locations investigated exceeds the factored applied loads, the deck is adequately reinforced for Design Case 1.

Design Case 2
[A13.4.1]

Design Case 2: Vertical forces specified in [A13.2]
Extreme Event Limit State

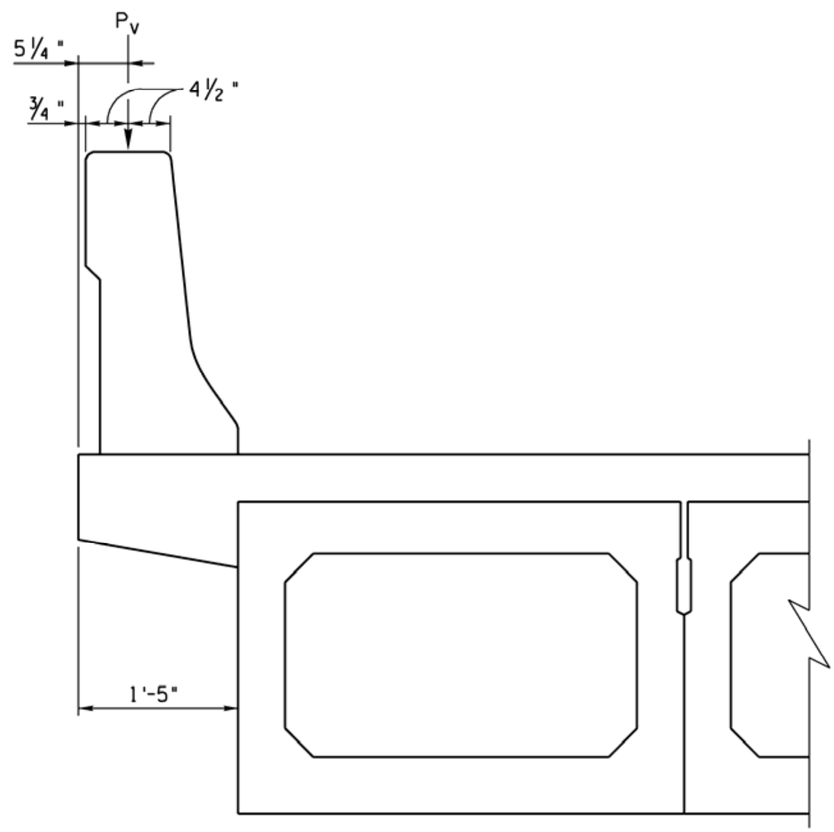


Figure 7

[A13.2-1]

This case represents a crashed vehicle on top of the barrier and is treated as an extreme event. The downward vertical force, F_v = l8.0 kips, is distributed over a length, F_l = 18.0 feet. The vehicle is assumed to be resting on top of the center of the barrier. See Figure 7.

At the face of exterior support:

DC Dead Loads = 0.12 + 0.29 = 0.41 ft-k
DW Dead Load = 0 ft-k

[3.6.1]

Vehicle
Collision = [18.0/18.0] [1.417 - (5.25/12)] = 0.98 ft-k

[A13.4.1]
Extreme Event II
[3.4.1]

The load factor for dead load shall be taken as 1.0.

$$M_u = 1.00(0.41) + 1.00(0) + 1.00(0.98) = 1.39 \text{ ft-k}$$

Flexural Resistance [5.7.3.2]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2} \right)$$

Try #5 reinforcing bars

d_s = 12.00 – 2.50 clr – 0.625 / 2 = 9.19 inches

Use #5 @ 12", the same reinforcing required for the interior span and overhang Design Case 1.

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \, \beta_1 b} = \frac{(0.310)\cdot(60)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(12)} = 0.491 \text{ in}$$

[5.7.3.2.3]

a = β_1c = (0.825)(0.491) = 0.41 inches

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{9.19}{0.491} - 1 \right) = 0.053$$

[5.5.4.2.1]

Since $\varepsilon_T > 0.005$ the member is tension controlled.

$$M_n = (0.310)\cdot(60)\cdot\left(9.19 - \frac{0.41}{2} \right) \div 12 = 13.93 \text{ ft-k}$$

[1.3.2.1]

φ = 1.00

$M_r = \phi M_n$ = (1.00)(13.93) = 13.93 ft-k

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the extreme limit state is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated this requirement.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification requires that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u.

$S_c = bh^2 / 6 = (12)(12.00)^2 / 6 = 288 \text{ in}^3$

$1.2M_{cr} = 1.2f_rS_c$ = 1.2(0.785)(288)/12 = 22.61 ft-k

1.33 M_u = (1.33)(1.39) = 1.85 ft-k < M_r = 13.93 ft-k

Therefore the minimum reinforcing criteria is satisfied.

Design Case 3

Design Case 3: The loads specified in [3.6.1] that occupy the overhang Strength and Service Limit State

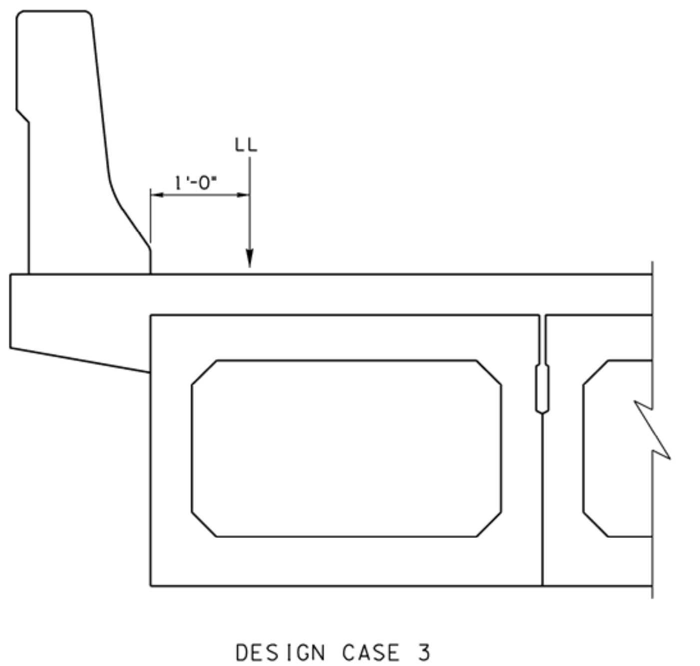
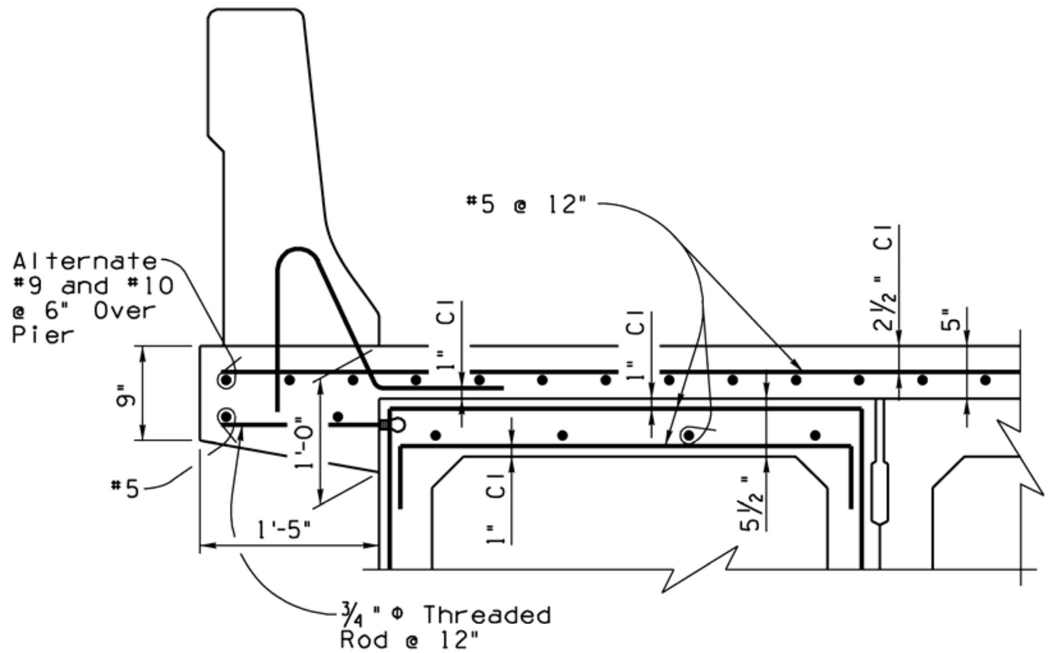


Figure 8

Due to the short overhang, the live load acting at a distance on one foot from the barrier face does not act on the overhang. Therefore, this case need not be investigated.

Figure 9 shows the required reinforcing in the deck and box beam slab.



TYPICAL SECTION

Figure 9

SUPERSTR DGN
Precast Prestressed
Box Beam

The composite section properties have been calculated subtracting the ½-inch wearing surface from the cast-in-place top slab thickness. However, this wearing surface has been included in weight calculations.

Step 1 – Determine Section Properties

Transformed section properties will be used for the structural design but gross section properties will be used for deflection and live load distribution calculations. The use of transformed section properties simplifies some calculations but complicates others requiring an iterative approach.

For a precast prestressed concrete beam the transformed section properties at transfer are used for determination of stresses due to prestressing at release and self-weight. The transformed section properties at service are used for non-composite externally applied loads. The transformed composite section properties are used for the composite dead loads and live loads. The net section properties are used to determine stresses due to time-dependent losses. The calculation of these properties is an iterative process since the required area of strands is a function of the number and location of the strands. These steps have been eliminated and the section properties will be shown for the final strand configuration.

For this problem the beam section properties will be calculated. The standard AASHTO box beams are 48 inch wide. However, local practice is to fabricate the beams one-half inch narrower so the effective spacing, including the ½ inch spacing between adjacent box beams, is 48 inches. For this problem two rows of strands are placed in the bottom slab as shown in Figure 11. To maintain the 2 inch edge distance to the center of the strands, the bottom slab thickness was increased to 6 inches.

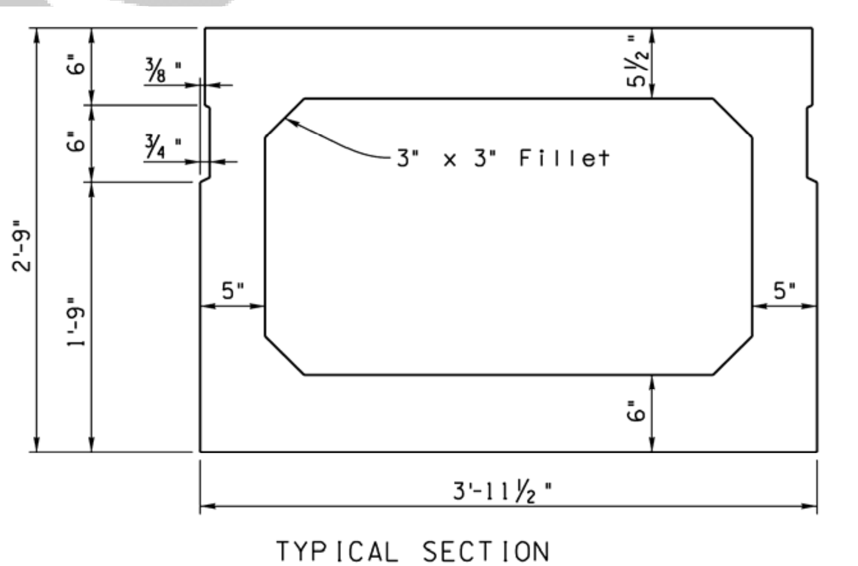


Figure 10

The section properties for the typical section shown in Figure 10 are shown below:

Box Beam Typical Section Gross Section Properties

Gross Section - Box Beam (33 inch deep)

	W	H	A	y	Ay	Io	A(y-yb)^2
Top	47.50	5.50	261.25	30.25	7903	659	52486
Web	10.00	21.50	215.00	16.75	3601	8282	98
Bottom	47.50	6.00	285.00	3.00	855	855	48730
Fillet	3.00	3.00	9.00	26.50	239	5	978
Fillet	3.00	3.00	9.00	7.00	63	5	741
Inset	0.75	6.00	-4.50	30.00	-135	-14	-872
Key	1.50	6.00	-9.00	24.00	-216	-27	-565
			765.75		12310	9765	101596

$A_g = 765.75\ in^2$
$y_b = 12310 / 765.75 = 16.076\ in$ $e = 16.076 – 2.824 = 13.252\ in$
$I_g = 9765 + 101{,}596 = 111{,}361\ in^4$
$r^2 = 111{,}361 / 765.75 = 145.43\ in^2$

Diaphragm Area

The area of the interior diaphragms for the box beam is required to determine the weight of the interior diaphragms.

Area = $(37.50)(21.50) – 4(1/2)(3.00)(3.00) = 788.25\ in^2$

Composite Properties

Composite Section - Box Beam & Deck

$n = 3861 / 4070 = 0.949$

Interior Box Beam

n	W	H	A	y	Ay	Io	A(y-yb)^2
			765.75	16.076	12310	111361	12554
0.949	48.00	4.50	204.98	35.25	7226	346	46892
			970.73		19536	111707	59446

$A_c = 970.73\ in^2$
$y_{cb} = 19536 / 970.73 = 20.125\ in$ $e = 20.125 – 2.824 = 17.301\ in$
$y_{ct} = 33.00 – 20.125 = 12.875\ in$
$I_c = 111{,}707 + 59{,}446 = 171{,}153\ in^4$
$r^2 = 171{,}153 / 970.73 = 176.31\ in^2$

Exterior Box Beam

n	W	H	A	y	Ay	Io	A(y-yb)^2
			765.75	16.076	12310	111361	25882
0.949	65.00	4.50	277.58	35.25	9785	468	49597
0.949	17.00	4.00	64.53	31.00	2000	86	5364
0.949	17.00	½*3.00	24.20	28.00	678	12	906
			1132.06		24773	111927	81749

A_c = 1132.06 in^2
y_{cb} = 24773 / 1132.06 = 21.883 in
y_{ct} = 33.00 – 21.883 = 11.117 in
I_c = 111,927 + 81,749 = 193,676 in^4

Torsional Inertia

The torsional inertia is required for determination of the live load distribution. This property is calculated for the composite section since that is the section the live load will see.

[C4.6.2.2.1-3]

s = center-to-center length for the composite section.
t = thickness of member.

Walls: s = 33 + 4.50 – 10.0 / 2 – 6.0 / 2 = 29.50 in
Slabs: s = 47.50 – 5.00 = 42.50 in

$$J = \frac{4A_o^2}{\sum \frac{s}{t}} = \frac{(4)\cdot[(42.50)\cdot(29.50)]^2}{\frac{29.50}{5.00}(2)+\frac{42.50}{6.00}+\frac{42.50}{10.00}} = 271{,}796 \ \text{in}^4$$

Volume/Surface Ratio

The surface area of the box beam is composed of the exterior surface and the interior surface. The interior surface is not exposed to the atmosphere and will be ignored.

Perimeter = 2(47.50 + 33) = 161 in

V/S = 765.75 / 161 = 4.76 in

The issue of whether to consider the interior surface area is currently not resolved. A case can be made to consider 50% of the interior surface area. This ratio is used in determining the creep restraint at supports. The ratio is also used in the refined method of prestress loss calculation but normal practice is to use the approximate method. Increasing the perimeter will reduce the V/S ratio and result in a higher creep.

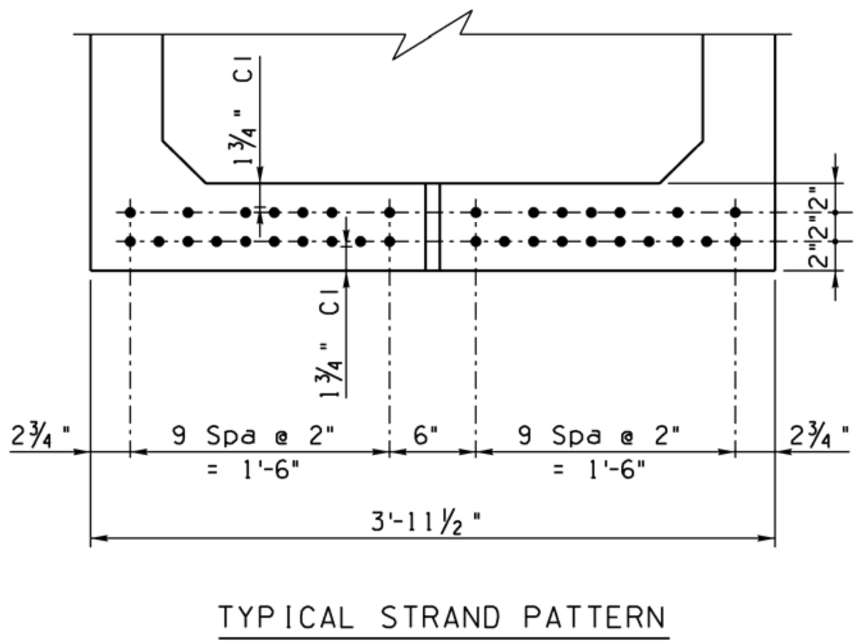


Figure 11

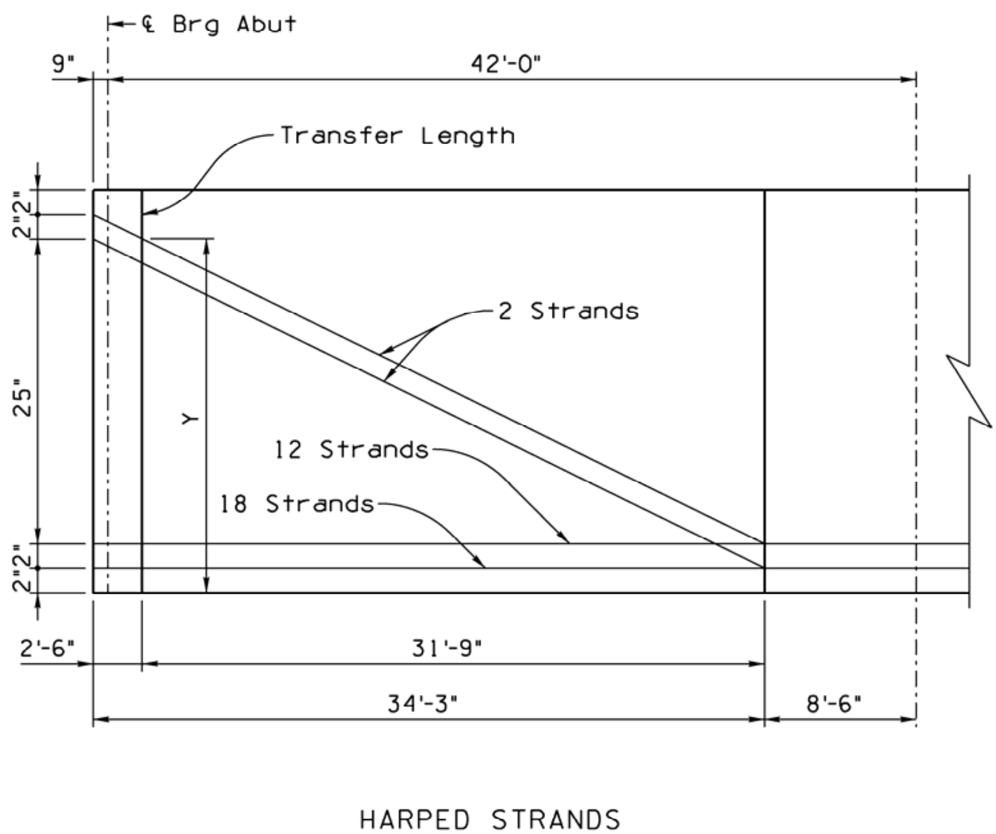


Figure 12

Midspan Transformed Properties

Transformed section properties are calculated at the midspan based on the strand pattern shown in Figure 11. The area of prestress strand and the center of gravity are calculated as follows:

A_{ps} = 0.153(34) = 5.202 in^2
c.g. = [20(2.0) + 14(4.0)] ÷ 34 = 2.824 in

Net Section - Box Beam

No.	As	A	y	Ay	Io	A(y-yb)^2
		765.75	16.076	12310	111,361	6
34	0.153	-5.20	2.824	-15	0	-926
		760.55		12295	111,361	-920

A_n = 760.55 in^2
y_{nb} = 12295 / 760.55 = 16.166 in e = 16.166 – 2.824 = 13.342 in
y_{nt} = 33.00 – 16.166 = 16.834 in
I_n = 111,361 - 920 = 110,441 in^4
r^2 = I / A = 110,441 / 760.55 = 145.21 in^2

Transformed Section - Box Beam (n = 7.46) at Transfer (f'$_{ci}$ = 4.4 ksi)

n	No.	As	A	y	Ay	Io	A(y-yb)^2
			760.55	16.166	12295	110441	318
7.46	34	0.153	38.81	2.824	110	0	6255
			799.36		12405	110441	6573

A_t = 799.36 in^2
y_{tb} = 12405 / 799.36 = 15.519 in e = 15.519 – 2.824 = 12.695 in
y_{tt} = 33.00 – 15.519 = 17.481 in
I_t = 110,441 + 6573 = 117,014 in^4
r^2 = I / A = 117,014 / 799.36 = 146.38 in^2

Transformed Section - Box Beam (n = 7.00) at Service (f'$_c$ = 5.0 ksi)

n	No.	As	A	y	Ay	Io	A(y-yb)^2
			760.55	16.166	12295	110441	282
7.00	34	0.153	36.41	2.824	103	0	5903
			796.96		12398	110441	6185

A_t = 796.96 in^2
y_{tb} = 12398 / 796.96 = 15.557 in e = 15.557 – 2.824 = 12.733 in
y_{tt} = 33.00 – 15.557 = 17.443 in
I_t = 110,441 + 6185 = 116,626 in^4

Composite Section - Box Beam & Deck

n = 3861 / 4070 = 0.949

Interior Box Beam

n	W	H	A	y	Ay	Io	$A(y-yb)^2$
			796.96	15.557	12398	116626	12937
0.949	48.00	4.50	204.98	35.25	7226	346	50294
			1001.94		19624	116972	63231

$A_c = 1001.94\ in^2$
$y_{cb} = 19624 / 1001.94 = 19.586$ in   e = 19.586 – 2.824 = 16.762 in
$y_{ct} = 33.00 – 19.586 = 13.414$ in
$I_c = 116{,}972 + 63{,}231 = 180{,}203\ in^4$

Exterior Box Beam

n	W	H	A	y	Ay	Io	$A(y-yb)^2$
			796.96	15.557	12398	116626	26949
0.949	65.00	4.50	277.58	35.25	9785	468	53462
0.949	17.00	4.00	64.53	31.00	2000	86	5982
0.949	17.00	½*3.0	24.20	28.00	678	12	1063
			1163.27		24861	117192	87456

$A_c = 1163.27\ in^2$
$y_{cb} = 24861 / 1163.27 = 21.372$ in   e = 21.372 – 2.824 = 18.548 in
$y_{ct} = 33.00 – 21.372 = 11.628$ in
$I_c = 117{,}192 + 87{,}456 = 204{,}648\ in^4$

Transfer Length

The section properties are also required near the ends of the beam at a distance equal to the transfer length from the end of the beam. Since transformed section properties are being used, the section properties will vary with the change in center of gravity of the strands. The transfer length of the bonded prestressing strands is 60 times the strand diameter. For 0.5 inch diameter strand the transfer length equals 30 inches. The centerline of bearing is 9 inches from the end. Therefore the critical location is 21 inches from the centerline of bearing.

See Figure 12 for a diagram of the harped strands. The rise in the top strand at the end of the transfer length is:

Y = 4.00 + (27.00)(31.75) / (34.25) = 29.029 in

$$cg = \frac{2\cdot(29.029)+2\cdot(27.029)+12\cdot(4.00)+18\cdot(2.00)}{34} = 5.768 \text{ in}$$

<u>Net Section</u> - Box Beam

No.	As	A	y	Ay	Io	$A(y\text{-}yb)^2$
		765.75	16.076	12310	111361	4
34	0.153	-5.20	5.768	-30	0	-560
		760.55		12280	111361	-556

A_n = 760.55 in^2
y_{nb} = 12280 / 760.55 = 16.146 in e = 16.146 – 5.768 = 10.378 in
y_{nt} = 33.00 – 16.146 = 16.854 in
I_n = 111,361 - 556 = 110,805 in^4

<u>Transformed Section</u> - Box Beam (n = 7.46) at Transfer

n	No.	As	A	y	Ay	Io	$A(y\text{-}yb)^2$
			760.55	16.146	12280	110805	192
7.46	34	0.153	38.81	5.768	224	0	3785
			799.36		12504	110805	3977

A_t = 799.36 in^2
y_{tb} = 12504 / 799.36 = 15.643 in e = 15.643 – 5.768 = 9.875 in
y_{tt} = 33.00 – 15.643 = 17.357 in
I_t = 110,805 + 3977 = 114,782 in^4

Transformed Section - Box Beam (n = 7.00) at Service

n	No.	As	A	y	Ay	Io	A(y-yb)^2
			760.55	16.146	12280	110805	171
7.00	34	0.153	36.41	5.768	210	0	3571
			796.96		12490	110805	3742

A_t = 796.96 in^2
y_{tb} = 12490 / 796.96 = 15.672 in e = 15.672 – 5.768 = 9.904 in
y_{tt} = 33.00 – 15.672 = 17.328 in
I_t = 110,805 + 3742 = 114,547 in^4

Composite Section - Box Beam & Deck

Interior Box Beam

n	W	H	A	y	Ay	Io	A(y-yb)^2
			796.96	15.672	12490	114547	12790
0.949	48.00	4.50	204.98	35.25	7226	346	49705
			1001.94		19716	114893	62495

A_c = 1001.94 in^2
y_{cb} = 19716 / 1001.94 = 19.678 in e = 19.678 – 5.768 = 13.910 in
y_{ct} = 33.00 – 19.678 = 13.322 in
I_c = 114,893 + 62,495 = 177,388 in^4

Exterior Box Beam

n	W	H	A	y	Ay	Io	A(y-yb)^2
			796.96	15.672	12490	114547	26616
0.949	65.00	4.50	277.58	35.25	9785	468	52855
0.949	17.00	4.00	64.53	31.00	2000	86	5884
0.949	17.00	½*3.0	24.20	28.00	678	12	1038
			1163.27		24953	115113	86393

A_c = 1163.27 in^2
y_{cb} = 24953 / 1163.27 = 21.451 in e = 21.451 – 5.768 = 15.683 in
y_{ct} = 33.00 – 21.451 = 11.549 in
I_c = 115,113 + 86,393 = 201,506 in^4

For an in-depth discussion of calculation of section properties comparing this approximate method with a more precise method refer to Appendix B.

Step 2 – Determine Loads and Stresses

The flexural design of the precast prestressed box beam is based on simple span positive moments. Normally moments, shears and stresses are calculated at tenth points using computer software programs. For this problem, only critical values will be determined.

Dead Load [3.5.1]

In LFRD design, the dead load are separated between DC loads and DW loads since their load factors differ. For precast girders, each load is also separated by the type of section property used to determine the stresses. The DC loads that use the transformed section properties at transfer include moments from the self-weight of the precast beam. The DC loads that use the transformed section properties at service include the moments from externally applied loads including the shear key concrete, build-up and the cast-in-place deck. The small overhangs on each side are equally distributed to all members for this narrow bridge since the shear keys are already cast and differential movement between beams is prohibited. The DC loads that use the composite transformed section properties include the barriers. The DW load that uses the composite transformed section properties includes the 0.025 ksf Future Wearing Surface and any utilities. The barrier and future wearing surface are distributed equally to all beams.

Loads

Self Weight	0.150(766 /144)	= 0.798 k/ft
Int Diaphragms	0.150(788.25 / 144)(1.00)	= 0.821 k each
Shear Key	0.150[(1.25)(6) + (2.00)(6)]/144	= 0.020 k/ft
Build-up	0.150(1.00 / 12)(4.00)	= 0.050 k/ft
Slab	0.150(5.00 / 12)(4.00)	= 0.250 k/ft
Overhang	0.150(10.50)(17)(2 / 7) / 144	= 0.053 k/ft
Barriers	0.355(2 / 7)	= 0.101 k/ft
FWS	0.025(28.00) / 7	= 0.100 k/ft

Midspan Moments

DC Loads – Transformed Section Properties at Transfer

Self Weight	$0.798(84.00)^2 \div 8$	= 704 ft-k
Int Diaphragms	0.821[(3/2)(42.00) - 21.00]	= <u>34</u> ft-k
		= 738 ft-k

DC Loads – Transformed Section Properties at Service

Shear Key	$0.020(84.00)^2 \div 8$	= 18 ft-k
Build-up	$0.050(84.00)^2 \div 8$	= 44 ft-k
Overhang	$0.053(84.00)^2 \div 8$	= 47 ft-k
Slab	$0.250(84.00)^2 \div 8$	= <u>221</u> ft-k
		= 330 ft-k

DC Loads – Composite Transformed Section Properties

Barriers $0.101(84.00)^2 \div 8 =$ 89 ft-k

DW Loads – Composite Transformed Section Properties

FWS $0.100(84.00)^2 \div 8 =$ 88 ft-k

Transfer Length Moments

At a distance x from the support, the moment from a uniform load is:

$M_x = (w)(x)(L - x) \div 2$

DC Loads – Transformed Section Properties at Transfer

Self Weight	0.798(1.75)(84.00 - 1.75)÷2	= 57 ft-k
Int Diaphragms	0.821(3/2)(1.75)	= 2 ft-k
		= 59 ft-k

DC Loads – Transformed Section Properties at Service

Shear Key	0.020(1.75)(84.00 – 1.75) ÷2	= 1 ft-k
Build-up	0.050(1.75)(84.00 – 1.75) ÷2	= 4 ft-k
Overhang	0.053(1.75)(84.00 - 1.75) ÷2	= 4 ft-k
Slab	0.250(1.75)(84.00 - 1.75) ÷2	= 18 ft-k
		= 27 ft-k

DC Loads – Composite Transformed Section Properties

Barriers	0.101(1.75)(84.00 - 1.75) ÷2	= 7 ft-k

DW Loads – Composite Transformed Section Properties

FWS	0.100(1.75)(84.00 - 1.75) ÷2	= 7 ft-k

Hold-Down Moments

DC Loads – Transformed Section Properties at Transfer

Self Weight	0.798(33.50)(84.00 - 33.50) ÷ 2	= 675 ft-k
Int Diaphragms	0.821[(3/2)(33.50) - 12.50]	= 31 ft-k
		= 706 ft-k

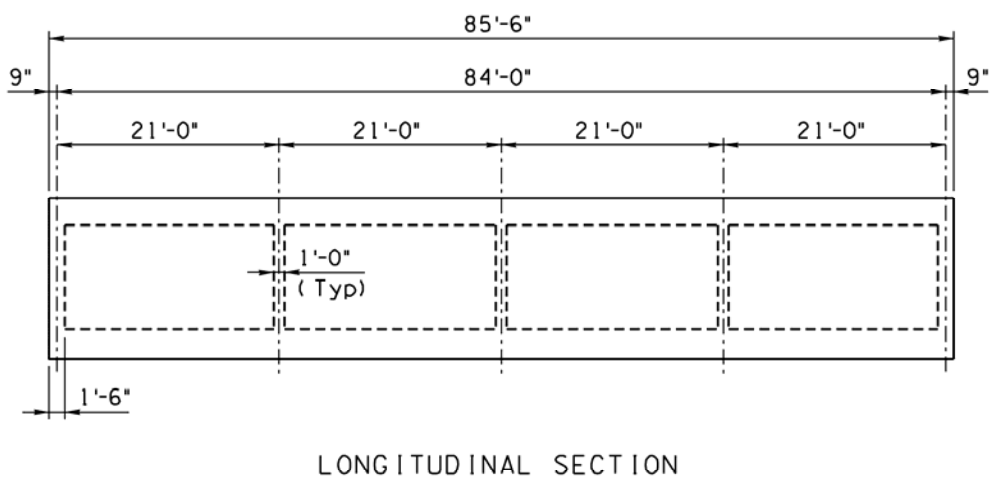


Figure 13

Live Load
[3.6.1]
[BDG]

The HL-93 live load in the LRFD specification differs from the HS-20-44 load in the Standard Specifications. For design of the precast prestressed box beam, ADOT calculates the live load moments assuming a simple span.

Midspan Moments

The maximum moment at midspan from the design lane load is caused by loading the entire span. The force effects from the design lane load shall not be subject to a dynamic load allowance. At midspan the moment equals the following:

Design Lane

$$M_{lane} = w \cdot (l)^2 \div 8 = 0.640 \cdot (84)^2 \div 8 = 564 \text{ ft-k}$$

The maximum design truck moment results when the truck is located with the middle axle at midspan. The truck live load positioned for maximum moment at midspan is shown below:

Design Truck

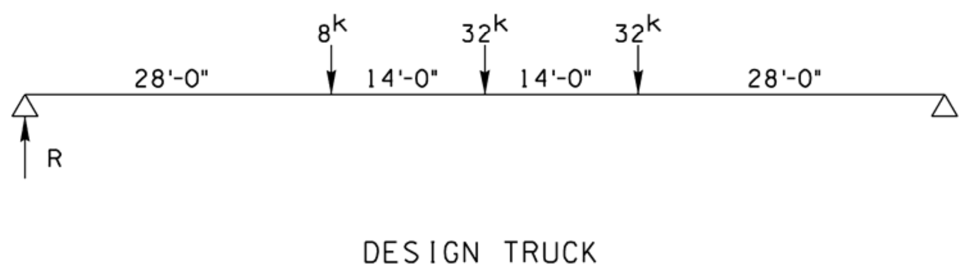


Figure 14

$$R = [8 \cdot (56) + 32 \cdot (42) + 32 \cdot (28)] \div 84 = 32.00 \text{ kips}$$
$$M_{truck} = 32.00 \cdot (42) - 8 \cdot (14) = 1232 \text{ ft-k}$$

The maximum design tandem moment results when the tandem is located with one of the axles at midspan. The tandem live load positioned for maximum moment is shown below:

Design Tandem

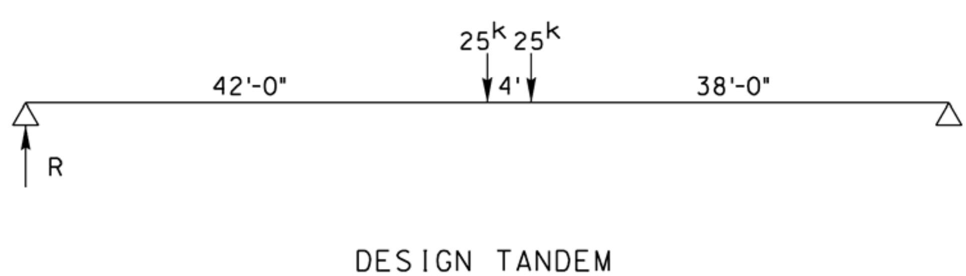


Figure 15

$$R = [25 \cdot (42) + 25 \cdot (38)] \div 84 = 23.81 \text{ kips}$$
$$M_{\tan dem} = 23.81 \cdot (42) = 1000 \text{ ft-k}$$

By inspection the moment from the combination of design truck and design lane load is higher than the combination of design tandem and design lane load.

[4.6.2.2.1-1]

The LRFD Specification has made major changes to the live load distribution factors. The first step is to determine the superstructure type from Table 4.6.2.2.1-1. For side-by-side box beams with a cast-in-place concrete overlay the typical cross section is identified as Type (f).

Interior Box Beam

Since the range of applicability for all variables is within the allowable, the live load distribution factor for moment for an interior beam with one lane loaded may be taken as:

	Applicable Range
N_b = number of beams = 7	$5 \le N_b \le 20$
b = width of beam (in) = 47.50 in	$35 \le b \le 60$
L = span length of beam (ft) = 84 ft.	$20 \le L \le 120$

I = moment of inertia of composite section (in^4) = 171,153 in^4
J = torsional inertia of composite section (in^4) = 271,796 in^4

[4.6.2.2.2b-1]

$$LL\ Distribution = k\left(\frac{b}{33.3L}\right)^{0.5}\left(\frac{I}{J}\right)^{0.25}$$

where $k = 2.5(N_b)^{-0.2} > 1.5$

$k = (2.5)(7)^{-0.2} = 1.694$

$$LL\ Distribution = 1.694 \cdot \left(\frac{47.50}{(33.3) \cdot (84.00)}\right)^{0.5}\left(\frac{171{,}153}{271{,}796}\right)^{0.25} = 0.197$$

The live load distribution factor for moment for an interior beam with two or more lanes loaded is:

$$LL\ Distribution = k\left(\frac{b}{305}\right)^{0.6}\left(\frac{b}{12.0L}\right)^{0.2}\left(\frac{I}{J}\right)^{0.06}$$

$$LL\ Distribution = 1.694\left(\frac{47.50}{305}\right)^{0.6}\left(\frac{47.50}{12.0 \cdot 84.00}\right)^{0.2}\left(\frac{171{,}153}{271{,}796}\right)^{0.06} = 0.293$$

Exterior Box Beam

The live load distribution factor for one design lane loaded for moment for an exterior beam is:

[4.6.2.2.2d-1]

$$\text{LL Distribution} = e\ g_{interior}$$

$$e = 1.125 + d_e / 30 > 1.0$$

[4.3]

d_e = distance from the exterior web of the exterior beam to the inside face of barrier in feet. A response from FHWA to our inquiry concerning the definition of d_e, states that d_e is measured to the center of the exterior web. For this problem d_e equals 2.5 inches or 0.21 feet.

$$e = 1.125 + 0.21 / 30 = 1.132$$

$$\text{LL Distribution} = 1.132(0.197) = 0.223$$

The live load distribution factor for two or more design lanes loaded for moment for an exterior beam is:

$$\text{LL Distribution} = e\ g_{interior}$$

$$e = 1.04 + d_e / 25 > 1.0$$

$$e = 1.04 + 0.21 / 25 = 1.048$$

$$\text{LL Distribution} = 1.048(0.293) = 0.307$$

Skew Effect
[4.6.2.2.2e]

Since the bridge is a right angle bridge the live load skew reduction factor is not applied.

Dynamic Load Allowance
[3.6.2]

The dynamic load allowance IM equals 33% for Strength and Service Limit States.

Dynamic load allowance applies to the truck or tandem but not to the design lane load. The dynamic load allowance has been included in the summation of live loads for one vehicle.

The maximum midspan moment from live load plus dynamic load allowance is:

$$M_{LL+IM} = [564 + 1.33(1232)](0.293) = 645 \text{ ft-k} \quad \text{Interior Beam}$$

$$M_{LL+IM} = [564 + 1.33(1232)](0.307) = 676 \text{ ft-k} \quad \text{Exterior Beam}$$

Transfer Length Moment

At the transfer length the live load moment is:

Lane	M = 0.640(1.75)(84.00 – 1.75) ÷ 2	= 46 ft-k
Truck	M = 1.75[(32)(82.25) + (32)(68.25) + 8(54.25)] ÷ 84	= 109 ft-k
Tandem	M = 1.75[(25)(82.25) + (25)(78.25)] ÷ 84	= 84 ft-k

M_{LL+IM} = [46 + 1.33(109)](0.293) = 56 ft-k   Interior Beam
M_{LL+IM} = [46 + 1.33(109)](0.307) = 59 ft-k   Exterior Beam

Limit States [3.4.1-1]

The LRFD Specification has made major changes to the group load combinations contained in [T3.4.1-1]. There are several limit states that must be considered in design of the superstructure. Limit states for this problem are as follows:

Midspan Moments

STRENGTH I – Basic load combination relating to the normal vehicular use of the bridge without wind.

$$M_u = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)$$

M_u = 1.25(738 + 330 + 89) + 1.50(88) + 1.75(645) = 2707 ft-k   Interior
M_u = 1.25(738 + 330 + 89) + 1.50(88) + 1.75(676) = 2761 ft-k   Exterior

SERVICE LIMIT STATES:

Midspan Stresses

Transformed Non-Composite at Transfer

M = 738 ft-k

$$f_t = \frac{(738)\cdot(12)\cdot(17.481)}{117{,}014} = 1.323\ \text{ksi}$$

$$f_b = \frac{(738)\cdot(12)\cdot(15.519)}{117{,}014} = -1.175\ \text{ksi}$$

Transformed Non-Composite at Service

M = 330 ft-k

$$f_t = \frac{(330)\cdot(12)\cdot(17.443)}{116{,}626} = 0.592\ \text{ksi}$$

$$f_b = \frac{(330)\cdot(12)\cdot(15.557)}{116{,}626} = -0.528\ \text{ksi}$$

Transformed Composite

Dead Load

M = 89 + 88 = 177 ft-k

$$f_t = \frac{(177)\cdot(12)\cdot(13.414)}{180{,}203} = 0.158 \text{ ksi}$$

$$f_b = \frac{(177)\cdot(12)\cdot(19.586)}{180{,}203} = -0.231 \text{ ksi}$$

LL + IM

M = 645 ft-k

$$f_t = \frac{(645)\cdot(12)\cdot(13.414)}{180{,}203} = 0.576 \text{ ksi}$$

$$f_b = \frac{(645)\cdot(12)\cdot(19.586)}{180{,}203} = -0.841$$

SERVICE I – Load combination relating to normal operational use of the bridge including wind loads to control crack width in reinforced concrete structures. For a precast member with a cast-in-place deck where transformed section properties are used each service state must be broken into subgroups depending upon the section properties used to determine the stress.

$M_s = 1.0(DC + DW) + 1.0(LL + IM)$

$\sum f_t = 1.0(1.323 + 0.592 + 0.158) + 1.0(0.576) = 2.649$ ksi

SERVICE III – Load combination relating only to tension in prestressed concrete superstructures with the objective of crack control.

$M_s = 1.0(DC + DW) + 0.80(LL + IM)$

$\sum f_b = 1.0(-1.175 - 0.528 - 0.231) + 0.80(-0.841) = -2.607$ ksi

Exterior Box Beam

Transformed Composite

Dead Load

M = 89 + 88 = 177 ft-k

$$f_t = \frac{(177)\cdot(12)\cdot(11.628)}{204{,}648} = 0.121 \text{ ksi}$$

$$f_b = \frac{(177)\cdot(12)\cdot(21.372)}{204{,}648} = -0.222 \text{ ksi}$$

LL + IM

M = 676 ft-k

$$f_t = \frac{(676)\cdot(12)\cdot(11.628)}{204{,}648} = 0.461 \text{ ksi}$$

$$f_b = \frac{(676)\cdot(12)\cdot(21.372)}{204{,}648} = -0.847 \text{ ksi}$$

SERVICE I

$\sum f_t =$ 1.0(1.323 + 0.592 + 0.121) + 1.0(0.461) = 2.497 ksi

SERVICE III

$\sum f_b =$ 1.0(-1.175 – 0.528 – 0.222) + 0.8(-0.847) = -2.603 ksi

Since the top and bottom midspan stresses are less for the exterior box beam than for the interior, the prestress design will be based on an interior box beam.

Transfer Length Stresses

Transformed Non-Composite at Transfer

M = 59 ft-k

$$f_t = \frac{(59)\cdot(12)\cdot(17.357)}{114{,}782} = 0.107\,\text{ksi}$$

$$f_b = \frac{(59)\cdot(12)\cdot(15.643)}{114{,}782} = -0.096\ \text{ksi}$$

Transformed Non-Composite at Service

M = 27 ft-k

$$f_t = \frac{(27)\cdot(12)\cdot(17.328)}{114{,}547} = 0.049\ \text{ksi}$$

$$f_b = \frac{(27)\cdot(12)\cdot(15.672)}{114{,}547} = -0.044\ \text{ksi}$$

Interior Box Beam

Transformed Composite

Dead Load: M = 7 + 7 = 14 ft-k

$$f_t = \frac{(14)\cdot(12)\cdot(13.322)}{177{,}388} = 0.013\ \text{ksi}$$

$$f_b = \frac{(14)\cdot(12)\cdot(19.678)}{177{,}388} = -0.019$$

LL + IM: M = 56 ft-k

$$f_t = \frac{(56)\cdot(12)\cdot(13.322)}{177{,}388} = 0.050$$

$$f_b = \frac{(56)\cdot(12)\cdot(19.678)}{177{,}388} = -0.075\ \text{ksi}$$

SERVICE I

$\sum f_t =$ 1.0(0.107 + 0.049 + 0.013) + 1.0(0.050) = 0.219 ksi

SERVICE III

$\sum f_b =$ 1.0(-0.096 – 0.044 – 0.019) + 0.8(-0.075) = -0.219 ksi

Prestress Design [5.9]

Step 3 – Determine Number of Strands

The design of a precast prestressed concrete beam involves making assumptions, calculating results, comparing the results to the assumptions and repeating the process until convergence occurs. The iterative process will not be shown in this example. Rather the final iteration with valid assumptions and calculations will be shown.

The required number of strands, the associated center of gravity, and long-term time-dependent losses, the release concrete strength and final concrete strength must be assumed. For this problem assume the following:

No. ½" Diameter Strands = 34
Time-Dependent Losses = 33.32 ksi
f'_{ci} = 4.4 ksi
f'_c = 5.0 ksi

The first calculation is to determine a strand pattern and associated center of gravity. For this problem 34 strands are required with 20 in the lower level and 14 in the second level. The center of gravity from the bottom equals:

$$c.g. = \frac{(20)\cdot(2)+(14)\cdot(4)}{34} = 2.824 \text{ in}$$

In this pattern 2 strands in each web are harped to within 2 inches of the top of the box beam with the hold-down point 8'-6" from the center of the beam. See Figure 12.

[5.9.4] [BDG]

With the above values the number of strands can be determined from the basic equation for stress for a prestressed member. While transformed section properties are used, note that the stresses from the time-dependent losses must be based on the net section properties. The number of strands is based on the tension limit of $0.0984\sqrt{f'_c}$ in the bottom fiber at midspan after all losses.

$$f_{bt} A_{str} NS\left(\frac{1}{A_t}+\frac{e_t y_{tb}}{I_t}\right) - f_{loss} A_{str} NS\left(\frac{1}{A_n}+\frac{e_n y_{nb}}{I_n}\right) + \sum f_b \geq -0.0948\sqrt{f'_c}$$

Solving for the number of strands, NS, results in:

$$NS \geq \frac{\left[-\sum f_b - 0.0948\sqrt{f'_c}\right]}{f_{bt} A_{str}\left(\frac{1}{A_t}+\frac{e_t y_{tb}}{I_t}\right) - f_{loss} A_{str}\left(\frac{1}{A_n}+\frac{e_n y_{nb}}{I_n}\right)}$$

Where:

$$f_{bt} A_{str}\left(\frac{1}{A_t}+\frac{e_t y_{tb}}{I_t}\right)=(200.27)\cdot(0.153)\cdot\left(\frac{1}{799.36}+\frac{(12.695)\cdot(15.519)}{117{,}014}\right)$$

$$= 0.089922$$

$$f_{loss} A_{str}\left(\frac{1}{A_n}+\frac{e_n y_{nb}}{I_n}\right)=(33.32)\cdot(0.153)\cdot\left(\frac{1}{760.55}+\frac{(13.342)\cdot(16.166)}{110{,}441}\right)$$

$$= 0.016659$$

Allowable Tension = $0.0948\sqrt{5.0} = -0.212$ ksi

$$NS \geq \frac{[-(-2.607)-0.212]}{0.089922-0.016659} = 32.69 \quad \text{Use 34 strands}$$

Step 4 – Determine Losses

Loss of Prestress [5.9.5]

Total losses in a prestressed precast member are due to relaxation before transfer, elastic shortening, and the time-dependent losses consisting of shrinkage, creep and relaxation losses.

Relaxation Loss Before Transfer

[5.9.5.3]
[BDG]

[5.9.5.4.4b-1]

The relaxation loss is broken up into two parts: the relaxation before transfer and the relaxation after transfer. The relaxation before transfer is the loss in stress from the time the strands are pulled until they are released. The equation for relaxation before transfer in the 2004 Specification is slightly different than the equation shown in the 1975 Commentary of the Standard Specifications with the denominator being changed from 45 to 40. For concrete release strengths less than 4.5 ksi, 18 hours may be assumed between time of concrete pour and time of strand release. Typically the strands will be pulled the day before the concrete is poured. Assume a total of 36 hours exists between time of stressing and time of release. The 2006 Interim Revisions deleted this loss without any explanation. However, when the strands are tensioned relaxation will occur until the strands are cut and the force transferred to the concrete. In some regions of the country fabricators overstress the strands initially to compensate for these losses but the fabricators in Arizona do not.

The relaxation loss before transfer for low-relaxation strands is:

$$\Delta f_{pR1} = \frac{\log(24.0t)}{40.0}\left[\frac{f_{pj}}{f_{py}}-0.55\right]f_{pj}$$

$$\Delta f_{pR1} = \frac{\log(24.0\cdot 1.50)}{40.0}\left[\frac{0.75}{0.90}-0.55\right]\cdot(0.75)\cdot(270) = 2.23 \text{ ksi}$$

The prestress stress before transfer = (0.75)(270) – 2.23 = 200.27 ksi

Elastic Shortening Losses Transformed Section Properties

Elastic shortening losses need not be determined to calculate concrete stresses when transformed section properties are used. However, elastic shortening losses are required for some calculations involving strand stresses and for the determination of time-dependent losses using the refined method.

An alternate to determining the elastic shortening losses directly, is to apply the self-weight of the member plus the prestress before transfer to the transformed section. The Commentary in Article C5.9.5.2.3a states that when calculating concrete stresses using transformed section properties, the effects of losses and gains due to elastic deformations are implicitly accounted for and the elastic shortening loss, $\Delta f_{pES,}$ should not be included in the prestressing force applied to the transformed section at transfer.

$e_t = 15.519 - 2.824 = 12.695$ in

The effective prestress force is the jacking stress minus the relaxation loss from time of stressing till time of transfer.

$P_{bt} = (200.27)(0.153)(34) = 1041.80$ k

The concrete stress at the centroid of the prestress steel using transformed section properties at transfer is:

$$f_{cgp} = (1041.80)\cdot\left[\frac{1}{799.36}+\frac{(12.695)^2}{117{,}014}\right]-\frac{(738)\cdot(12)\cdot(12.695)}{117{,}014}$$

$f_{cgp} = 2.738 - 0.961 = 1.777$ ksi

The elastic shortening loss is then determined as follows:

[5.9.5.2.3a-1]

$$\Delta f_{pES} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{3818}\cdot(1.777) = 13.26\,\text{ksi}$$

This method of determining the elastic shortening loss eliminates the need to estimate losses or use a trial and error approach. The method is direct, simple and produces the same elastic shortening loss as the Commentary equation using the net section properties as demonstrated on the following page.

When using transformed section properties the elastic shortening in not included as a loss in the prestress. Therefore this step is not required and is only shown for educational purposes. Refer to Appendix E of this examplefor a comparision of losses and stresses using different section properties for a further clarification on this issue.

Elastic Shortening [5.9.5.2.3a]

Net Section Properties

When transformed section properties are not used, elastic shortening losses can be calculated directly with a rather lengthy equation in lieu of a trial and error method. The equation for calculation of elastic shortening in the LRFD Commentary [C5.9.5.2.3a-1] is correct as long as the variable f_{pbt} includes the relaxation before transfer. The equation shown in the Commentary has been modified by dividing both the numerator and denominator by the area of the beam. This modification eliminates the need to work with large numbers improving the accuracy of the calculations. The net section properties are used in this calculation.

[C5.9.5.2.3a-1] Modified

$$\Delta f_{pES} = \frac{f_{pbt} A_{ps}\left(r^2 + e_m{}^2\right) - e_m M_g}{A_{ps}\left(r^2 + e_m{}^2\right) + \dfrac{I \cdot E_{ci}}{E_p}}$$

The elastic shortening loss will be calculated at the midspan.

Midspan Losses

<u>0.5 Span</u>

e_m = 16.166 – 2.824 = 13.342 in

A_{ps} = (34)(0.153) = 5.202 in^2

$$A_{ps}\left(r^2 + e_m{}^2\right) = (5.202)\cdot\left(145.21 + (13.342)^2\right) = 1681$$

$$\frac{I \cdot E_{ci}}{E_p} = \frac{(110{,}441)\cdot(3818)}{28{,}500} = 14{,}795$$

$$\Delta f_{pES} = \frac{(200.27)\cdot(1681) - 13.342\cdot(738)\cdot(12)}{1681 + 14{,}795} = 13.26 \text{ ksi}$$

Calculate f_{cgp} and verify the elastic shortening loss.

$$f_{cgp} = (5.202)\cdot[200.27 - 13.26]\cdot\left(\frac{1}{760.55} + \frac{(13.342)^2}{110{,}441}\right) - \frac{(738)\cdot(12)\cdot(13.342)}{110{,}441} = 1.777 \text{ ksi}$$

[5.9.5.2.3b-1]

$$\Delta f_{pES} = \frac{E_p}{E_{ci}}\cdot f_{cgp} = \left[\frac{28500}{3818}\right]\cdot(1.777) = 13.26 \text{ ksi OK}$$

Approximate Time-Dependent Losses Interim 2006

For standard precast pretensioned members the long-term prestress losses due to creep of concrete, shrinkage of concrete and relaxation of prestress steel may be estimated as follows:

[5.9.5.3-1]

$$\Delta f_{pLT} = 10.0\frac{f_{pi}A_{ps}}{A_g}\gamma_h\gamma_{st} + 12.0\gamma_h\gamma_{st} + \Delta f_{pR}$$

in which:

[5.9.5.3-2]

$$\gamma_h = 1.7 - 0.01H = 1.7 - 0.01(40) = 1.30$$

[5.9.5.3-3]

$$\gamma_{st} = \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 4.4} = 0.926$$

f_{pi} = prestressing steel stress immediately prior to transfer.

f_{pi} = (0.75)(270) – 2.23 = 200.27 ksi

Δf_{pR} = an estimate of relaxation loss taken as 2.5 ksi for low relaxation strand.

$$\Delta f_{pLT} = 10.0 \cdot \frac{(200.27)\cdot(5.202)}{766}\cdot(1.30)\cdot(0.926) + 12.0\cdot(1.30)\cdot(0.926) + 2.50$$

Δf_{pLT} = 33.32 ksi

The variables in the approximate equation do not vary along the span so the time-dependent losses will be constant.

The final loss excluding elastic shortening is shown below:

[5.9.5.3]
[DBG]

$\Delta f_{pT} = \Delta f_{pRbt} + \Delta f_{pES} + \Delta f_{pLT}$

Δf_{pT} = 2.23 + 0 + 33.32 = 35.55 ksi

The refined method of determining time-dependent losses is shown in Appendix C, of this example problem. At the time this problem was developed, Bridge Group was in the process of evaluating this method of loss calculation. For the purpose of this example, the approximate losses will be used.

[

Prestress Strand Stress [Table 5.9.3-1]

Step 5 – Check Allowable Stress in Strands

There are two limits for stress in prestress strands for pretensioned members. The first allowable limit is immediately prior to transfer. It is important to note that Table 5.9.3-1 has an error. Under Immediately prior to transfer the table includes the prestressing force plus the elastic shortening loss. The elastic shortening loss should not be included. This check is on the stress in the strands just prior to transfer. At this time these is no elastic shortening only relaxation before transfer since the stress has not been transferred to the concrete. The strands are usually pulled to a stress equal to 0.75 f_{pu}.

(1) $f_{pj} = 0.75\ f_{pu} - (2.23) / (270) = 0.742\ f_{pu} < 0.75\ f_{pu}$ OK.

The second stress limit is a service limit state after all losses. The dead load (excluding self-weight) and live load plus dynamic load allowance is considered.

$f_{pe} = 0.75f_{pu} - (13.26 + 35.55) / (270) = 0.569\ f_{pu}$ after all losses

At service limit state added dead load and live load plus dynamic allowance stresses are added to the strand stress since the strands are bonded.

$$f_{service} = \left[\frac{(330)\cdot 12\cdot(12.733)}{116{,}626} + \frac{(89+88+645)\cdot 12\cdot(16.762)}{180{,}203}\right] \cdot \frac{28{,}500}{4070} = 9.452 \text{ ksi}$$

Strand stress = $0.569\ f_{pu} + (9.452) / (270) f_{pu} = 0.604\ f_{pu}$

(2) Strand stress = $0.604\ f_{pu} < 0.80\ f_{py} = 0.80(0.90)\ f_{pu} = 0.720\ f_{pu}$

Since the two criteria for stress in the strand are met, the jacking coefficient of 0.75 is satisfactory.

Step 6 – Verify Initial Concrete Strength

Once the amount of prestressing steel is determined from tension criteria, the resulting concrete stress and required concrete strength can be determined. Service I limit state is used to determine the initial concrete compressive stress. The concrete stress in compression before time dependent losses is limited to:

[5.9.4.1.1]
[5.9.4]
[BDG]

$$Allowable\ Compression = 0.60 \cdot f'_{ci} = 0.60 \cdot (4.4) = 2.640\ ksi$$

The basic equation for stress in concrete follows:

$$f_s = A_{ps} f_{si} \left(\frac{1}{A} + \frac{e_m y}{I} \right) + \frac{\sum (\gamma M) y}{I}$$

<u>Hold-Down Point (33.50 feet from CL Brg)</u>

f_{si} = 200.27 ksi

F_{pi} = (5.202)(200.27) = 1041.80 kips

$$f_b = \frac{(706)\cdot(12)\cdot(15.519)}{117{,}014} = -1.124\ \text{ksi}$$

$$f_b = (1041.80)\cdot\left[\frac{1}{799.36} + \frac{(12.695)\cdot(15.519)}{117{,}014}\right] - 1.124 = 1.933\ \text{ksi}$$

<u>Transfer Length (1.75 feet from CL Brg)</u>

$$f_b = \frac{(59)\cdot(12)\cdot(15.643)}{114{,}782} = -0.096\ \text{ksi}$$

$$f_b = (1041.80)\cdot\left[\frac{1}{799.36} + \frac{(9.875)\cdot(15.643)}{114{,}782}\right] - 0.096 = 2.609\ \text{ksi}$$

Additional refinement of the example is not likely to reduce the required release strength of f'_{ci} = 4.4 ksi. Since the release strength is not excessive the design is adequate. Note that a reduction in the release strength will affect the modulus of elasticity and the elastic shortening loss. Since, the initial concrete stresses are less than the allowable compressive stress, f'_{ci} = 4.4 ksi is acceptable.

Step 7 – Temporary Tension at Ends

The stresses at the end of the beam and at the hold-down points of the precast beams must be checked to ensure that the eccentricity is limited to keep any tension within the allowable limits. As with the compressive check, the end critical location will be at the end of the transfer length.

The allowable tension in the top of the precast beam without additional mild reinforcement equals:

[5.9.4]
[BDG]

$$\text{Allowable Tension} = 0.0948\sqrt{f'_{ci}} = 0.0948\sqrt{4.4} = 0.199 \text{ ksi} < 0.200 \text{ ksi}$$

Hold-Down Point (33.50 feet from CL Brg)

$$f_t = \frac{(706)\cdot(12)\cdot(17.481)}{117{,}014} = 1.266 \text{ ksi}$$

$$f_t = (1041.80)\cdot\left[\frac{1}{799.36} - \frac{(12.695)\cdot(17.481)}{117{,}014}\right] + 1.266 = 0.593 \text{ ksi}$$

Transfer Length (1.75 feet from CL Brg)

$$f_t = \frac{(59)\cdot(12)\cdot(17.357)}{114{,}782} = 0.107 \text{ ksi}$$

$$f_t = (1041.80)\cdot\left[\frac{1}{799.36} - \frac{(9.875)\cdot(17.357)}{114{,}782}\right] + 0.107 = -0.145 \text{ ksi}$$

Since the tension is less than the allowable, the criteria is satisfied without adding mild reinforcing in the top of the beam.

Step 8 – Determine Final Concrete Strength

The required final concrete strength is determined after all prestress losses at the midspan and the transfer length. The Service I Limit State load combination is used for each of the three compressive load cases.

[Table 5.9.4.2.1-1]

Prestress after transfer (Use transformed properties at transfer):

Midspan

$F_i = (5.202)[(0.75)(270) – 2.23] = 1041.80$ k

$$f_t = (1041.80)\cdot\left(\frac{1}{799.36} - \frac{(12.695)\cdot(17.481)}{117{,}014}\right) = -0.673 \text{ ksi}$$

$$f_b = (1041.80)\cdot\left(\frac{1}{799.36} + \frac{(12.695)\cdot(15.519)}{117{,}014}\right) = 3.057 \text{ ksi}$$

Prestress time-dependent losses (Use net properties):

$F_{loss} = (5.202)(33.32) = 173.33$ k

$$f_t = (-173.33)\cdot\left(\frac{1}{760.55} - \frac{(13.342)\cdot(16.834)}{110{,}441}\right) = 0.125 \text{ ksi}$$

$$f_b = (-173.33)\cdot\left(\frac{1}{760.55} + \frac{(13.342)\cdot(16.166)}{110{,}441}\right) = -0.566 \text{ ksi}$$

<u>Case I</u> – Permanent Loads plus Effective Prestress

Allowable Compression = 0.45 f'_c = (0.45)(5.0) = 2.250 ksi

f_t = 1.0(-0.673 + 0.125 + 1.323 + 0.592 + 0.158) = 1.525 ksi < 2.250 ksi

<u>Case II</u> – One-half the Case I loads plus LL + IM

Allowable Compression = 0.40 f'_c = (0.40)(5.0) = 2.000 ksi

f_t = ½(1.525) + 1.0(0.576) = 1.339 ksi < 2.000 ksi Allowable

Case III – Effective Prestress, Permanent Loads and Transient Loads

Allowable Compression = 0.60 φ_w f'$_c$ = 0.60(1.00)(5.0) = 3.000 ksi

The reduction factor φ_w shall be taken equal to 1.0 when the wall slenderness ratio λ_w is not greater than 15. The critical slenderness ratio involves the bottom slab.

$$\lambda_w = \frac{X_u}{t} = \frac{(47.50 - 10.00)}{6.00} = 6.3 \leq 15$$

Since the ratio is less than the allowable, the equivalent rectangular stress block can be used and φ_w = 1.00.

f_t = 1.0(-0.673 + 0.125) + 2.649 = 2.101 ksi < 3.000 ksi Allowable

Transfer Length

Transfer Length

Since the relaxation loss before transfer and the time-dependent losses do not vary along the length of the beam the effective prestress force is the same as determined at midspan.

Prestress after transfer (Use transformed properties at transfer):

$$f_b = (1041.80)\cdot\left(\frac{1}{799.36} + \frac{(9.875)\cdot(15.643)}{114{,}782}\right) = 2.705 \text{ ksi}$$

Prestress time-dependent losses (Use net properties):

$$f_b = (-173.33)\cdot\left(\frac{1}{760.55} + \frac{(10.378)\cdot(16.146)}{110{,}805}\right) = -0.490 \text{ ksi}$$

Case I – Permanent Loads plus Effective Prestress

f_b = 1.0(2.705 – 0.490 –0.096 –0.044 – 0.019) = 2.056 ksi < 2.250 ksi Allowable

Since the live load causes tension in the bottom fiber, Cases 2 and 3 will have a smaller compressive stress and therefore will not control.

Step 9 – Determine Final Concrete Tension

Determination of the tension in the concrete is a Service III Limit State. This step is not required since the number of strands was determined based on the tension in the bottom fiber in Step 3 ensuring that the criteria is satisfied. The allowable tension after all losses is limited to:

[5.9.4]
[BDG]

$$Allowable\ Tension = 0.0948\sqrt{f'_c} = 0.0948\sqrt{5.0} = 0.212\ ksi$$

The basic equation for stress in concrete is:

$$f = A_{ps} f_{se}\left[\frac{1}{A} + \frac{e_m y}{I}\right] - \frac{\sum(\gamma M)y}{I}$$

<u>Bottom fiber at Midspan</u>

f_b = 1.0(3.057 – 0.566) –2.607 = -0.116 ksi < -0.212 ksi

Since the tension is less than the allowable tension the criteria is satisfied.

Fatigue Limit State [5.5.3.1]

Fatigue of the reinforcement need not be checked for fully prestressed components designed to have extreme fiber tensile stress due to Service III Limit State with the tensile stress limit specified in Table 5.9.4.2.2-1.

Flexural Resistance [5.7.3]

Step 10 – Flexural Resistance

The flexural resistance of the box beam must exceed the applied factored loads. Strength I is used.

$M_r = \varphi M_n < \sum\gamma M = M_u$

Midspan

STRENGTH I: $\sum\gamma M = M_u$ = 2761 ft-k (exterior beam)

$A_{ps} = (0.153)(34) = 5.202\ in^2$

[5.7.3.1.1-1]

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

[5.7.3.1.1-2]

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 2\left(1.04 - \frac{243}{270}\right) = 0.28 \text{ for low relaxation strand}$$

For a rectangular section without mild reinforcing steel:

d_p = 33.00 + 5.00 – 0.5 w.s. – 2.824 = 34.68 inches

[5.7.3.1.1-4]

$$c = \frac{A_{ps}f_{pu}}{0.85f'_c\beta_1 b + kA_{ps}\frac{f_{pu}}{d_p}}$$

$$c = \frac{(5.202)\cdot(270)}{0.85\cdot(4.5)\cdot(0.825)\cdot(48.00) + 0.28\cdot(5.202)\cdot\frac{270}{34.68}} = 8.63 < t_{slab} = 10.0”$$

The depth of the stress block is such that it crosses two layers of concrete with different strengths. This effect can be handled by using β_1 and f'_c for the lower strength concrete. A more complex method of producing a weighted average is shown in the Specification. However, the effort required by this method is not worth the extra effort required by the refinement.

[C 5.7.2.2]

Since the stress block depth is less than the composite slab thickness, the section is treated as a rectangular section:

$$a = c\beta_1 = (8.63)\cdot(0.825) = 7.12\ in$$

The tensile strain must be calculated as follows:

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1\right) = 0.003 \cdot \left(\frac{34.68}{8.63} - 1\right) = 0.009$$

[C 5.5.4.2-1]

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 1.00$.

$$f_{ps} = (270) \cdot \left(1 - (0.28) \cdot \frac{8.63}{34.68}\right) = 251.19\ ksi$$

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2}\right)$$

$$M_n = (5.202) \cdot (251.19) \cdot \left(34.68 - \frac{7.12}{2}\right) \div 12 = 3389 \text{ ft-k}$$

$\varphi M_n = 1.0(3389) = 3389$ ft-k > $M_u = 2761$ ft-k

$\therefore$ Section is adequate for flexural resistance

Minimum Reinforcing [5.7.3.3.2]

There is also a minimum amount of reinforcement that must be provided in a section. The amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

1.2 M_{cr}
1.33 M_u

The cracking moment is determined on the basis of elastic stress distribution and the modulus of rupture of the concrete.

$$M_{cr} = S_c(f_r + f_{cpe}) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c f_r$$

Use gross section properties.

$$S_c = \frac{I}{y_b} = \frac{171{,}153}{20.125} = 8504 \text{ in}^3$$

$$S_{nc} = \frac{I}{y_b} = \frac{111{,}361}{16.076} = 6927 \text{ in}^3$$

Use transformed section properties at transfer to determine initial stress:

$$f_i = (1041.80)\cdot\left[\frac{1}{799.36} + \frac{(12.695)\cdot(15.519)}{117{,}014}\right] = 3.057 \text{ ksi}$$

Use net section properties to determine stress from prestress losses:

$$f_{loss} = (-173.33)\cdot\left(\frac{1}{760.55} + \frac{(13.342)\cdot(16.166)}{110{,}441}\right) = -0.566 \text{ ksi}$$

$$M_{cr} = (8504)\cdot(0.827 + 3.057 - 0.566) \div 12 - (738 + 330)\cdot\left(\frac{8504}{6927} - 1\right)$$

M_{cr} = 2108 ft-k

$S_c f_r$ = (8504)(0.827) ÷ 12 = 586 ft-k minimum

1.2 M_{cr} = (1.2)(2108) = 2530 < φM_n = 3389 ft-k

Therefore the minimum reinforcing requirement is satisfied.

Positive Moment Continuity Connection

The girders must be connected at the bottom to resist any positive moment at the supports. This moment is caused by restraining the girder as the ends tend to rotate due to creep and shrinkage. Usually strands are extended and hooked up into the cast-in-place diaphragm to resist this moment.

The LRFD Specification does not contain any direction as to design for these forces. Therefore, the design for the positive moment connection will follow the procedure outlined in the PCA publication "Design of Continuous Highway Bridges with Precast, Prestressed Concrete Girders", August 1969. Figures referenced PCA in this section refer to Figures in that publication. The end rotations from dead load, prestress and differential shrinkage from the cast-in-place slab are restrained by the continuity connection with appropriate creep factors considered. In addition negative moments will result from the composite dead load such as the barriers. For a three span bridge, positive moments will result from live loads in remote spans.

Creep Factor

Since creep is time dependent, the amount of positive restraint moment induced depends on the time when the continuity connection is made. The sooner the connection is made the higher the restraint moments will be. For design purposes 30 days will be used. For design, the ultimate creep value of 0.386×10^{-6} may be taken from PCA Figure 5 for a concrete with an initial modulus of 3818 ksi using the 20-year creep curve as the ultimate creep. This value must be modified to adjust for the effect of age when the girders are prestressed and for the volume/surface ratio.

Assuming the prestress is transferred to the concrete at day one, the creep adjustment factor is 1.80 from PCA Figure 6. The beam volume/surface ratio 4.76 is used ignoring the interior surface area. The creep adjustment factor for volume/surface ratio is 1.16 from PCA Figure 7.

The amount of creep that has occurred before the connection is made at an assumed time of 30 days is 40 percent from PCA Figure 8. This means that 60 percent of the creep occurs after the connection is made contributing to the restraint moment.

The adjusted creep strain is:

$$\varepsilon_s = (0.386 \times 10^{-6})(1.80)(1.16)(0.60) = 0.484 \times 10^{-6}$$

The effects of creep under prestress and dead load can be evaluated by standard elastic analysis methods by assuming the elements were cast and prestressed as a monolithic continuous girder. The variable φ is the ratio of creep strain to elastic strain. This value can be determined by multiplying the creep strain by the modulus of concrete as follows:

$$\varphi = (0.484 \times 10^{-6})(3818 \times 10^{3}) = 1.848$$

The continuity moments are then multiplied by the following factor to account for creep:

$$Creep\ Factor = \left(1 - e^{-\varphi}\right) = \left(1 - e^{-1.848}\right) = 0.842$$

Non-Composite DL and P/S

Once the girders are restrained, additional creep rotation from the non-composite dead load of the girder, diaphragms and deck slab will cause a restraint moment. This restraint moment is the moment at the support resulting from the analysis of a continuous beam with the weight of the girder, diaphragms and slab adjusted by the dead load creep factor. From the continuous beam analysis the resulting dead load moments are show below:

Girder	-588 ft-k
Diaphragms	-27 ft-k
Slab & NC DL	-274 ft-k
	-889 ft-k

CR: Adjusted DL = (-889)(0.842) = -749 ft-k

The final prestress force is applied to the continuous beam resulting in a positive support moment of 1270 ft-k.

CR: Adjusted P/S = (1270)(0.842) = 1069 ft-k

Composite DL LL + IM

The barrier will cause negative moments at the piers while the live load plus dynamic load allowance will cause a positive moment. However, there is no creep modification factor for these loads since the loads are applied after the bridge has been made continuous and are not the result of creep restraint. The following pier moments result:

DC: Barrier	-74 ft-k
LL + IM	85 ft-k

Differential Shrinkage

The remaining force is the differential shrinkage caused by the time delay between casting the box beam and placing the deck. During this time, the box beam shortens due to shrinkage. When the deck is cured the deck and box beam will shorten together. However, the deck must undergo all its shrinkage while the box beam has already seen much of its shortening. The deck will shorten more relative to the box beam causing a positive moment along the span. This results in a negative restraint moment at the support.

When test data is not available, the ultimate shrinkage of concrete at a relative humidity of 50 percent can be estimated as 0.600×10^{-3}. This value must be corrected for humidity variances. For a relative humidity of 40 percent the correction factor is 1.09 from PCA Figure 10.

Assuming a 30 day lapse between casting the box beams and placing the deck, the box beam will have undergone 40 percent of its shrinkage as seen from PCA Figure 8. This means that the box beam/deck system will see a differential shrinkage equal to 40 percent of the total shrinkage. The differential shrinkage strain is:

$$\varepsilon_s = (0.600 \times 10^{-3})(1.09)(0.40) = 0.262 \times 10^{-3}$$

The equation for the differential shrinkage moment applied to the box beam along its entire length is:

$$M_{ds} = \varepsilon_s E_b A_b \left(y_t + \frac{t}{2} \right)$$

ε_s = differential shrinkage strain
E_b = elastic modulus for the deck slab concrete = 3861 ksi
A_b = area of deck slab = (4.50)(48) = 216 in^2
y_t = distance to the top of beam from the centroid of the gross composite section = 12.875 in

$$M_{ds} = \left(0.262x10^{-3}\right)\cdot\left(3861\right)\cdot\left(216\right)\cdot\left(12.875 + \frac{4.50}{2}\right) \div 12$$

M_{ds} = 275 ft-k

For a 3 span continuous beam with equal spans, the support moment will equal 1.20 times the uniformly applied moment. Therefore the support moment equals (-1.20)(275) = –330 ft-k.

The negative support moment due to differential shrinkage is adjusted for creep by the following factor:

$$Creep\ Factor = \frac{\left(1-e^{-\varphi}\right)}{\varphi} = \frac{\left(1-e^{-1.848}\right)}{1.848} = 0.456$$

The support moment for differential shrinkage must be adjusted by the above creep factor resulting in:

CR: Adjusted Differential Shrinkage = (0.456)(-330) = -151 ft-k

Combining the above loads results in the following:

Service I Limit State

Limit States
[Table 3.4.1-1]

M_s = 1.0(DC) + 1.0(LL + IM) + 1.0(SH + CR)
M_s = 1.0(– 74) + 1.0(85) +1.0(-749 + 1069 -151) = 180 ft-k

Strength I Limit State

M_u = 0.90DC + 1.75(LL + IM) + 0.5(SH + CR)
M_u = 0.90(– 74) + 1.75(85) + 0.5(-749 + 1069 - 151) = 167 ft-k

The prestress strands will be extended to resist the positive moment. The strands will be designed based on the criteria in Report No. FHWA-RD-77-14, "End Connections of Pretensioned I-Beam Bridge", November 1974.

Try extending 8 strands with four each in the bottom two rows.

A_s = 8(0.153) = 1.224 in^2
c.g. = 3.00 in
d = 38.00 – 0.5 ws – 3.00 = 34.50 inch

$$p = \frac{A_s}{bd} = \frac{1.224}{(48.00)\cdot(34.50)} = 0.000739$$

np = (7) (0.000739) = 0.00517

Service I
Limit State

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.00517)+(0.00517)^2} - 0.00517 = 0.097$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.097}{3} = 0.968$$

$$f_s = \frac{M_s}{A_s jd} = \frac{(180)\cdot(12)}{(1.224)\cdot(0.968)\cdot(34.50)} = 52.84\ \text{ksi}$$

From test data in the research report the recommended development length due to service loads for strands bent 90° over a reinforcing bar is:

$L_e = 0.228f_s + 8.25"$ when $L_{pb} \le 8.25"$

$$L_e = 0.225\left[f_s - \frac{L_{pb} - 8.25}{0.472}\right] + L_{pb} \text{ when } L_{pb} > 8.25"$$

Normally the gap between girders is 12 inches. With two rows of strands extended the lower one is extended 10 inches while the upper one is extended 8 inches. To simplify the design assume that both rows are extended only 8 inches. Therefore L_{pb}, the length to the bend, is ≤ 8.25 inches.

$L_e = (0.228)(52.84) + 8.25 = 20.3$ inches

Strength I Limit State [Table 3.4.1-1]

The strength limit state must also be checked. From the research report an upper limit of 150 ksi is placed on the stress in the strand with the required development length as follows:

$L_e = 0.163f_{ps} + 8.25$ when $L_{pb} \le 8.25"$

$$L_e = 0.163\left[f_{ps} - \frac{L_{pb} - 8.25}{0.337}\right] + L_{pb} \text{ when } L_{pb} > 8.25"$$

Try a 21 inch extension and rearrange the equation to solve for f_{ps}.

$$f_{ps} = \frac{L_e - 8.25}{0.163} = \frac{21 - 8.25}{0.163} = 78.22 \text{ ksi} \le 150 \text{ ksi}$$

$$a = \frac{A_s f_{ps}}{0.85 f'_c b} = \frac{(1.224)\cdot(78.22)}{0.85\cdot(4.5)\cdot(48.00)} = 0.52 \text{ in}$$

$$\varphi M_n = \phi A_s f_{ps}\left(d - \frac{a}{2}\right) = (0.90)\cdot(1.224)\cdot(78.22)\cdot\left(34.50 - \frac{0.52}{2}\right) \div 12$$

$\varphi M_n = 246$ ft-k > $M_u = 167$ ft-k

Therefore, the service and strength limit states are satisfied by extending 8 strands a total of 21 inches.

Negative Moment Continuity Reinforcement

The precast prestressed box beam behaves as a simple span under self-weight and the non-composite dead loads. However, this bridge type is made continuous to eliminate the expansion joints and improve the riding surface of the deck. Continuity is provided by designing an adequate amount of mild reinforcing steel in the top slab of the deck to resist the negative moments from the composite dead loads, live load plus dynamic load allowance and any creep or shrinkage restraint moment.

To maximize the negative moment, the restraint moment should be determined at a time of 120 days. This longer time will produce a greater negative shrinkage restraint moment than the normal 60 days assumed for the deck pour and the 30 days assumed for positive connection design. The method of determining the shrinkage restraint forces was shown in the previous section on positive moment continuity connection.

Creep Factor

The only variable that changes from the previous calculation is the time used for creep. The amount of creep that has occurred before the connection is made, at an assumed time of 120 days, is 65 percent from PCA Figure 8. This means that 35 percent of the creep occurs after the connection is made contributing to the restraint moment.

The adjusted creep strain is:

$$\varepsilon_s = \left(0.386\ x\ 10^{-6}\right)\cdot\left(1.80\right)\cdot\left(1.16\right)\cdot\left(0.35\right) = 0.282\ x\ 10^{-6}$$

The variable φ is determined as follows:

$$\varphi = (0.282 \text{ x } 10^{-6})(3818 \text{ x } 10^{3}) = 1.077$$

The continuity moments are then multiplied by the following factor to account for creep:

$$Creep\ Factor = \left(1 - e^{-\varphi}\right) = \left(1 - e^{-1.077}\right) = 0.659$$

CR: Adjusted DL = (-889)(0.659) = -586 ft-k

CR: Adjusted P/S = (1270)(0.659) = 837 ft-k

Differential Shrinkage

Assuming a 120 day lapse between casting the box beams and placing the deck, the box beam will have undergone 65 percent of its shrinkage as seen from PCA Figure 8. This means that the box beam/deck system will see a differential shrinkage equal to 65 percent of the total shrinkage. The differential shrinkage strain is:

$$\varepsilon_s = (0.600 \text{ x } 10^{-3})(1.09)(0.65) = 0.425 \text{ x } 10^{-3}$$

The differential shrinkage moment equals:

$M_{ds} = (0.425 \times 10^{-3})(3861)(216)(12.875 + 4.50 / 2) \div 12$
$M_{ds} = 447$ ft-k

The negative support moment due to differential shrinkage is adjusted for creep by the following factor:

$$Creep\ Factor = \frac{\left(1-e^{-\varphi}\right)}{\varphi} = \frac{\left(1-e^{-1.077}\right)}{1.077} = 0.612$$

From a continuous beam analysis of uniformly applied moment from the differential shrinkage, the support moment equals (-1.20)(447) = –536 ft-k. This value must be adjusted by the creep factor of 0.612 resulting in a moment equal to (0.612)(-536) = -328 ft-k.

A continuous beam analysis is made for DC, DW and LL+IM for a configuration with spans of 85’-3”, 86’-6”, 85’-3”. For an interior girder the live load from one vehicle is modified by the distribution factor of 0.293 and the skew reduction factor of 1.0. The sum of the creep and shrinkage moments equal -586 + 837 –328 = -77 ft-k

Strength I Limit State

For the Strength I Limit State where DC and DW moments are positive the FWS is ignored and the following equation applies:

$M_u = 0.90(DC) + 1.75(LL + IM) + 0.5(SH + CR)$

When both DC and DW are negative the following equation applies:

$M_u = 1.25(DC) + 1.50(DW) + 1.75(LL + IM) + 0.5(SH + CR)$

A summary of negative moments for an interior box beam follows:

Span 1

	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DC (Barriers)	58	55	44	25	-1	-34	-74
DW (FWS)	58	54	43	25	-1	-34	-74
One Vehicle	-349	-436	-524	-611	-809	-1052	-1764
LL + IM	-102	-128	-154	-179	-237	-308	-517
SH & CR	-31	-39	-46	-54	-62	-69	-77
Service I	-75	-112	-156	-208	-301	-445	-742
Strength I	-142	-194	-253	-318	-449	-667	-1147

Span 2

	0.0	0.1	0.2	0.3	0.4	0.5
DC (Barriers)	-74	-40	-14	5	16	20
DW (FWS)	-74	-40	-14	5	16	20
One Vehicle	-1764	-1177	-960	-724	-624	-525
LL + IM	-517	-345	-281	-212	-183	-154
CU & SH	-77	-77	-77	-77	-77	-77
Service I	-742	-502	-386	-284	-244	-211
Strength I	-1147	-752	-569	-405	-344	-290

One Vehicle = design lane plus design truck or design tandem for one lane including dynamic load allowance.

LL + IM = One Vehicle times the distribution factor of 0.293.

A comparison of the restraint moments for 30 days versus 120 days reveals an interesting trend. At 30 days, the time used to determine the positive continuity moment connection, the restraint moment is –749 + 1069 –151 = 169 ft-k. At 120 days, the time used to determine the negative continuity moment, the restrain moment is –586 + 837 –328 = -77 ft-k. While the dead load and prestress creep moments reduce with increased time, the negative restraint moment due to differential shrinkage increases. Since the time of erection and deck pour is not controllable, it is reasonable to bracket the extremes for design.

Negative Moment Design

Even though the top surface serves as the deck the allowable stress in the longitudinal direction is not limited to 24 ksi as is the case for transverse reinforcing. Thus the service limit state may not control the design.

Service I Limit State [Table 3.4.1-1]

$$M_s = 1.0(M_{DC} + M_{DW}) + 1.0(M_{LL+IM}) + 1.0(M_{SH+CR})$$

Composite Loads:

M_{LL+IM} = (-1764)(0.293) = -517 ft-k

M_s = 1.0(-74 - 74) + 1.0(-517) +1.0(-77) = -742 ft-k

Allowable Stress

Try #10 reinforcing bars

d_s = 38.00 – 2.50 clear – 0.625 – 1.27 / 2 = 34.24 inches

Since there is no direct check for the allowable stress, assume a stress of 36 ksi maximum with the understanding that his step may have to be repeated.

$$A_s \approx \frac{M_s}{f_s j d_s} = \frac{(742)\cdot(12)}{(36.0)\cdot(0.9)\cdot(34.24)} = 8.03\ \text{in}^2$$

Try alternating #9 and #10 @ 6 inches

A_s = (8)(1.00 + 1.27) ÷2 = 9.08 in^2 per box beam

Determine stress block depth assuming a rectangular section.

$$p = \frac{A_s}{bd_s} = \frac{9.08}{(47.50)\cdot(34.24)} = 0.00558$$

np = 7(0.00558) = 0.03908

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.03908) + (0.03908)^2} - 0.03908 = 0.243$$

kd = (0.243)(34.24) = 8.33 inch > 6.00 inch bottom slab thickness

Since the depth of the stress block exceeds the depth of the bottom slab, the section must be treated as a T-section.

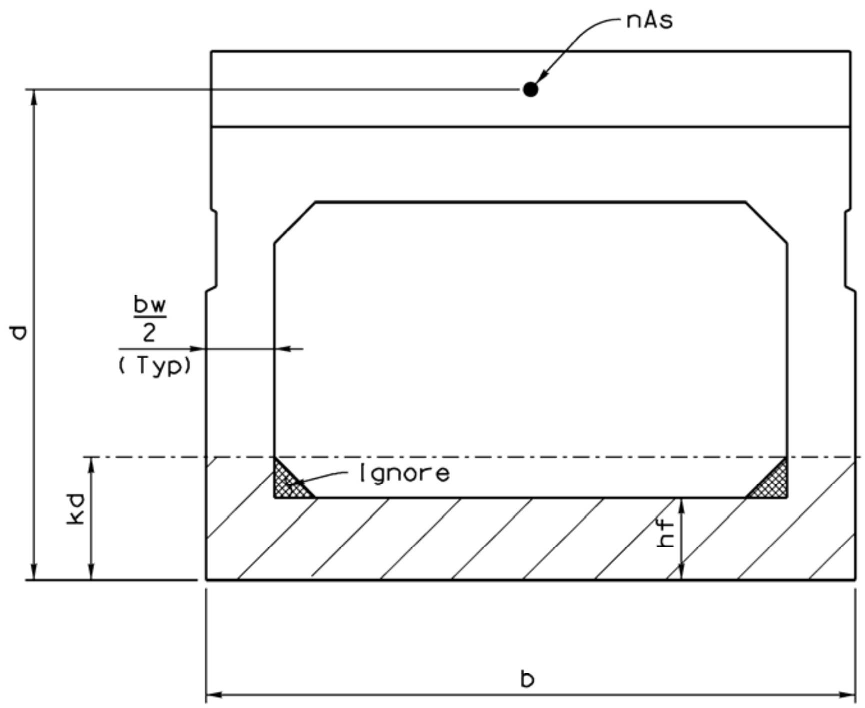


Figure 16

Transform the area of reinforcing into an equivalent area of concrete and take moments about the neutral axis ignoring the fillets. The following equation results:

$nA_s(d - kd) = (b - b_w)h_f(kd - h_f \div 2) + b_w(kd)^2 \div 2$

$(b_w \div 2)(kd)^2 + [(b - b_w)h_f + nA_s](kd) - (b - b_w)h_f^2 \div 2 - nA_sd = 0$

Solving the quadratic equation for kd results in the following coefficients:

$A = b_w \div 2 = 10.00 \div 2 = 5.00$

$B = (b - b_w)h_f + nA_s = (47.50 - 10.00)(6.00) + 7(9.08) = 288.56$

$C = -(b - b_w)h_f^2 \div 2 - nA_sd = -(47.50 - 10)(6.00)^2 \div 2 - 7(9.08)(34.24)$
$= -2851$

$$kd = \frac{-288.56 + \sqrt{(288.56)^2 - 4 \cdot (5.00) \cdot (-2851)}}{2 \cdot (5.00)} = 8.60 \text{ in}$$

Determine the moment of inertia of the cracked section about the neutral axis as follows:

$$I_{cr} = nA_s\left(d - kd\right)^2 + \frac{b_w\left(kd\right)^3}{3} + \frac{\left(b - b_w\right)h_f^{\,3}}{12} + \left(b - b_w\right)h_f\left(kd - \frac{h_f}{2}\right)^2$$

I_{cr} = (7)(9.08)(34.24 – 8.60)^2 + (10.00)(8.60)^3÷3 + (47.50 – 10)(6.00)^3÷12 + (47.50 – 10)(6.00)(8.60 – 6.00 ÷2)^2 = 51,636 in^4

y_t = 34.24 – 8.60 = 25.64 in

$$f_s = \frac{(7)\cdot(742)\cdot(12)\cdot(25.64)}{51{,}636} = 30.95 \text{ ksi}$$

There is not a direct stress limit in the LRFD Specification. The Bridge Group limit of 24 ksi for decks applies to the transverse reinforcing not the longitudinal reinforcing. However, the reinforcing stress is required in determining the maximum allowable reinforcing spacing.

Control of Cracking [5.7.3.4]

For all concrete components in which the tension in the cross-section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in Equation 5.7.3.4-1 shall be satisfied.

f_{sa} = 0.80f_r = 0.80($0.24\sqrt{f'_c}$) = 0.80(0.509) = 0.407 ksi

$$f_{cr} = \frac{M_s y_t}{I_g} = \frac{(742)\cdot(12)\cdot(12.875 + 4.5)}{171{,}153} = 0.904\ ksi\ > f_{sa} = 0.407 \text{ ksi}$$

Since the service limit state cracking stress exceeds the allowable, the spacing, s, of mild steel reinforcing in the layer closest to the tension force shall satisfy the following:

[Equation 5.7.3.4-1]

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

where

γ_e = 0.75 for Class 2 exposure condition for decks

d_c = 2.5 clear + 0.625 + 1.27 ÷ 2 = 3.76 inches

f_s = 30.95 ksi

h = 38.00 inches

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{3.76}{0.7 \cdot (38.00 - 3.76)} = 1.16$$

$$s \le \frac{(700)\cdot(0.75)}{(1.16)\cdot(30.95)} - (2)\cdot(3.76) = 7.10 \text{ in}$$

Since the spacing of 6.0 inches is less than 7.10 inches, the cracking criteria is satisfied.

Strength I Limit State [3.4.1]

Factored moment for Strength I is as follows:

$$M_u = \gamma_{DC} M_{DC} + \gamma_{DW} M_{DW} + 1.75 M_{LL+IM} + 0.5 M_{SH+CR}$$

$$M_u = 1.25\cdot(-74) + 1.50\cdot(-74) + 1.75\cdot(-517) + 0.5\cdot(-77) = -1147 \text{ ft-k}$$

Verify section type by calculating depth of rectangular stress block.

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b_w}$$

$$c = \frac{(9.08)\cdot(60)}{0.85\cdot(5.0)\cdot(0.800)\cdot(47.50)} = 3.37 \text{ in} < 6.00 \text{ in bottom slab}$$

Since the depth is less than the bottom slab depth the section is treated as a rectangular section.

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \phi M_n$$

[Equation5.7.3.2.2-1]

$$M_n = A_s f_y \left(d_s - \frac{a}{2} \right)$$

[5.7.3.2.3]

$$a = \beta_1 c = (0.800)\cdot(3.37) = 2.70 \text{ in}$$

$$M_n = (9.08)\cdot(60)\cdot\left(34.24 - \frac{2.70}{2}\right) \div 12 = 1493 \text{ ft-k}$$

The tensile strain must be calculated as follows:

$$\varepsilon_T = 0.003\left(\frac{d_t}{c}-1\right) = 0.003\cdot\left(\frac{34.24}{3.37}-1\right) = 0.027$$

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 0.90$ for the reinforced member.

[5.5.4.2.1]

$$M_r = (0.90)\cdot(1493) = 1344 \text{ ft-k}$$

Since the flexural resistance, M_r = 1344 ft-k, is greater than the factored moment, M_u = 1147 ft-k, the strength limit state is satisfied.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u. Use the composite gross section properties for this calculation.

S_c = (171,153) / (12.875 + 4.50) = 9851 in^3

$$1.2M_{cr} = 1.2 f_r S_c = (1.2)\cdot(0.785)\cdot(9851)\div 12 = 773 \text{ ft-k}$$
$$1.2M_{cr} = 773 \le M_r = 1344 \text{ ft-k}$$

Therefore the minimum reinforcement limit is satisfied.

Fatigue

The stress range in the continuous reinforcing over the pier must be checked for fatigue.

Fatigue Load [3.6.1.4]

The fatigue load shall be one design truck but with a constant 30.0 feet between the 32.0 kip axles. The dynamic load allowance of 15 percent shall be applied to the fatigue load. From the live load generator, the maximum fatigue truck moment at the support is:

Negative LL + IM = -605 ft-k
Positive LL + IM = 152 ft-k

The live load distribution for an interior girder with one design lane loaded is 0.197.

Multiple Presence Factor [3.6.1.1.2]

The multiple presence factor is already included in the distribution factor. Therefore the force effect should be divided by 1.20 when investigating fatigue.

Fatigue Limit State [3.4.1]

$M_f = 0.75M_{LL+IM} = (0.75)(-605)(0.197) / 1.20 = -74$ ft-k
$M_f = 0.75M_{LL+IM} = (0.75)(152)(0.197) / 1.20 = 19$ ft-k

$$f_{range} = \frac{(7)\cdot(74+19)\cdot(12)\cdot(25.64)}{51{,}636} = 3.88 \text{ ksi}$$

[5.5.3.2-1]

$$f_f = 21 - 0.33 f_{\min} + 8\left(\frac{r}{h}\right)$$

When the actual value of r / h is not known use a value of 0.30.

DC + DW + LL+IM:

$$f_{\min} = \frac{(7)\cdot(74+74-19)\cdot(12)\cdot(25.64)}{51{,}636} = 5.38 \text{ ksi}$$

$$f_f = 21 - (0.33)\cdot(5.38) + 8\cdot(0.30) = 21.62 \text{ ksi}$$

Since the stress range of 5.38 ksi is less than the allowable fatigue stress of 21.62 ksi, the fatigue criteria for the reinforcing is satisfied.

The superstructure is adequately reinforced for negative moment using #9 and #10 alternating at 6 inches over the pier. This reinforcing can be reduced along the span based on the negative moment diagram considering the service, strength and fatigue limit states. The reinforcing anchorage requirements in 5.14.1.2.7b shall be satisfied.

Shear
[5.8]

The LRFD method of shear design is a complete change from the methods specified in the Standard Specifications and that used by ADOT. For this example an in-depth shear design will be performed at the critical locations in Span 1 near the abutment and pier.

Critical Section
[5.8.3.2]

d_v
[5.8.2.9]

The critical location is located a distance d_v from the face of the support. This creates a problem in that d_v is largest of three values, two of which are a function of distance from the support. To eliminate the iterative process in determining the critical shear location, a simplification is required. It is recommended that the equation, $d_v = 0.72$ h be used to determine the critical shear location. Since d_v is the larger of the three values determined in Step 3, using $d_v = 0.72h = (0.72)(33.0 + 4.50) / 12 = 2.25$ feet will be conservative.

Step 1 – Determine Shear

Shears and moments from a simple span analysis will be slightly higher near the abutment and will be used in that analysis. Shears and moments from a continuous beam analysis will be used near the pier. The more critical design will be used in the final solution to obtain a symmetrical shear reinforcing pattern for the box beam.

Abutment Shear

For a uniform load "w" distributed along the entire span "L" the shear at a distance x from the support is $V_x = (w)(L / 2 – x)$. Simple span shears 2.25 feet from the abutment are determined as follows:

Box Beam	$V_{Crit} = 0.798\ [(84.00) / 2 – 2.25]$	= 31.7 kips
Diaphragms	$V_{Crit} = 0.821(3 / 2)$	= 1.2 kips
Non-Comp	$V_{Crit} = 0.373\ [(84.00) / 2 – 2.25]$	= 14.8 kips
Barriers	$V_{Crit} = 0.101\ [(84.00) / 2 - 2.25]$	= 4.0 kips
DC	$V_{Crit} = 31.7 + 1.2 + 14.8 + 4.0$	= 51.7 kips
DW	$V_{Crit} = 0.100\ [(84.00) / 2 - 2.25]$	= 4.0 kips

Design Lane	$V_{Crit} = [0.640)(84.00 - 2.25)^2 / 2] \div 84.00$	= 25.5 k
Design Truck	$V_{Crit} = [32(81.75) + 32(67.75) + 8(53.75)] \div 84$	= 62.1 k
Design Tandm	$V_{Crit} = [25(81.75) + 25(77.75)] \div 84$	= 47.5 k

Vehicle $V_{Crit} = 25.5 + 1.33(62.1) = 108.1$ kips

Interior Box Beam

The live load distribution factor for shear will be determined based on the provisions the more critical of an interior or exterior beam. The distribution of live load per lane for shear for an interior beam for one design lane loaded is:

LL Distribution
[4.6.2.2.1]

$$LL\ Distribution = \left(\frac{b}{130L}\right)^{0.15}\left(\frac{I}{J}\right)^{0.05} = \left(\frac{47.50}{130 \cdot 84.00}\right)^{0.15}\left(\frac{171{,}153}{271{,}796}\right)^{0.05} = 0.432$$

Live Load Distribution [4.6.2.2.1]

The distribution for two or more design lanes loaded is:

$$LL\ Distribution = \left(\frac{b}{156}\right)^{0.4}\left(\frac{b}{12.0L}\right)^{0.1}\left(\frac{I}{J}\right)^{0.05}\left(\frac{b}{48}\right)$$

[4.6.2.2.3a-1]

where $\frac{\text{b}}{48} = \frac{47.5}{48} = 0.990 \le 1.0$

$$LL\ Distribution = \left(\frac{47.50}{156}\right)^{0.4}\left(\frac{47.50}{12.0 \cdot 84.00}\right)^{0.1}\left(\frac{171{,}153}{271{,}796}\right)^{0.05}(1.0) = 0.447$$

Exterior Box Beam

The distribution of live load per lane for shear for an exterior beam for one design lane loaded is:

$g = e\ g_{interior}$

$e = 1.25 + \frac{d_e}{20} \ge 1.0$ where d_e has been previously calculated = 0.21 feet

.

$$e = 1.25 + \frac{0.21}{20} = 1.26$$

g = (1.26)(0.432) = 0.544

The distribution of live load per lane for shear for an exterior beam with two or more design lanes loaded is:

$$g = e\ g_{interior}\left(\frac{48}{b}\right) where\ \frac{48}{b} \le 1.0$$

$$e = 1 + \left[\frac{d_e + \frac{b}{12} - 2.0}{40}\right]^{0.5} \ge 1.0$$

$$e = 1 + \left[\frac{0.21 + \frac{47.5}{12} - 2.0}{40}\right]^{0.5} = 1.233$$

g = (1.233)(0.447)(1.0) = 0.551 ⇐ Critical

Skew Effect [4.6.2.2.3c-1]

For skewed bridges, the shear shall be adjusted to account for the effects of skew. For a right angle bridge the correction factor equals one.

LL+IM

V_{Crit} = (108.1)(0.551)(1.00) = 59.6 kips

V_u = 1.25(51.7) + 1.50(4.0) + 1.75(59.6) = 174.9 kips

Step 2 – Determine Analysis Model

Sectional Model [5.8.3]

The sectional model of analysis is appropriate for the design of typical bridge webs where the assumptions of traditional beam theory are valid. Where the distance from the point of zero shear to the face of the support is greater than 2d the sectional model may be used. Otherwise, the strut-and-tie model should be used. Assume a 12 inch long bearing pad.

For Simple spans:

Point of Zero Shear to Face of Support = 84.00 / 2 – 0.50 = 41.50 ft

2d = 2(3.125) = 6.25 ft < 41.50 ft

Therefore sectional model may be used.

Step 3 – Shear Depth, d_v

The shear depth is the maximum of the following criteria:

1) $d_v = 0.9\, d_e$ where $d_e = \dfrac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = d_p$ when $A_s = 0$

From Figure 12 at a distance 2.25 feet from centerline bearing:

Y = 4.00 + (27.00)(31.25) / 34.25 = 28.635 in

$$c.g. = \frac{(2)\cdot(28.635)+(2)\cdot(26.635)+(12)\cdot(4.00)+(18)\cdot(2.00)}{34} = 5.72 \text{ in}$$

$$d_p = 37.50 - 5.72 = 31.78\ in$$

$$d_v = 0.9 d_p = 0.9(31.78) = 28.60 \text{ in}$$

2) 0.72h = 0.72(37.50) = 27.00 in

[C5.8.2.9-1]

$$3)\ d_v = \frac{M_n}{A_s f_y + A_{ps} f_{pu}}$$

At the critical location, 2.25 feet from the abutment, the 4 harped strands are in the compression zone and are ignored for strength calculations. Since the compression zone extends beyond the deck and into the beam, the width of the beam will be used for b.

$A_s = (0.153)(30) = 4.590\ \text{in}^2$

[5.7.3.1.1-4]

$$c = \frac{A_{ps} f_{pu}}{0.85 f'_c\ \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}}$$

When the cast-in-place deck has a different strength than the precast member, the lower concrete strength may be used in the flexural analysis when the compression block extends into the precast member.

$$c = \frac{(4.590)\cdot(270)}{0.85\cdot(4.5)\cdot(0.825)\cdot(47.50) + 0.28\cdot(4.590)\cdot\dfrac{270}{31.78}} = 7.71\ \text{in}$$

$a = c\beta_1 = (7.71)(0.825) = 6.36$ in

[5.7.3.1.1-1]

$$f_{ps} = (270)\cdot\left[1-(0.28)\cdot\frac{7.71}{31.78}\right] = 251.66\ \text{ksi}$$

The effect of the bond on the strand stress at the end of the girder must be considered. The embedment length, l_{px} = 9.00 + 2.25(12) = 36 inches is greater than the transfer length of 30 inches. The development length must also be satisfied as follows:

[5.11.4.2-1]

$$l_d \geq k\left(f_{ps} - \frac{2}{3} f_{pe}\right) d_b$$

k = 1.6 for pretensioned members with a depth greater than 24.0 inches.

f_{pe} = effective stress in the prestress steel after losses.
f_{pe} = (0.75)(270) – 2.23 - 13.26 – 33.32 = 153.69 ksi

f_{ps} = average stress in prestress steel at the time for which the nominal resistance of the member is required = 251.66 ksi.

$$l_d \geq (1.6)\cdot\left(251.66-\frac{2}{3}\cdot 153.69\right)\cdot(0.5) = 119.4 \text{ inches} = 9.95 \text{ feet}$$

Since the strand is not fully developed, the flexural resistance is reduced. This is a complex problem to solve. AASHTO has an equation to determine the stress in the strand as a function of embedment length as shown below:

[5.11.4.2-4]

$$f_{px} = f_{pe} + \frac{(l_{px} - 60d_b)}{(l_d - 60d_b)}(f_{ps} - f_{pe})$$

$$f_{px} = 153.69 + \frac{(36.00 - 60\cdot 0.5)}{(119.4 - 60\cdot 0.5)}\cdot(251.66 - 153.69) = 160.27 \text{ ksi}$$

This reduced strand stress will result in a change in the neutral axis and resulting stress block depth. The problem now becomes how to determine the location of the neutral axis while in a transition zone between working stress and ultimate strength. The Specification does not provide any clear direction as to how to proceed. Considering the complex issues involved it is recommended that this third method of determing the shear depth not be used for sections within the development length of the strands for pretensioned members.

Based on the above, the shear depth, $d_{v,}$ controlled by criteria 1, equals 28.60 inches.

Step 4 – Calculate, V_p

Due to the strand upturn at the hold-down points, some of the prestress force is in the upward vertical direction and directly resists the applied shear. Since the critical section for shear is located beyond the transfer length, the effective prestress force is used. Since the critical section for shear is near the transfer length, the transfer length losses are used. See Figure 12 for the angle of the cable path.

$$\alpha = \frac{(27.00)}{12\cdot(34.25)} = 0.06569 \text{ radians}$$

V_p = (4)(0.153) [(0.75)(270) – 2.23 – 13.26 - 33.32] (0.06569) = 6.2 kips

[5.8.3.3-2]

Step 5 – Check Shear Width, b_v

The LRFD Specification requires that web width be checked for minimum width to protect against crushing.

$V_u <= \varphi V_n = \varphi(0.25f'_c b_v d_v + V_p)$

$$\text{Required } b_v = \frac{\frac{174.9}{0.9} - 6.2}{(0.25)\cdot(5.0)\cdot(28.60)} = 5.26 \text{ inches}$$

Available b_v = 5.00(2 webs) = 10.00 inches, ok

Step 6 – Evaluate Shear Stress

Calculate the shear stress as follows:

[5.8.2.9-1]

$$v_u = \frac{V_u - \varphi V_p}{\varphi b_v d_v} = \frac{174.9 - 0.90\cdot(6.2)}{0.90\cdot(10.00)\cdot(28.60)} = 0.658 \text{ ksi}$$

$$\frac{v_u}{f'_c} = \frac{0.658}{5.0} = 0.132$$

Step 7 – Estimate Crack Angle θ

The LRFD method of shear design involves several cycles of iteration. The first step is to estimate a value of θ, the angle of inclination of diagonal compressive stress. Since the formula is not very sensitive to this estimate assume that θ = 26.5 degrees. This simplifies the equation for the first cycle by setting the coefficient 0.5 cotθ = 1.0.

Step 8 – Calculate strain, ε_x

There are two formulae for the calculation of strain for sections containing at least the minimum amount of transverse reinforcing. The first formula is used for positive values of strain indicating tensile stresses, while the second formula is used for negative values of strain indicating compressive stresses.

Formula for ε_x for positive values:

[5.8.3.4.2-1]

$$\varepsilon_x = \left[\frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}}{2(E_sA_s + E_pA_{ps})} \right]$$

Formula for e_x for negative values:

[5.8.3.4.2-2]

$$\varepsilon_x = \left[\frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}}{2(E_cA_c + E_sA_s + E_pA_{ps})} \right]$$

where:

General Procedure [5.8.3.4.2]

A_c = area of concrete on the flexural tension side of the member. The flexural tension side is the portion of the member in tension from flexure with a depth of half the composite member depth of (37.50) / 2 = 18.75 inches.
A_c = (47.50)(6) + (10.0)(12.75) + (2)(1/2)(3)(3) = 421.5 in^2

A_{ps} = area of prestressing steel on the flexural tension side of the member.
A_{ps} = 0.153(30) = 4.590 in^2

A_s = area of nonprestressed steel on the flexural tension side of the member. A_s = 0.

f_{po} = a parameter taken as the modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete. For the usual levels of prestressing, a value of 0.7 f_{pu} will be appropriate. Since f_{po} is an effective stress and the critical location is further from the beam end than the transfer length, the full stress is utilized.

f_{po} = 0.70(270) = 189 ksi

N_u = factored axial force taken as positive if tensile.
N_u = 0 kips

V_u = factored shear force.
V_u = 174.9 kips

M_u = factored moment but not to be taken less than $V_u d_v$.

For a uniformly distributed load, the moment at a distance x from the support is $M_x = (w)(x)(L - x) \div 2$. The moments at the critical shear location are calculated below:

Box Beam	M_{Crit} = 0.798(2.25)(84.00 – 2.25) / 2 = 73 ft-k
Diaphragm	M_{Crit} = 0.821(3 / 2)(2.25) = 3 ft-k
Non-Comp	M_{Crit} = 0.373(2.25)(84.00 - 2.25) / 2 = 34 ft-k
Barriers	M_{Crit} = 0.101(2.25)(84.00 - 2.25) / 2 = 9 ft-k

DC M_{Crit} = 73 + 3 + 34 + 9 = 119 ft-k

DW M_{Crit} = 0.100(2.25)(84.00 – 2.25) / 2 = 9 ft-k

The live load moment corresponding to the shear is determined by multiplying the critical shear by the critical distance.

Design Lane	M_{Crit} = (25.5)(2.25) = 57 ft-k
Design Truck	M_{Crit} = (62.1)(2.25) = 140 ft-k
Design Tandm	M_{Crit} = (47.5)(2.25) = 107 ft-k

LL+IM M_{Crit} = [57 + 1.33(140)](0.307) = 75 ft-k

M_{Crit} = 1.25(119) + 1.50(9) + 1.75(75) = 294 ft-k

M_u = 294 ft-k but not less than $V_u d_v$ = (174.9)(28.60) / 12 = 417 ft-k

[5.8.3.4.2-1]

$$\varepsilon_x = \left[\frac{\frac{|417 \cdot 12|}{28.60} + 0 + 1.0 \cdot |174.9 - 6.2| - (4.590) \cdot (189)}{2(29000 \cdot 0 + 28500 \cdot 4.590)}\right]$$

$\varepsilon_x = -0.00200 = -2.00 \times 1000^{-3}$

Since the value is negative the second formula must be used.

[5.8.3.4.2-2]

$$\varepsilon_x = \left[\frac{\frac{|417 \cdot 12|}{28.60} + 0 + 1.0|174.9 - 6.2| - (4.590) \cdot (189)}{2(4070 \cdot 421.5 + 28500 \cdot 4.590)}\right]$$

$\varepsilon_x = -0.000142 = -0.142 \times 1000^{-3}$

[5.8.3.4.2-1]

Now go into [Table 5.8.3.4.2-1] to read the values for θ and β. From the previously calculated value of v_u / f'_c = 0.132, enter the ≤ 0.150 row and from the calculated value of ε_x = -0.142 x 1000, enter the ≤ -0.10 column. The new estimate for values is shown below:

θ = 23.3 degrees
β = 2.79

With the new value of θ, the strain must be recalculated.

$$\varepsilon_x = \left[\frac{\dfrac{|417 \cdot 12|}{28.60} + 0 + 0.5 \cdot |174.9 - 6.2| \cot(23.3) - 4.590 \cdot (189)}{3{,}692{,}640} \right]$$

ε_x = -0.000135 = -0.135 x 1000

With this new estimate for strain, reenter the table and determine new values for θ and β. Since our new values are the same as assumed, our iterative portion of the design is complete.

Step 9 - Calculate Concrete Shear Strength, V_c

The nominal shear resistance from concrete, V_c, is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316 \beta \sqrt{f'_c} b_v d_v$$

$$V_c = 0.0316 \cdot (2.79) \cdot \sqrt{5.0} \cdot (10.00) \cdot (28.60) = 56.4 \text{ kips}$$

Step 10 - Determine Required Vertical Reinforcement, V_s

[5.8.3.3-4]

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s} = \frac{A_v f_y d_v \cot\theta}{s} \; where\ \alpha = 90°$$

$$V_u \le V_R = \phi V_n = \phi(V_c + V_s + V_p)$$

$$V_s = \frac{V_u}{\phi} - V_c - V_p = \frac{A_v f_y d_v \cot\theta}{s}$$

$$s = \frac{A_v f_y d_v \cot\theta}{\frac{V_u}{\phi} - V_c - V_p} = \frac{(0.62)\cdot(60)\cdot(28.60)\cdot\cot(23.3)}{\frac{174.9}{0.90} - 56.4 - 6.2} = 18.8 \text{ in}$$

Minimum Reinforcing

The minimum transverse reinforcing requirement is satisfied by limiting the maximum allowable spacing to the following:

[5.8.2.5]

$$s_{\max} \le \frac{A_v f_y}{0.0316\sqrt{f'_c}\,b_v}$$

$$s_{\max} \le \frac{(0.62)\cdot(60)}{0.0316\,\sqrt{5.0}\,(10.00)} = 52.6 \text{ in}$$

Maximum Spacing [5.8.2.7]

The maximum spacing of transverse reinforcing is limited by the following:

For $v_u < 0.125 f'_c \Rightarrow s_{\max} = 0.8 d_v \le 24.0$

For $v_u \ge 0.125 f'_c \Rightarrow s_{\max} = 0.4 d_v \le 12.0$

$v_u = 0.658$ ksi $\ge 0.125(5.0) = 0.625$ ksi

$s_{max} = 0.4(28.60) = 11.4$ inches $\Leftarrow$ Critical

Use #5 stirrups at 11 inch spacing

Determine the stirrup resistance as follows:

$$V_s = \frac{(0.62)\cdot(60)\cdot(28.60)\cot(23.3)}{11} = 224.6 \text{ kips}$$

The shear strength is the lesser of:

[5.8.3.3-1] $V_n = V_c + V_s + V_p = [56.4 + 224.6 + 6.2] = 287.2$ kips

[5.8.3.3-2] $V_n = 0.25 f'_c b_v d_v + V_p = [0.25(5.0)(10.00)(28.60) + 6.2] = 363.7$ kips

$\varphi V_n = (0.90)(287.2) = 258.5 \text{ k} > V_u = 174.9 \text{ k}$

Step 11 - Longitudinal Reinforcement

In addition to transverse reinforcement, shear requires a minimum amount of longitudinal reinforcement. The requirement for longitudinal reinforcement follows:

[5.8.3.5-1]

$$A_s f_y + A_{ps} f_{ps} \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

Where V_s is limited to $V_u / \varphi = 174.9 / 0.90 = 194.3$ kips

The effect of the bond on the strand at the end of the girder must be considered. The development length equals the following:

[5.11.4.2-1]

$$l_d \geq k\left(f_{ps} - \frac{2}{3} f_{pe}\right)d_b$$

k = 1.6 for pretensioned members with a depth greater than 24.0 inches.

f_{pe} = effective stress in the prestress steel after losses.
f_{pe} = (0.75)(270) – 2.23 – 13.26 – 33.32 = 153.69 ksi

f_{ps} = average stress in prestress steel at the time for which the nominal resistance of the member is required = 251.66 ksi.

$$l_d \geq (1.6)\cdot\left(251.66 - \frac{2}{3}\cdot 153.69\right)\cdot(0.5) = 119.4 \text{ inches}$$

[5.11.4.2-4]

$$f_{px} = f_{pe} + \frac{(l_{px} - 60d_b)}{(l_d - 60d_b)}(f_{ps} - f_{pe})$$

$$f_{px} = 153.69 + \frac{(36.00 - 60\cdot 0.5)}{(119.4 - 60\cdot 0.5)}\cdot(251.66 - 153.69) = 160.27 \text{ ksi}$$

Considering only the prestressing steel in the tension side of the member yields the following:

$$(4.590)\cdot(160.27) \geq \frac{|417\cdot 12|}{(28.60)\cdot(1.00)} + \left(\left|\frac{174.9}{0.9} - 6.2\right| - 0.5\cdot 194.3\right)\cot(23.3)$$

736 kips > 386 kips

In addition, at the inside edge of the bearing area of simple end supports to the section of critical shear, the longitudinal reinforcement on the flexural tension side of the member shall satisfy:

[5.8.3.5-2]

$$A_s f_y + A_{ps} f_{ps} \geq \left(\frac{V_u}{\varphi_v} - 0.5V_s - V_p \right) \cot\theta$$

Assuming a 12 inch long bearing pad, the development length is 9 + 6 = 15 inches. The transfer length is 30 inches so the effective prestress stress accounting for development is:

f_{ps} = (15 / 30)(153.69) = 76.85 ksi

$$(4.590)\cdot(76.85) \geq \left(\frac{174.9}{0.90} - 0.5\cdot(194.3) - 6.2 \right) \cot(23.3)$$

353 kips > 211 kips

Therefore the bottom prestressing strands are adequate for longitudinal reinforcement without the addition of mild reinforcing.

Critical Shear At Pier

The shear calculations will also be shown in Span 1 near the pier where a large negative moment is present in addition to the large shear in basically a reinforced concrete section. The critical location is located a distance d_v from the face of the support. The equation, $d_v = 0.72$ h = 2.25 feet will be used to determine the critical shear location near the pier.

Determine Shear

The shears based on a continuous beam analysis were determined for Span 1 from a computer program as follows:

	0.9	1.0	Unit
DC (Barriers)	-4.3	-5.2	k
DW (FWS)	-4.3	-5.1	k
LL+IM Vehicle	-111.9	-123.9	k
SH & CR	-0.9	-0.9	k

Shears are determined 2.25 feet from the simple span support and 3.50 feet from the continuous support to be at the same location as follows:

Box Beam $V_{Crit} = 0.798\ [(84.00) / 2 - 2.25] = 31.7$ kips
Diaphragms $V_{Crit} = 0.821(3 / 2) = 1.2$ kips
Non-Comp $V_{Crit} = 0.373\ [(84.00 / 2 - 2.25] = 14.8$ kips
Barriers $V_{Crit} = 5.2 - 0.101(3.50) = 4.8$ kips
DC $V_{Crit} = 31.7 + 1.2 + 14.8 + 4.8 = 52.5$ kips

DW $V_{Crit} = 5.1 - 0.100(3.50) = 4.8$ kips
Vehicle $V_{Crit} = 123.9 - (123.9 - 111.9)(3.50) / 8.525 = 119.0$ kips

LL+IM $V_{Crit} = (119.0)(0.551)(1.00) = 65.6$ kips

$$V_u = 1.25(52.5) + 1.50(4.8) + 1.75(65.6) + 0.5(0.9) = 188.1 \text{ kips}$$

Shear Depth, d_v

For the shear design, the harped strands will be conservatively ignored in the determination of the shear depth. The shear depth is the maximum of the following criteria:

1) $d_v = 0.9\ d_e$ where $d_e = \dfrac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = d_s$ when $A_{ps} = 0$

$$d_s = 38.00 - 2.50\ clr - 0.625 - 1.27 / 2 = 34.24\ in$$

$$d_v = 0.9d_s = 0.9 \cdot (34.24) = 30.82 \text{ in}$$

2) 0.72h = 0.72(37.50) = 27.00 in

3) $$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{pu}}$$

From the negative moment continuity connection design, the moment capacity equals 1493 ft-k.

$$d_v = \frac{1493 \cdot (12)}{(9.08) \cdot (60) + 0} = 32.89 \text{ in}$$

Based on the above, the shear depth, d_v, is controlled by criteria 3 and equals 32.89 inches.

Calculate, V_p

Due to symmetry the upward shear force near the pier equals that near the abutment. The critical section is beyond the transfer length so the full effective prestress force may be used.

V_p = (4)(0.153) [(0.75)(270) – 2.23 - 13.26 - 33.32] (0.06569) = 6.2 kips

Check Shear Width, b_v

The LRFD Specification requires that web width be checked for minimum width to protect against crushing.

$V_u <= \varphi V_n = \varphi(0.25f'_c b_v d_v + V_p)$

$$\text{Required } b_v = \frac{\frac{188.1}{0.9} - 6.2}{(0.25) \cdot (5.0) \cdot (32.89)} = 4.93 \text{ inches}$$

Available b_v = 5.00(2 webs) = 10.00 inches, ok

Evaluate Shear Stress

$$v_u = \frac{V_u - \varphi V_p}{\varphi b_v d_v} = \frac{188.1 - 0.90 \cdot (6.2)}{0.90 \cdot (10.00) \cdot (32.89)} = 0.617 \text{ ksi}$$

$$\frac{v_u}{f'_c} = \frac{0.617}{5.0} = 0.123$$

Calculate strain, ε_x

A_c = area of concrete on the flexural tension side of the member.
A_c = (0.949)(48)(4.50) + (47.50)(5.5) + (10.0)(8.75) + (2)(1/2)(3)(3)
= 562.73 in^2

A_{ps} = area of prestressing steel on the flexural tension side of the member.
A_{ps} = 0 in^2 (Neglect the 4 harped strands for simplicity).

A_s = area of nonprestressed steel on the flexural tension side of the member. A_s = 9.08 in^2

f_{po} = 0.70(270) = 189 ksi

N_u = factored axial force taken as positive if tensile.
N_u = 0 kips

V_u = factored shear force.
V_u = 188.1 kips

M_u = factored moment, but not to be taken less than $V_u d_v$.

The moment at the critical shear location is required. The moments from the continuous beam analysis at the critical location near the pier follow:

	0.9	1.0	Units
DC (Barriers)	-34	-74	ft-k
DW (FWS)	-34	-74	ft-k
One Vehicle	-1052	-1764	ft-k
SH & CR	-69	-77	ft-k

Box Beam M_{Crit} = 0.798(2.25)(84.00 – 2.25) / 2 = 73 ft-k
Diaphragm M_{Crit} = 0.821(3 / 2)(2.25) = 3 ft-k
Non-Comp M_{Crit} = 0.373(2.25)(84.00 – 2.25) / 2 = 34 ft-k
Barriers M_{Crit} = -74 + 4.8(3.50) - 0.101$(3.50)^2$ / 2 = -58 ft-k

DW M_{Crit} = -74 + 4.8(3.50) – 0.100$(3.50)^2$ / 2 = -58 ft-k

LL + IM M_{Crit} = [-1764 + (1764 - 1052)(3.50) / 8.525](0.307)
= -452 ft-k

SH + CR M_{Crit} = [-77 + (77 – 69)(3.50) / 8.525] = -74 ft-k

M_u = 0.90(73 + 3 + 34) + 1.25(-58) + 1.50(-58) + 1.75(-452) + 0.5(-74)
= -889 ft-k
M_u = 889 ft-k but not less than $V_u d_v$ = (188.1)(32.89) / 12 = 516 ft-k

$$\varepsilon_x = \left[\frac{\dfrac{|889 \cdot 12|}{32.89} + 0 + 1.0 \cdot |188.1 - 6.2| - (0) \cdot (189)}{2(29000 \cdot 9.08 + 28500 \cdot 0)} \right]$$

$\varepsilon_x = 0.000961 = 0.961 \text{ x } 1000^{-3}$

Since the value is positive the first formula must be used.

Now go into [Table 5.8.3.4.2-1] to read the values for θ and β. From the previously calculated value of v_u / f'_c = 0.123, enter the ≤ 0.125 row and from the calculated value of ε_x = 0.961 x 1000, enter the ≤ 1.00 column. The new estimate for values is shown below:

θ = 37.0 degrees
β = 2.13

With the new value of θ, the strain must be recalculated.

$$\varepsilon_x = \left[\frac{\dfrac{|889 \cdot 12|}{32.89} + 0 + 0.5 \cdot |188.1 - 6.2| \cot(37.0) - 0 \cdot (189)}{526{,}640} \right]$$

$\varepsilon_x = 0.000845 = 0.845 \text{ x } 1000^{-3}$

With this new estimate for strain, reenter the table and determine new values for θ and β. Since our new values are the same as assumed, our iterative portion of the design is complete.

Calculate Concrete Shear Strength, V_c

The nominal shear resistance from concrete, V_c, is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316 \beta \sqrt{f'_c}\, b_v d_v$$

$$V_c = 0.0316 \cdot (2.13) \cdot \sqrt{5.0} \cdot (10.00) \cdot (32.89) = 49.5 \text{ kips}$$

Determine Required Vertical Reinforcement, V_s

$$s = \frac{A_v f_y d_v \cot\theta}{\frac{V_u}{\phi} - V_c - V_p} = \frac{(0.62)\cdot(60)\cdot(32.89)\cdot\cot(37.0)}{\frac{188.1}{0.90} - 49.5 - 6.2} = 10.6 \text{ in}$$

Minimum and Maximum Shear Reinforcing

Based on minimum reinforcing requirements s_{max} = 52.6 inches. Based on the maximum spacing requirements:

$v_u = 0.617 < 0.125(5.0) = 0.625$ so $s_{max} = 0.8d_v = 0.8(32.89) = 26.3$ inches but less than 24 inches.

Use #5 stirrups at 10 inch spacing

$$V_s = \frac{(0.62)\cdot(60)\cdot(32.89)\cot(37.0)}{10} = 162.4 \text{ kips}$$

The shear strength is the lesser of:

[5.8.3.3-1] $V_n = V_c + V_s + V_p = [49.5 + 162.4 + 6.2] = 218.1$ kips

[5.8.3.3-2] $V_n = 0.25f'_c b_v d_v + V_p = [0.25(5.0)(10.00)(32.89) + 6.2] = 417.3$ kips

$\varphi V_n = (0.90)(218.1) = 196.3 \text{ k} > V_u = 188.1 \text{ k}$

Longitudinal Reinforcement

In addition to vertical reinforcement, shear requires a minimum amount of longitudinal reinforcement. The requirement for longitudinal reinforcement follows:

[5.8.3.5-1]

$$A_s f_y + A_{ps} f_{ps} \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

Where V_s is limited to $V_u / \varphi = 188.1 / 0.90 = 209.0$ kips

Considering the mild reinforcing steel yields the following:

$$(9.08)\cdot(60) \geq \frac{|889\cdot 12|}{(32.89)\cdot(0.90)} + \left(\left|\frac{188.1}{0.9} - 6.2\right| - 0.5\cdot(162.4)\right)\cot(37.0)$$

545 kips > 522 kips

Therefore the mild reinforcing steel, consisting of alternating #9 and #10 at 6 inches, is adequate.

Interface Shear Transfer
[5.8.4]

For precast box beams, the cast-in-place deck is cast separately. Thus the shear transfer across this surface must be investigated. For this example, the shear transfer will be investigated at the critical shear location near the abutment. Only the composite dead loads and live load plus dynamic load allowance are considered.

$V_u = 1.25(4.0) + 1.50(4.0) + 1.75(59.6) = 115.3$ k

d_e = the distance between the centroid of the steel in the tension side of the beam to the center of the compression block in the deck. For simplicity d_e may be taken as the distance between the centroid of the tension steel and the midthickness of the deck. Since this location is within the development length of the strand, use the approximate method.

$d_e = d_p - a / 2 = 31.78 - 10.00 / 2 = 26.78$ in

[C5.8.4.1-1]

$$V_{uh} = \frac{V_u}{d_e} = \frac{115.3}{26.78} = 4.31 \text{ k/in}$$

The nominal shear resistance of the interface plane is:

[5.8.4.1-1]

$V_n = cA_{cv} + \mu[A_{vf}f_y + P_c]$

A_{cv} = the area of concrete engaged in shear transfer.
= 47.50 in^2/in

A_{vf} = the area of shear reinforcement crossing the shear plane.
= (0.31)(2 webs) / (11 in spacing) = 0.0564 in^2/in

[5.8.4.2]

For concrete placed against clean, hardened concrete with surface intentionally roughened to an amplitude of 0.25 in

$\mu = 1.0\ \lambda = 1.0(1.0) = 1.0$
$c = 0.100$ ksi

$V_n = 0.100(47.50) + 1.0[(0.0564)(60) + 0] = 8.13$ k/in

[5.8.4.2]

The nominal shear resistance used in the design shall not be greater than the lesser of:

[5.8.4.1-2]

$$V_n \leq 0.2 f'_c A_{cv} = 0.2 \cdot (4.5) \cdot (47.50) = 42.75 \text{ k/in}$$

[5.8.4.1-3]

$$V_n = 0.8 A_{cv} = 0.8 \cdot (47.50) = 38.00 \text{ k/in}$$

Since the nominal capacity of 8.13 k/in is less than 38.00 k/in, the maximum allowed, the nominal horizontal shear resistance is 8.13 k/in. The horizontal shear strength is:

$\varphi V_n = (0.90)(8.13) = 7.32$ k/in > $V_{uh} = 4.31$ k/in ok

Minimum Shear Reinforcing

[5.8.4.1-4]

The minimum area reinforcing crossing the interface is:

$$A_{vf} \geq \frac{0.050 b_v}{f_y} = \frac{(0.050)\cdot(47.50)}{60} = 0.0396\ in^2/in$$

For #5 @ 11 inches $A_{vf} = (0.31)(2) / 11 = 0.0564$ in^2/in

∴ The minimum criteria is satisfied.

Pre-Tensioned Anchor Zone [5.10.10]

The design of anchor zone for precast members is simple. Bursting reinforcing is required to resist a force equal to 4 percent of the total prestress force at a unit stress of 20 ksi. The first reinforcing bar should be placed as close to the face as possible with all the required reinforcing placed within a distance equal to the member depth divided by 4.

$$A_s = (0.04)\cdot\frac{(0.75)\cdot(270)\cdot(34)\cdot(0.153)}{20.0} = 2.11\ \text{in}^2$$

Use two 4-legged #5 stirrups. $A_s = (2)(4)(0.31) = 2.48$ in^2

[BDG]

The first stirrup should be placed 2 inches from the beam end. The remaining stirrups should be placed within a distance = 33.00 / 4 = 8.25 inches from the end. Use 6 inch spacing between the two stirrups. The requirements in Article 5.10.10.2 for confinement are not deemed appropriate for box beam members.

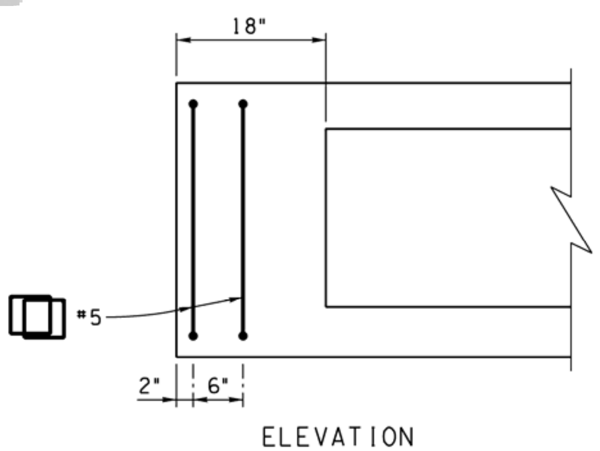


Figure 17

Deflections [5.7.3.6]

Deflections must be calculated so a camber can be put in the superstructure to provide for a smooth riding surface and the build-up can be estimated to determine the appropriate seat elevations.

Determination of the beam deflections is a tedious task complicated by the fact that the deck not only makes the beam composite but also continuous. If the continuity is not considered properly a rough ride can result.

There are four stages the beam experiences. The first stage is the release deflection when the prestress force is transferred to the beam. The release deflection is the summation of the deflections caused by self-weight of the beam and the prestress force under relaxation before transfer and elastic shortening losses.

The second stage is the initial deflection where some time dependent losses have occurred in the prestress force but where the concrete has also experienced creep.

The third and fourth stages comprise the final deflection used to determine screeds. The third stage consists of the addition of non-composite loads with the beam simply supported. The fourth stage includes the addition of all other loads including the effects from time-dependent prestress losses and additional concrete creep under a composite continuous beam.

For calculations for deflection, the prismatic gross section properties and prestress losses at midspan will be used. While this may not be technically correct, it will provide sufficiently accurate deflections considering all the unknowns.

Release Deflection

<u>Release Deflection:</u>

The deflection at midspan from self-weight is a combination of uniform loads from the typical section and concentrated loads from the three equally spaced diaphragms. At this time the beam concrete is at its release strength.

$$\Delta_{beam} = \frac{5wl^4}{384EI} = \frac{5 \cdot (0.798) \cdot (84.00)^4 \cdot (1728)}{384 \cdot (3818) \cdot (111{,}361)} = 2.102 \text{ in}$$

$$\Delta_{diaph} = \frac{19PL^3}{384EI} = \frac{19 \cdot (0.821) \cdot (84.00)^3 \cdot (1728)}{384 \cdot (3818) \cdot (111{,}361)} = 0.098 \text{ in}$$

The deflection from prestressing is more complicated but can be determined using moment area theory. The deflection at midspan from prestressing is:

$$\Delta_{p/s} = \frac{P}{24EI}\left[e_m\left(2L^2 + 4aL - 4a^2\right) + e_e\left(L^2 - 4aL + 4a^2\right)\right]$$

P_r = [(0.75)(270) – 2.23 – 13.26](0.153)(34) = 972.83 kips

e_m = 16.076 – 2.824 = 13.352 in

From Figure 15 at the centerline of bearing:
Y = 4.00 + 27.00(33.50) / 34.25 = 30.409 in

$$c.g. = \frac{(2)\cdot(30.409) + (2)\cdot(28.409) + (12)\cdot(4.00) + (18)\cdot(2.00)}{34} = 5.930 \text{ in}$$

e_e = 16.076 – 5.930 = 10.146 in

$2L^2 + 4aL - 4a^2 = 2(84)^2 + 4(8.50)(84) - 4(8.50)^2 = 16679$

$L^2 - 4aL + 4a^2 = (84)^2 - 4(8.50)(84) + 4(8.50)^2 = 4489$

$$\Delta_{p/s} = \frac{(972.83)\cdot(144)}{24\cdot(3818)\cdot(111{,}361)}\cdot\left[(13.352)\cdot(16679) + (10.146)\cdot(4489)\right]$$

$\Delta_{p/s}$ = 3.683 in

The deflection of the beam at release equals the algebraic sum of the above deflections:

Δ_R = 2.102 + 0.098 – 3.683 = -1.483 in upward

Initial Deflection

Initial Deflection:

The initial deflection accounts for a loss of prestress over time and creep in the concrete. The deflection from the beam based on its final concrete strength is:

$$\Delta_{beam} = \frac{5wl^4}{384EI} = \frac{5\cdot(0.798)\cdot(84.00)^4\cdot(1728)}{384\cdot(4070)\cdot(111{,}361)} = 1.972 \text{ in}$$

$$\Delta_{diaph} = \frac{19PL^3}{384EI} = \frac{19\cdot(0.821)\cdot(84.00)^3\cdot(1728)}{384\cdot(4070)\cdot(111{,}361)} = 0.092 \text{ in}$$

The deflection from prestressing is complicated by the fact that some prestress loss has occurred. For this problem the time from transfer of prestressing force till deck pour is assumed to be 60 days. An overall creep factor of 2.0 is applied with the assumption that 40% of the creep and 50% of the time-dependent losses have occurred at the time of deck placement.

Time-Dependent P/S Loss (60 days) = (0.50)(33.32) = 16.66 ksi

The effective prestress force at 60 days is:

P_{60} = [(0.75)(270) – 2.23 – 13.26 – 16.66](34)(0.153) = 886.16 k

$$\Delta_{P/S} = \frac{(886.16)\cdot(144)}{24\cdot(4070)\cdot(111{,}361)}\cdot\left[(13.352)\cdot(16679)+(10.146)\cdot(4489)\right]$$

$\Delta_{P/S}$ = 3.147 in

The deflection of the beam at 60 days equals the algebraic sum of the dead load and prestress deflections times a creep factor. For an overall creep of 2.00, use a modifier of 1.00 + (0.40)(2.00) = 1.80.

Δ_i = 1.80(-3.147 + 1.972 + 0.092) = -1.949 in upward

Final Deflection

Final Deflection:

The final deflection accounts for the remainder of the time-dependent prestress loss and concrete creep. The final deflection is added to the profile grade to determine the screed elevations. The final deflections must be separated between those that occur while the beam is acting as a simple span and those that occur on the composite continuous beam.

Instantaneous Simple Span

The simple span deflections will occur instantaneously and will consist of the weight of the shear key, build-up and deck with a uniform load equal to 0.373 k/ft.

$$\Delta_{DL} = \frac{5wl^4}{384EI} = \frac{5\cdot(0.373)\cdot(84.00)^4\cdot(1728)}{384\cdot(4070)\cdot(111{,}361)} = 0.922 \text{ in}$$

Instantaneous Continuous Spans

The barriers will be cast with the now composite beam acting as a continuous member. A standard continuous beam program is used to determine the deflection from the uniform barrier load. The resulting deflection is 0.088 inches. The above deflections are immediate with the resulting deflection equal to 0.922 + 0.088 = 1.010 inches.

Time-Dependent Couninuous Spans

The remaining deflections are creep related and occur over time. These deflections are more complicated to determine due to continuity and a computer program is normally required. Input values are shown in Figure 18 but detailed calculations are not.

Long-term deflections include creep of the girder, barriers, slab, final prestress force, loss of prestress and differential shrinkage. Different creep factors are applied to the different loads. The girder dead load and prestress have already seen a creep factor of 0.80 with a creep factor of 2.00 – 0.80 = 1.20 remaining for the long-term effects. For loads applied at 60 days or later, a creep factor of 1.0 is used. Therefore a creep factor of 1.00 is applied to the loss of prestress, slab, barrier and differential shrinkage.

A summary of output from the continuous beam analysis is shown below:

	Deflection	Creep	Total
Box Beam	0.730	1.20	0.876
Barriers	0.088	1.00	0.088
Final P/S	-0.989	1.20	-1.187
Loss of P/S	0.137	1.00	0.137
Slab	0.325	1.00	0.325
Diff Shr	0.250	1.00	0.250
Total			0.489

The final deflection at midspan equals 1.010 + 0.489 = 1.499 inches for Span 1. The midspan deflection for Span 2 will be smaller since both ends of the span are continuous and partially restrained.

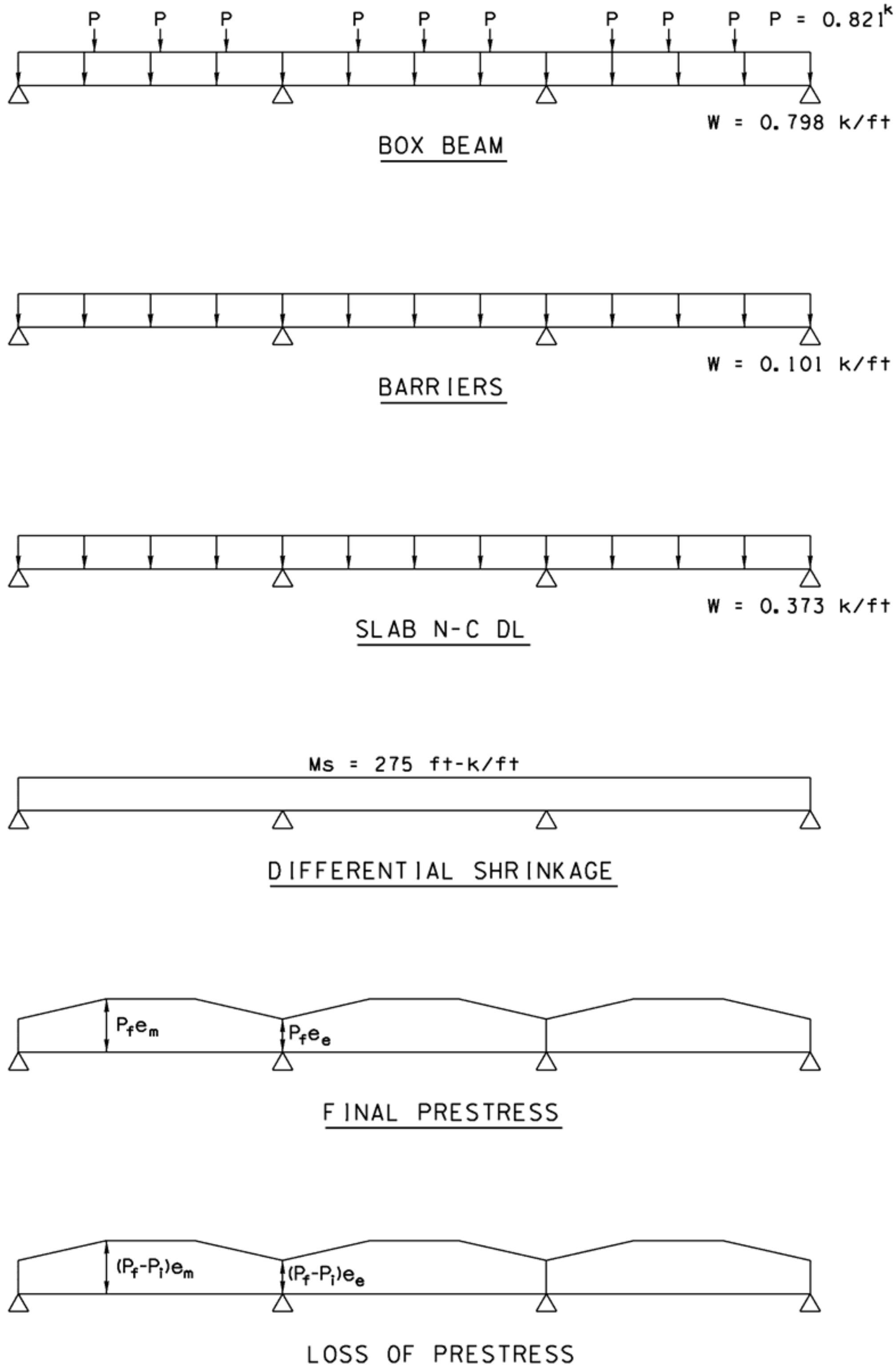

P P P P P P P P P P = 0.821^k
W = 0.798 k/ft
BOX BEAM
W = 0.101 k/ft
BARRIERS
W = 0.373 k/ft
SLAB N-C DL
Ms = 275 ft-k/ft
DIFFERENTIAL SHRINKAGE
$P_f e_m$
$P_f e_e$
FINAL PRESTRESS
$(P_f-P_i)e_m$
$(P_f-P_i)e_e$
LOSS OF PRESTRESS

Figure 18

APPENDIX A

PRECISE OVERHANG ANALYSIS

A simplified method of determining the adequacy of an overhang subjected to both tension and flexure is included in the example. This appendix shows the more complex and precise method along with the assumptions made to derive the approximate simplified equation.

Tension and Flexure [5.7.6.2]

The solution of the deck design problem involves determining the resistance of the deck overhang to a combination of tension and flexure. Members subjected to eccentric tension loading, which induces both tensile and compressive stresses in the cross section, shall be proportioned in accordance with the provisions of Article 5.7.2.

[5.7.2]

Assumptions for a valid analysis for an extreme event limit state are contained in Article 5.7.2. Factored resistance of concrete components shall be based on the conditions of equilibrium and strain compatibility and the following:

Strain is directly proportional to the distance from the neutral axis.

For unconfined concrete, the maximum usable strain at the extreme concrete compressive fiber is not greater than 0.003.

The stress in the reinforcement is based on a stress-strain curve of the steel or on an approved mathematical representation.

Tensile strength of the concrete is ignored.

The concrete compressive stress-strain distribution is assumed to be a rectangular stress block in accordance with Article 5.7.2.2.

The development of the reinforcing is considered.

[1.3.2.1]

While the article specifies the use of the reduction factors in Article 5.5.4.2, that requirement only applies to a strength limit state analysis. For an extreme event limit state, the resistance factor shall be taken as 1.0.

The above assumptions as shown in Figures A-1, A-2 and A-3 were used in the development of the equations for resistance from tension and flexure that occur with a vehicular collision with a traffic railing.

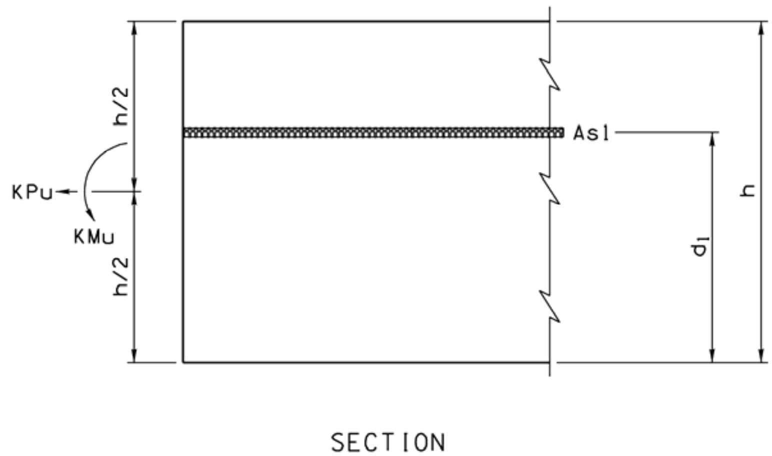


Figure A-1

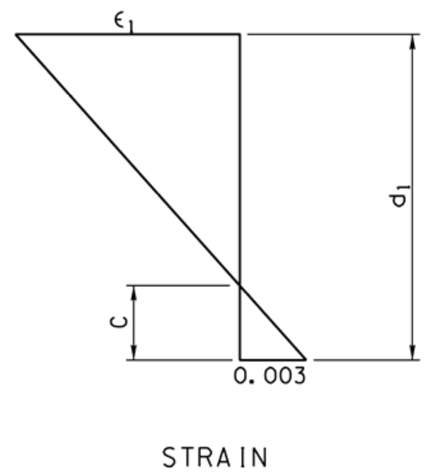


Figure A-2

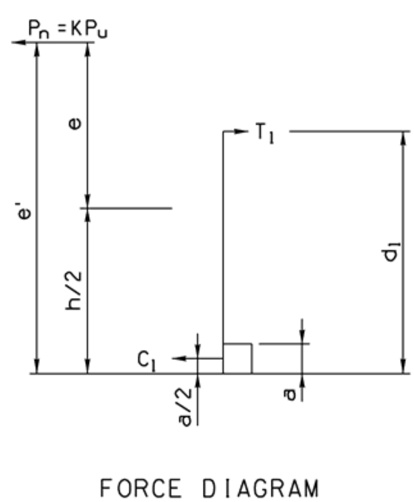


Figure A-3

The design of the deck overhang is complicated because both a bending moment and a tension force are applied. The problem can be solved using equilibrium and strain compatibility. The following trial and error approach may be used:

1. Assume a stress in the reinforcing
2. Determine force in reinforcing
3. Solve for k, the safety factor
4. Determine values for 'a' and 'c'
5. Determine corresponding strain
6. Determine stress in the reinforcing
7. Compare to assumed value and repeat if necessary

Face of Barrier Location 1 Figure 4

The design horizontal force in the barrier is distributed over the length L_b equal to L_c plus twice the height of the barrier. See Figures 5 and 6.

$L_b = 11.86 + 2(2.67) = 17.20$ ft
$P_u = 54.83 / 17.20 = \underline{3.188 \text{ k/ft}} < 3.261$ k/ft connection strength.

Dimensions

h = 12.00 in
$d_1 = 12.00 - 2.50 \text{ clr} - 0.625 / 2 = 9.19$ in

Moment at Face of Barrier

Deck $= 0.150(9.00 / 12)(1.42)^2 \div 2 = 0.11$ ft-k
$0.150(3.00 / 12)(1.42)^2 \div 6 = \underline{0.01 \text{ ft-k}}$
$= 0.12$ ft-k

Barrier $= 0.355(0.817) = 0.29$ ft-k

Collision $= 3.188[2.67 + (12.00/12) / 2] = 10.11$ ft-k

[A13.4.1] Extreme Event II [3.4.1]

The load factor for dead load shall be taken as 1.0.
$M_u = 1.00(0.12 + 0.29) + 1.00(10.11) = 10.52$ ft-k

$e = M_u / P_u = (10.52)(12) / (3.188) = 39.60$ in

1. Assume Stress

Assume the top layer of reinforcing yields and $f_{S1} = 60$ ksi

2. Determine Force

Determine resulting force in the reinforcing:

$T_1 = (0.310)(60) = 18.60$ k

Strength Equation

Solving the equations of equilibrium by summing the forces on the section and summing the moments about the soffit and setting them equal to zero yields the following two equations. See Figure A-3.

Sum forces in horizontal direction

Eqn 1: $-kP_u + T_1 + T_2 - C_1 = 0$ where $C_1 = 0.85f'_c ab$

Sum of moments

Eqn 2: $kP_u(e') - T_1(d_1) - T_2(d_2) + C_1(a/2) = 0$

Solving the above equations for k, the ratio of strength to applied force and moment, results in a quadratic equation with the following coefficients:

$$A = \frac{P_u^{\ 2}}{1.70 f'_c\, b}$$

$$B = P_u\left(e + \frac{h}{2} - \frac{T_1}{0.85 f'_c\, b}\right)$$

$$C = -T_1 d_1 + \frac{(T_1)^2}{1.70 f'_c\, b}$$

Solution

Substituting in specific values yields:

$$A = \frac{(3.188)^2}{1.70\cdot(4.5)\cdot(12)} = 0.110712$$

$$B = (3.188)\cdot\left(39.60 + \frac{12.00}{2} - \frac{(18.60)}{0.85\cdot(4.5)\cdot(12)}\right) = 144.0809$$

$$C = -(18.60)\cdot(9.19) + \frac{(18.60)^2}{1.70\cdot(4.5)\cdot(12)} = -167.1654$$

3. Determine k Safety Factor

Solution of the quadratic equation yields the value k, the safety factor.

$$k = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

$$k = \frac{-144.0809 + \sqrt{(144.0809)^2 - 4\cdot(0.110712)\cdot(-167.1654)}}{2\cdot(0.110712)} = 1.159$$

Since the value of k is greater than one the deck is adequately reinforced at this location.

4. Determine 'a' and 'c'

Calculate the depth of the compression block from Eqn 1. See Figure A-3.

$$a = \frac{(T_1 - kP_u)}{0.85 f'_c\, b}$$

$$a = \frac{(18.60 - (1.159)\cdot(3.188))}{0.85\cdot(4.5)\cdot(12)} = 0.32\ \text{in}$$

$$c = \frac{a}{\beta_1} = \frac{0.32}{0.825} = 0.39\ \text{in}$$

5. Strain
6. Stress

Determine the resulting strain in the top layer of reinforcing. See Figure A-2.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

$\varepsilon_1 = 0.003(d_1 / c\text{-}1) = 0.003(9.19 / 0.39 - 1) = 0.0677$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

7. Verify Assumption

Since the top layer of reinforcing yields the assumptions made in the analysis are valid.

Maximum Strain

The LRFD Specification does not have an upper limit on the amount of strain in a reinforcing bar. ASTM does require that smaller diameter rebar have a minimum elongation at tensile strength of 8 percent. This appears to be a reasonable upper limit for an extreme event state where $\varphi = 1.00$. For this example the strain of 6.8 percent is below this limit.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section. This step is not necessary for design but is included for educational purposes.

$\varphi P_n = \varphi k P_u = (1.00)(1.159)(3.188) = 3.69$ k

Solve for equilibrium from Figure A-3 by substituting M_n for $kP_u e$ and taking moments about the center of the compression block:

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (18.60)\cdot\left(9.19 - \frac{0.32}{2}\right)$$
$$- (1.159)\cdot(3.188)\cdot\left(\frac{12.00}{2} - \frac{0.32}{2}\right) = 146.38 \text{ in-k}$$

φM_n = (1.00)(146.38) / 12 = 12.20 ft-k

The factor of safety for flexure is 12.20 / 10.52 = 1.159 the same as for axial strength. Thus this method provides both a tensile and flexural strength with the same safety factor.

Since the simplified method yields a greater strength, it would appear that the simplified analysis method produces non-conservative results. However, the simplified method uses a safety factor of 1.0 for axial load leaving more resistance for flexure. As the applied load approaches the ultimate strength the two methods will converge to the same result. In this example since the safety factor is close to one, the two procedures produce similar results.

Interior Support Location 2 Figure 4

The deck slab must also be evaluated at the interior point of support. For this location only the top reinforcing in the cast-in-place slab will be considered. At this location the design horizontal force is distributed over a length L_{s2} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the interior support. See Figures 4, 5 and 6. Using a distribution of 30 degree from the face of the barrier to the interior support results in the following:

L_{S2} = 11.86 + 2(2.67) + (2)[tan(30)](0.42) = 17.68 ft
P_u = 54.83 / 17.68 = 3.101 k/ft

Dimensions

h = 10.50 in
d_1 = 10.50 – 2.50 clr – 0.625 / 2 = 7.69 in

Moment at Interior Support

For dead loads use the maximum negative moments for the interior cells used in the interior deck analysis.
.

DC = 0.14 ft-k
DW = 0.02 ft-k
Collision = 3.101[2.67 + (10.50 / 12) / 2] = 9.64 ft-k

The load factor for dead load shall be taken as 1.0.

[A13.4.1]
Extreme Event II
[3.4.1]

M_u = 1.00(0.14) + 1.00(0.02) + 1.00(9.64) = 9.80 ft-k
e = M_u / P_u = (9.80)(12) / (3.101) = 37.92 in

1. Assume Stress

Assume the top layer of reinforcing yields and f_{S1} = 60 ksi.

2. Determine Force

Determine resulting force in the reinforcing:

T_1 = (0.310)(60) = 18.60 k

For the interior face, the box beam concrete strength should be used. Using the previously derived equations for safety factor yields the following:

$$A = \frac{(3.101)^2}{1.70 \cdot (5.0) \cdot (12)} = 0.094276$$

$$B = (3.101) \cdot \left(37.92 + \frac{10.50}{2} - \frac{(18.60)}{0.85 \cdot (5.0) \cdot (12)} \right) = 132.7392$$

$$C = -(18.60) \cdot (7.69) + \frac{(18.60)^2}{1.70 \cdot (5.0) \cdot (12)} = -139.6422$$

3. Determine k Safety Factor

Solution of the quadratic equation yields the value k, the safety factor.

$$k = \frac{-132.7392 + \sqrt{(132.7392)^2 - 4\cdot(0.094276)\cdot(-139.6422)}}{2\cdot(0.094276)} = 1.051$$

Since the value of k is greater than one, the deck is adequately reinforced at this location.

4. Determine ‘a’ and ‘c’

Calculate the depth of the compression block.

$$a = \frac{(T_1 - kP_u)}{0.85 f'_c\, b}$$

$$a = \frac{(18.60 - (1.051)\cdot(3.101))}{0.85\cdot(5.0)\cdot(12)} = 0.30 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{0.30}{0.800} = 0.38 \text{ in}$$

Since c = 0.38 inches is less than the box beam slab depth of 5.50 inches our assumption of using 5.0 ksi concrete strength is valid.

5. Strain

Determine the resulting strain in the top layer of reinforcing. See Figure 8.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

6. Stress

$\varepsilon_1 = 0.003(d_1 / c - 1) = 0.003(7.69 / 0.38 - 1) = 0.0577$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and f_{s1} = 60 ksi

7. Verify Assumption

Since the top layer of reinforcing yields the assumption made in the analysis is valid.

Maximum Strain

The maximum strain of 5.8 percent is less than the ADOT limit of 8 percent and is therefore satisfactory.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section.

$\varphi P_n = \varphi k P_u = (1.00)(1.051)(3.101) = 3.26$ k

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (18.60)\cdot\left(7.69 - \frac{0.38}{2}\right) - (1.051)\cdot(3.101)\cdot\left(\frac{10.50}{2} - \frac{0.38}{2}\right)$$

M_n = 123.01 in-k

φM_n = (1.00)(123.01) / 12 = 10.25 ft-k

The factor of safety for flexure is 10.25 / 9.80 = 1.046 approximately the same as for axial strength.

Simplified Method

A simplified method of analysis is available based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{18.60 - 3.101}{(0.85)\cdot(5.0)\cdot(12)} = 0.30$ in

$$\varphi M_n = (1.00)\cdot\left[(18.60)\cdot\left(7.69 - \frac{0.30}{2}\right) - (3.101)\cdot\left(\frac{10.50}{2} - \frac{0.30}{2}\right)\right] \div 12$$

φM_n = 10.37 ft-k

APPENDIX B

PRECISE TRANSFORMED SECTION PROPERTIES

The approximate method of calculating transformed section properties assumes that all the strands may be replaced with a single strand with an area equal to the area of all the strands acting at the center of gravity of the strand pattern. The precise method assumes the strands in each row may be replaced with a single strand with an area equal to the area of all strands in that row acting at the distance of that row from the bottom.

Both methods produce the same values for the area and location of the neutral axis. At the midspan, where all the strands are closely spaced near the bottom, the moment of inertia of the two methods is very close. Near the ends of the girder where the strands are harped there are some differences in the moment of inertia. However, these differences are small and can be ignored.

The approximate method is easier to calculate and produces satisfactory results. However, the precise method could be used to determine the stresses at the ends of the girder. This could be used to reduce the required initial concrete strength for designs requiring high release strengths.

Midspan Transformed Properties

The following transformed section properties are calculated at the midspan based on the strand pattern shown in Figure 14:

Net Section - Box Beam

No.	As	A	y	Ay	Io	$A(y-yb)^2$
		765.75	16.076	12310	111,361	6
14	0.153	-2.14	4.00	-9	0	-317
20	0.153	-3.06	2.00	-6	0	-614
		760.55		12295	111,361	-925

$A_n = 760.55\ in^2$
$y_{nb} = 12295 / 760.55 = 16.166$ in
$y_{nt} = 33.00 – 16.166 = 16.834$ in
$I_n = 111{,}361 - 925 = 110{,}436\ in^4$

Transformed Section - Box Beam (n = 7.46) at Transfer (f'_{ci} = 4.4 ksi)

n	No.	As	A	y	Ay	Io	$A(y-yb)^2$
			760.55	16.166	12295	110436	318
7.46	14	0.153	15.98	4.00	64	0	2120
7.46	20	0.153	22.83	2.00	46	0	4172
			799.36		12405	110436	6610

$A_t = 799.36\ in^2$
$y_{tb} = 12405 / 799.36 = 15.519$ in
$y_{tt} = 33.00 – 15.519 = 17.481$ in
$I_t = 110{,}436 + 6610 = 117{,}046\ in^4$

Transformed Section - Box Beam (n = 7.00) at Service (f'_c = 5.0 ksi)

n	No.	As	A	y	Ay	Io	$A(y-yb)^2$
			760.55	16.166	12295	110436	282
7.00	14	0.153	14.99	4.00	60	0	2002
7.00	20	0.153	21.42	2.00	43	0	3937
			796.96		12398	110436	6221

$A_t = 796.96\ in^2$
$y_{tb} = 12398 / 796.96 = 15.557$ in
$y_{tt} = 33.00 – 15.557 = 17.443$ in
$I_t = 110{,}436 + 6221 = 116{,}657\ in^4$

Composite Section - Box Beam & Deck

$n = 3861 / 4070 = 0.949$

n	W	H	A	y	Ay	Io	$A(y-yb)^2$
			796.96	15.557	12398	116657	12937
0.949	48.00	4.50	204.98	35.25	7226	346	50294
			1001.94		19624	117043	63231

$A_c = 1001.94\ in^2$
$y_{cb} = 19624 / 1001.94 = 19.586$ in
$y_{ct} = 33.00 – 19.586 = 13.414$ in
$I_c = 117{,}043 + 63{,}231 = 180{,}234\ in^4$

Transfer Length Transformed Properties

At transfer length from the beam end:

Net Section - Box Beam

No.	As	A	y	Ay	Io	$A(y-yb)^2$
		765.75	16.076	12310	111361	4
2	0.153	-0.31	29.029	-9	0	-51
2	0.153	-0.31	27.029	-8	0	-37
12	0.153	-1.84	4.00	-7	0	-271
18	0.153	-2.75	2.00	-6	0	-550
		760.54		12280	111361	-905

$A_n = 760.54\ in^2$
$y_{nb} = 12280 / 760.54 = 16.146$ in
$y_{nt} = 33.00 – 16.146 = 16.854$ in
$I_n = 111{,}361 - 905 = 110{,}456\ in^4$

Transformed Section - Box Beam (n = 7.46) at Transfer (f'_{ci} = 4.4 ksi)

n	No.	As	A	y	Ay	Io	A(y-yb)^2
			760.54	16.146	12280	110456	192
7.46	2	0.153	2.28	29.029	68	0	409
7.46	2	0.153	2.28	27.029	63	0	296
7.46	12	0.153	13.70	4.00	47	0	1857
7.46	18	0.153	20.54	2.00	42	0	3823
			799.34		12504	110456	6577

A_t = 799.34 in^2
y_{tb} = 12504 / 799.34 = 15.643 in
y_{tt} = 33.00 – 15.643 = 17.357 in
I_t = 110,456 + 6577 = 117,033 in^4

Transformed Section - Box Beam (n = 7.00) at Service (f'_c = 5.0 ksi)

n	No.	As	A	y	Ay	Io	A(y-yb)^2
			760.54	16.146	12280	110456	171
7.00	2	0.153	2.14	29.029	62	0	382
7.00	2	0.153	2.14	27.029	58	0	276
7.00	12	0.153	12.85	4.00	51	0	1751
7.00	18	0.153	19.28	2.00	39	0	3604
			796.95		12490	110456	6184

A_t = 796.95 in^2
y_{tb} = 12490 / 796.95 = 15.672 in
y_{tt} = 33.00 – 15.672 = 17.328 in
I_t = 110,456 + 6184 = 116,640 in^4

Composite Section - Box Beam & Deck (Interior Box Beam)

n = 3861 / 4070 = 0.949

n	W	H	A	y	Ay	Io	A(y-yb)^2
			796.96	15.672	12490	116640	12789
0.949	48.00	4.50	204.98	35.25	7226	346	49705
			1001.93		19716	116986	62494

A_c = 1001.93 in^2
y_{cb} = 19716 / 1001.93 = 19.678 in
y_{ct} = 33.00 – 19.678 = 13.322 in
I_c = 116,986 + 62,494 = 179,480 in^4

Comparison Properties
Midspan Transfer Length

	Precise	Approx	Precise	Approx
Transformed				
At	796.96	796.96	796.95	796.96
yb	15.557	15.557	15.672	15.672
It	116,657	116,626	116,640	114,547
Composite				
Ac	1001.94	1001.94	1001.94	1001.94
yb	19.586	19.586	19.678	19.678
Ic	180,234	180,203	179,480	177,388

These results show that at the midspan where the strands are closely spaced, the results are nearly identical but where the strands are more widely spread at the ends differences appear. The precise method should produce higher moment of inertias except where the harped strands are near the neutral axis where the approximate method will produce higher values. Use of the precise method could reduce the required concrete strength near the ends for some designs but the extra effort to calculate them is usually not warranted

APPENDIX C

REFINED PRESTRESSED LOSSES

The refined prestress losses are based on the following time line of event.

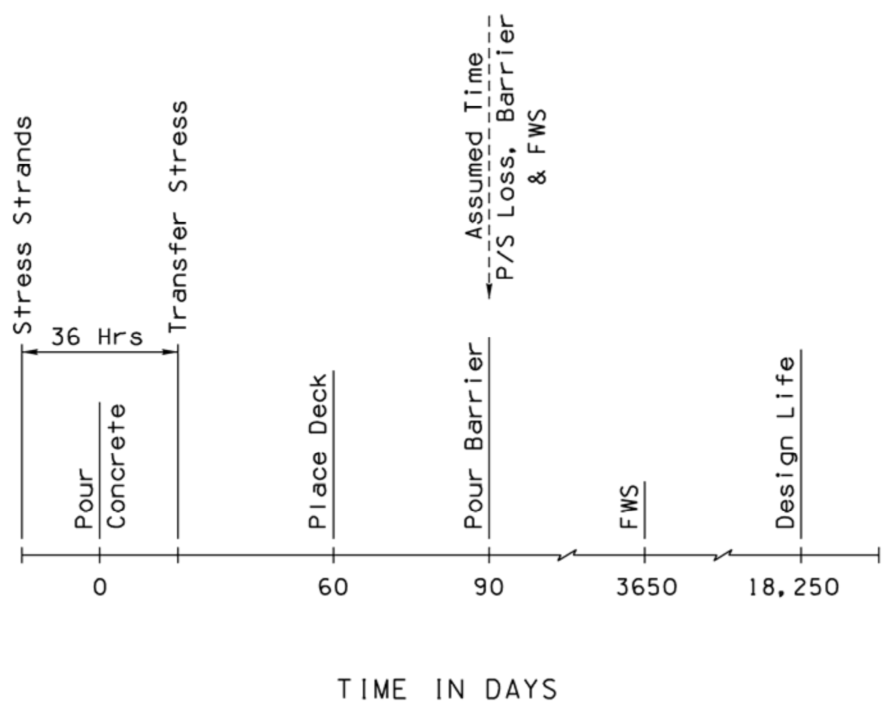


Figure C-1

Refined Time-Dependent Losses - 2006 [5.9.5.4]

For precast pretensioned members more accurate values of creep, shrinkage and relaxation related losses may be determined as follows:

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df}$$

The current Specification has a negative sign in front of Δf_{pSS}. However, this term is a gain in stress with analysis resulting in a negative sign for the term. Thus the sign should be a positive sign so the result is subtracted from the other values.

Δf_{pSR} = prestress loss due to shrinkage of girder concrete between transfer and deck placement.

Δf_{pCR} = prestress loss due to creep of girder concrete between transfer and deck placement.

Δf_{pR1} = prestress loss due to relaxation of prestressing strands between time of transfer and deck placement.

Δf_{pSD} = prestress loss due to shrinkage of girder concrete between time of deck placement and final time.

Δf_{pCD} = prestress loss due to creep of girder concrete between time of deck placement and final time.

Δf_{pR2} = prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time.

Δf_{pSS} = prestress loss due to shrinkage of deck composite section.

$(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id}$ = sum of time-dependent prestress losses between transfer and deck placement.

$(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df}$ = sum of time-dependent prestress losses after deck placement.

Shrinkage of Girder Concrete [5.9.5.4.2a] [5.9.5.4.2a-1]

The prestress loss due to shrinkage of girder concrete between time of transfer and deck placement shall be determined as follows:

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id}$$

Where:

ε_{bid} = concrete shrinkage strain of girder between the time of transfer and deck placement per Eq. 5.4.2.3.3-1.

[5.9.5.4.2a-2]

$$K_{id} = \frac{1}{1 + \frac{E_p}{E_{ci}} \frac{A_{ps}}{A_g} \left(1 + \frac{A_g e^2{}_{pg}}{I_g}\right) \left[1 + 0.7\psi_b(t_f, t_i)\right]}$$

K_{id} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between transfer and deck placement.

To determine the value of ε_{bid} the shrinkage strain of the girder must be determined at the time of deck placement. The basic equation for shrinkage is:

[5.4.2.3.3-1]

$\varepsilon_{sh} = -k_{vs}\, k_{hs}\, k_f\, k_{td}\, 0.48 \times 10^{-3}$

[5.4.2.3.2-2]

$k_{vs} = 1.45 - 0.13(V/S) = 1.45 - 0.13(4.76) = 0.831$

[5.4.2.3.3-2]

$k_{hs} = 2.00 - 0.014H = 2.00 - 0.014(40) = 1.440$

[5.4.2.3.2-4]

$$k_f = \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 4.4} = 0.926$$

[5.4.2.3.2-5]

$$k_{td} = \frac{t}{61 - 4f'_{ci} + t}$$

At time of deck placement, assumed at 60 days:

$$k_{td} = \frac{60}{61 - (4)\cdot(4.4) + 60} = 0.580$$

$\varepsilon_{bid} = -(0.831)(1.440)(0.926)(0.580)(0.48 \times 10^{-3}) = 0.308 \times 10^{-3}$

To determine the value of K_{id} the girder creep must be determined at final age as follows:

$\psi_b(t_f, t_i)$ = girder creep coefficient at final time due to loading introduced at transfer per Eq. 5.4.2.3.2-1

t_f = final age = (50 years)(365 days / year) = 18,250 days
t_i = age at transfer = 1 day.

The time between pouring of the concrete and transfer may be taken as 18 hours for release strengths up to 4.5 ksi. Use one day.

[5.4.2.3.2-1]

$\psi_b(t_f, t_i) = 1.9\ k_{vs}\ k_{hc}\ k_f\ k_{td}\ t_i^{-0.118}$

[5.4.2.3.2-3]

$k_{hc} = 1.56 - 0.008H = 1.56 - (0.008)(40) = 1.240$

[5.4.2.3.2-5]

$$k_{td} = \frac{18{,}250}{61-(4)\cdot(4.2)+18{,}250} = 0.998 \text{ Use 1.0 for design}$$

$\psi_b(t_f, t_i) = (1.9)(0.831)(1.240)(0.926)(1.0)(1)^{-0.118} = 1.813$

Use gross section properties for calculation of K_{id}.

$$K_{id} = \frac{1}{1+\frac{28500}{3818}\frac{5.202}{766}\left(1+\frac{(766)\cdot(13.252)^2}{111{,}361}\right)[1+(0.7)\cdot(1.813)]}$$

$K_{id} = 0.797$

The prestress loss due to shrinkage between the time of transfer and the time of deck placement can be determined as follows:

$$\Delta f_{pSR} = (0.000308)\cdot(28500)\cdot(0.797) = 7.00\text{ ksi}$$

Creep of Girder Concrete
[5.9.5.4.2b]

The prestress loss due to creep of girder concrete between time of transfer and deck placement is determined as follows:

[5.9.5.4.2b-1]

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \psi_b(t_d, t_i) K_{id}$$

f_{cgp} = concrete stress at center of gravity of prestressing tendons as determined for elastic shortening loss = 1.777 ksi

$\psi_b(t_d, t_i)$ = girder creep coefficient at time of deck placement due to loading introduced at transfer per Eq. 5.4.2.3.2-1

$\psi_b(t_f, t_i) = 1.9\ k_{vs}\ k_{hc}\ k_f\ k_{td}\ t_i^{-0.118}$

$\psi_b(t_d, t_i) = (1.9)(0.831)(1.240)(0.926)(0.580)(1)^{-0.118} = 1.052$

All the remaining variables have already been determined. The prestress loss due to creep is calculated as follows:

$$\Delta f_{pCR} = \frac{28500}{3818}\cdot(1.777)\cdot(1.052)\cdot(0.797) = 11.12\text{ ksi}$$

Relaxation of Prestressing Strands [5.9.5.4.2c]

The prestress loss due to relaxation of prestressing strands between time of transfer and deck placement is determined as follows:

[5.9.5.4.2c-1]

$$\Delta f_{pR1} = \frac{f_{pt}}{K_L}\left(\frac{f_{pt}}{f_{py}} - 0.55\right)$$

f_{pt} = stress in prestressing strands immediately after transfer but shall not be less than $0.55f_{py}$ = 0.55(243) = 133.65 ksi.

f_{pt} = (0.75)(270) – 2.23 – 13.26 = 187.01 ksi

K_L = 30 for low relaxation strands.

$$\Delta f_{pR1} = \frac{187.01}{30} \cdot \left(\frac{187.01}{243} - 0.55\right) = 1.37\,\text{ksi}$$

Time-Dependent Losses Prior to Deck Placement

The sum of these first three losses is the time-dependent loss prior to placement of deck.

$(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id}$ = 7.00 + 11.12 + 1.37 = 19.49 ksi

The losses from the time of deck placement to final time consist of four components: shrinkage of concrete, creep of concrete, relaxation of prestressing strand and shrinkage of deck concrete.

Shrinkage of Girder Concrete [5.9.5.4.3a]

The prestress loss due to shrinkage of the girder concrete between time of deck placement and final time is determined as follows:

[5.9.5.4.3a-1]

$$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df}$$

where:

ε_{bdf} = shrinkage strain of girder between time of deck placement and final time per Eq. 5.4.2.3.3-1.

K_{df} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between deck placement and final time.

[5.9.5.4.3a-2]

$$K_{df} = \frac{1}{1 + \frac{E_p}{E_{ci}} \frac{A_{ps}}{A_c} \left(1 + \frac{A_c e^2{}_{pc}}{I_c}\right)\left[1 + 0.7\psi_b\left(t_f, t_i\right)\right]}$$

e_{pc} = eccentricity of strands with respect to centroid of the net composite section.

A_c = area of section calculated using the net composite concrete section properties of the girder and the deck and the deck-to-girder modular ratio.

I_c = moment of inertia of section calculated using the net composite concrete section properties of the girder and the deck and the deck-to-girder modular ratio at service.

Use gross section properties to calculate K_{df}. The Specification states that net composite section properties should be used. However, that is not consistent with usage of gross section properties for K_{id}. Instead of introducing a new type of section properties gross composite section properties will be used.

Use gross composite section properties for calculation of K_{df}.

$$K_{df} = \frac{1}{1+\frac{28500}{3818}\frac{5.202}{971}\cdot\left(1+\frac{(971)\cdot(17.301)^2}{171{,}153}\right)\cdot[1+(0.7)\cdot(1.813)]}$$

$K_{df} = 0.803$

The value of shrinkage at the time of deck placement has already been calculated. The shrinkage at final time is calculated as follows:

$\varepsilon_{sh} = -k_{vs}\, k_{hs}\, k_f\, k_{td}\, 0.48 \times 10^{-3}$

$\varepsilon_{sh} = -(0.831)(1.440)(0.926)(1.0)(0.48 \times 10^{-3}) = 0.532 \times 10^{-3}$

The difference in strain between time of deck placement and final time is:

$\varepsilon_{bid} = 0.532 \times 10^{-3} - 0.308 \times 10^{-3} = 0.224 \times 10^{-3}$

The prestress loss due to shrinkage after deck placement is:

$\Delta f_{pSR} = \varepsilon_{bdf}\, E_p\, K_{df} = (0.000224)(28500)(0.803) = 5.13$ ksi

Creep of Girder Concrete [5.9.5.4.3b]

The prestress loss due to creep of girder concrete between time of deck placement and final time is determined as:

[5.9.5.4.3b-1]

$$\Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{cgp}\left[\psi_b(t_f,t_i)-\psi_b(t_d,t_i)\right]K_{df} + \frac{E_p}{E_c}\Delta f_{cd}\psi_b(t_f,t_d)K_{df} \geq 0.0$$

where:

$\psi_b(t_f\,,\, t_d)$ = girder creep coefficient at final time due to loading at deck placement per Eq. 5.4.2.3.2-1.

$\psi_b(t_f\,,\, t_d) = (1.9)(0.831)(1.240)(0.926)(1.0)(60)^{-0.118} = 1.118$

Δf_{cd} = change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement, combined with deck weight and superimposed loads.

The long-term time-dependent loss between transfer and deck placement is 19.49 ksi. This change in stress will result in the following tensile forces in the concrete at the centroid of the prestressing using net section properties.

$\Delta P = \Delta f_{pid}\, A_{ps} = (19.49)(5.202) = 101.39$ k

$$\Delta f_{cd1} = (101.39)\cdot\left(\frac{1}{760.55} + \frac{(13.342)^2}{110{,}441}\right) = -0.297\ \text{ksi}$$

The addition of the weight of the deck and superimposed loads will result in the following tensile forces in the concrete based on transformed section properties.

$$\Delta f_{cd2} = \frac{(330)\cdot(12)\cdot(12.733)}{116{,}626} + \frac{(89+88)\cdot(12)\cdot(16.762)}{180{,}203} = -0.630\,\text{ksi}$$

Δf_{cd} = -0.297 – 0.630 = -0.927 ksi

The resulting creep loss equals:

$$\Delta f_{pCD} = \frac{28500}{3818}\cdot(1.777)\cdot[1.813-1.052]\cdot(0.803)$$

$$+\frac{28500}{4070}\cdot(-0.927)\cdot[1.118]\cdot(0.803) = 8.11-5.83 = 2.28 \geq 0.0\,\text{ksi}$$

Δf_{pCD} = 2.28 ksi

Relaxation of Prestressing Strands [5.9.5.4.3c] [5.9.5.4.3c-1]

The prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time is determined as follows:

$$\Delta f_{pR2} = \Delta f_{pR1} = 1.37\,\text{ksi}$$

Shrinkage of Deck Concrete [5.9.5.4.3d]

The prestress gain due to shrinkage of deck composite section is determined as follows:

[5.9.5.4.3d-1]

$$\Delta f_{pSS} = \frac{E_p}{E_c} \Delta f_{cdf} K_{df} \left[1 + 0.7\psi_b \left(t_f, t_d\right)\right]$$

in which:

$$\Delta f_{cdf} = \frac{\varepsilon_{ddf} A_d E_{cd}}{\left[1 + 0.7\psi_d \left(t_f, t_d\right)\right]} \left(\frac{1}{A_c} + \frac{e_{pc} e_d}{I_c}\right)$$

Δf_{cdf} = change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete.

ε_{ddf} = shrinkage strain of deck concrete between placement and final time per Eq. 5.4.2.3.3-1

The basic equation for shrinkage is:

[5.4.2.3.3-1]

$\varepsilon_{sh} = -k_{vs} k_{hs} k_f k_{td} \, 0.48 \times 10^{-3}$

The volume-to-surface ratio is determined as follows:

$V = (4.50)(48) = 216 \text{ in}^2$

S = 48 top +48 bottom – 48 top flange = 48 in

V / S = 216 / 48 = 4.50 in

$k_{vs} = 1.45 - 0.13(V/S) = 1.45 - 0.13(4.50) = 0.865$

$k_{hs} = 2.00 - 0.014H = 2.00 - 0.014(40) = 1.440$

Since there is no specified release strength for the deck, use 80% f'_c.

$$k_f = \frac{5}{1 + f'_{ci}} = \frac{5}{1 + (0.80)\cdot(4.5)} = 1.087$$

$$k_{td} = \frac{t}{61 - 4f'_{ci} + t} = \frac{18{,}250}{61 - (4)\cdot(0.80)\cdot(4.5) + 18{,}250} = 0.997 \text{ Use } 1.0$$

$\varepsilon_{ddf} = -(0.865)(1.440)(1.087)(1.0)\,(0.48 \times 10^{-3}) = 0.650 \times 10^{-3}$

Other variables are defined as follows:

A_d = area of deck concrete = (4.50)(48) = 216 in^2

E_{cd} = modulus of elasticity of deck concrete = 3861 ksi

e_d = eccentricity of deck with respect to the transformed gross composite section, taken as negative in common construction.

e_d = -12.875 – 4.50 / 2 = -15.125 in

$\psi_d(t_f, t_d)$ = creep coefficient of deck concrete at final time due to loading introduced shortly after deck placement per Eq. 5.4.2.3.2-1

The deck will start to apply shrinkage effects to the beam as soon as the deck sets. Therefore use t_i = 1 day.

[5.4.2.3.2-1]

$\psi_d(t_f, t_d) = 1.9\ k_{vs}\, k_{hc}\ k_f\ k_{td}\ t_i^{-0.118}$

$\psi_d(t_f, t_d) = (1.9)(0.865)(1.240)(1.087)(1.0)(1)^{-0.118} = 2.215$

[5.9.5.4.3d-2]

$$\Delta f_{cdf} = \frac{(0.000650)\cdot(216)\cdot(3861)}{[1+(0.7)\cdot(2.215)]}\cdot\left(\frac{1}{970.73}+\frac{(17.301)\cdot(-15.125)}{171{,}153}\right) = -0.106$$

$$\Delta f_{pSS} = \frac{28500}{4070}\cdot(-0.106)\cdot(0.803)\cdot[1+0.7\cdot(1.118)] = -1.06\,\text{ksi}$$

Time-Dependent Losses After Deck Placement

The sum of the time-dependent prestress losses after deck placement is:

$(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df}$ = 5.13 + 2.28 + 1.37 – 1.06 = 7.72 ksi

Time-Dependent Losses

The final sum of time-dependent losses is:

Δf_{pLT} = 19.49 + 7.72 = 27.21 ksi
compared to the approximate time-dependent loss of 33.32 ksi.

APPENDIX D

APPROXIMATE PRESTRESS LOSSES

DEVELOPMENT OF EQUATION

The approximate method for determining time-dependent prestress losses is derived from the refined method shown in Appendix B. The time dependent loss is the sum of the losses before deck placement and those after deck placement. The following equation results:

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df}$$

Substituting the appropriate equation for each loss yields:

$$\Delta f_{pLT} = \varepsilon_{bid} E_p K_{id} + \frac{E_p}{E_{ci}} f_{cgp} \psi_b(t_d, t_i) K_{id} + \Delta f_{pR1}$$
$$+ \varepsilon_{bdf} E_p K_{df} + \frac{E_p}{E_{ci}} f_{cgp} \left[\psi_b(t_f, t_i) - \psi_b(t_d, t_i)\right] K_{df}$$
$$+ \frac{E_p}{E_c} \Delta f_{cd} \psi_b(t_f, t_d) K_{df} + \Delta f_{pR2} + \Delta f_{pSS}$$

Shrinkage Loss

The shrinkage loss is the sum of the shrinkage loss before deck placement and that after deck placement.

$$\text{Shrinkage} = \varepsilon_{bid} E_p K_{id} + \varepsilon_{bdf} E_p K_{df}$$

Assumption 1

Based on investigation of many examples, assume that $K_{id} = K_{df} = 0.8$

The equation simplifies to:

$$\text{Shrinkage} = 0.8E_p(\varepsilon_{bid} + \varepsilon_{bdf})$$

Where $\varepsilon_{bid} + \varepsilon_{bd}$ is the total shrinkage strain as shown below:

$$\varepsilon_{sh} = -k_{vs} k_{hs} k_f k_{td} 0.48 \times 10^{-3}$$

Assumption 2

Assume that the volume to surface ratio is 3.5, then

$$k_{vs} = 1.45 - 0.13(V/S) = 1.45 - 0.13(3.5) = 1.0$$

$k_{td} = 1.0$ for final time

$$\varepsilon_{sh} = -k_{vs} k_{hs} k_f k_{td} 0.48 \times 10^{-3} = (1.0) k_{hs} k_f (1.0) 0.48 \times 10^{-3}$$
$$\varepsilon_{sh} = k_{hs} k_f 0.48 \times 10^{-3}$$

The equation for shrinkage loss reduces to:

$$\text{Shrinkage} = 0.8E_p(\varepsilon_{bid} + \varepsilon_{bdf}) = 0.8(28{,}500) k_{hs} k_f 0.48 \times 10^{-3}$$
$$\text{Shrinkage} = 10.94 k_{hs} k_f \Rightarrow \underline{\text{Use} = 12.0 k_{hs} k_f}$$

Creep Loss

The creep loss is the sum of the creep loss before deck placement plus the creep loss after deck placement.

$$\text{Creep} = \frac{E_p}{E_{ci}} f_{cgp}\psi_b(t_d, t_i)K_{id} + \frac{E_p}{E_{ci}} f_{cgp}\left[\psi_b(t_f, t_i) - \psi_b(t_d, t_i)\right]K_{df} + \frac{E_p}{E_c}\Delta f_{cd}\psi_b(t_f, t_d)K_{df}$$

Again assume that $K_{id} = K_{df} = 0.8$ and combine terms resulting in:

$$\text{Creep} = \frac{E_p}{E_{ci}} f_{cgp}\psi_b(t_f, t_i)\cdot(0.8) + \frac{E_p}{E_c}\Delta f_{cd}\psi_b(t_f, t_d)\cdot(0.8)$$

Where:

$\psi_b(t_f, t_i) = 1.9\ k_{vs}\ k_{hc}\ k_f\ k_{td}\ t_i^{-0.118}$

Assume the following:

Volume to surface ratio equals 3.5 $\Rightarrow k_{vs} = 1.0$

Assumption 3

The load is applied at one day $\Rightarrow t_i^{-0.118} = 1.0$

$\psi_b(t_f, t_i) = 1.9\ (1.0)\ k_{hc}\ k_f\ (1.0)\ (1)^{-0.118} = 1.9\ k_{hc}\ k_f$

Assumption 4

Assume that the creep factor for loads applied after the deck pour equals:

$\psi_b(t_f, t_d) = 0.4\ \psi_b(t_f, t_i)$

Assume the following:

Assumption 5

1) $\frac{E_p}{E_{ci}} = 7$ based on $f'_{ci} = 5.0$ ksi

Assumption 6

2) $\frac{E_p}{E_c} = 6$ based on $f'_c = 6.8$ ksi

The equation for creep loss then reduces to the following:

$\text{Creep} = (7)f_{cgp}(1.9\ k_{hc}\ k_f)(0.8) + (6)\Delta f_{cd}(0.4)(1.9\ k_{hc}\ k_f)(0.8)$

$\text{Creep} = 10.64\ k_{hc}\ k_f\ f_{cgp} + 3.65\ k_{hc}\ k_f\ \Delta f_{cd}$

This is similar in form to the current prestress creep equation. Additional assumptions are now made to eliminate the two terms f_{cgp} and Δf_{cd}.

Assume the following:

Assumption 7

1) Final stress at the cg of the strands = 0

Assumption 8

2) Moment from the girder, deck placement and live load are equal, resulting in $\sum M = 3M_g$.

Assumption 9

3) $\left(1+\dfrac{Ae^2}{I}\right)=2$

Assumption 10

4) The effective prestress equals 80% of the initial prestress.

Sum the stresses at the c.g. of the strands as follows:

$$f_{cg} = \frac{3M_g e_p}{I_g} - 0.8P_i\left(\frac{1}{A_g}+\frac{e^2}{I_g}\right) = 0$$

$$\frac{3M_g e_p}{I_g} = \frac{0.8P_i}{A_g}\left(1+\frac{Ae_p{}^2}{I_g}\right) = \frac{0.8P_i}{A_g}(2.0)$$

Girder Moment

$$M_g = \frac{1.6P_i I_g}{3A_g e_p}$$

$$f_{cgp} = \frac{P_i}{A_g}\left(1+\frac{A_g e_p{}^2}{I_g}\right) - \frac{M_g e_p}{I_g}$$

$$f_{cgp} = \frac{0.8P_i}{A_g}(2) - \frac{M_g e_p}{I_g} = \frac{1.6P_i}{A_g} - \frac{1.6P_i I_g}{3A_g e_p}\cdot\frac{e_p}{I_g} = \frac{3.2P_i}{3A_g}$$

Deck Stress

$$\Delta f_{cd} = \frac{M_g e_p}{I_g} = \frac{1.6P_i I_g}{3A_g e_p}\cdot\frac{e_p}{I_g} = \frac{-1.6P_i}{3A_g}$$

Creep = 10.64 k_{hc} k_f ($3.2P_i$ / $3A_g$) + 3.65 k_{hc} k_f ($-1.6P_i$ / $3A_g$)

Creep = 9.40 (P_i / A_g) k_{hc} k_f

Rounding and Substituting f_{pi} A_{ps} = P_i Yields $\underline{10.0(f_{pi}\ A_{ps} / A_g)\ k_{hc}\ k_f}$

Assumption 11 Ignore the gain in prestress from the deck shrinkage.

Relaxation Loss
Assumption 12 Assume the relaxation from the prestressing strands equals 2.5 ksi for low relaxation strands.

The resulting equation is then:

$$\Delta f_{pLT} = 10.0 \frac{f_{pi} A_{ps}}{A_g} \gamma_h \gamma_{st} + 12.0 \gamma_h \gamma_{st} + \Delta f_{pR}$$

in which:

$\gamma_h = 1.7 - 0.01H$ is an average humidity factor for shrinkage and creep.

$$\gamma_{st} = \frac{5}{1 + f'_{ci}}$$

A summary of the assumptions made in the development of the approximate formula are listed below with the corresponding value from this example.

	Assumption	Actual
Assumption 1	$K_{id} = 0.8$ $K_{df} = 0.8$	$K_{id} = 0.797$ $K_{df} = 0.803$
Assumption 2	$V / S = 3.5$ $k_{VS} = 1.0$	$V / S = 4.76$ $k_{VS} = 0.831$
Assumption 3	$t_i = 1.0$	$t_i = 1.0$
Assumption 4	$\psi_b(t_f, t_d) = 0.4\ \psi_b(t_f, t_i)$	$\psi_b(t_f, t_d) = 0.617\ \psi_b(t_f, t_i)$
Assumption 5	$E_p / E_{ci} = 7$	$E_p / E_{ci} = 7.46$
Assumption 6	$E_p / E_c = 6$	$E_p / E_c = 7.00$
Assumption 7	$f_{cgp} = 0$	$f_{cgp} = -0.069$ ksi
Assumption 8	$M_g = M_d = M_{ll}$	$M_g = 738$ ft-k $M_d = 507$ ft-k $M_{ll} = 645$ ft-k

Assumption 9	$1 + A\,e^2 / I = 2$	$1 + A\,e^2 / I = 2.21$ gross $1 + A\,e^2 / I = 2.70$ gross composite
Assumption 10	$P_{eff} = 0.80\,P_i$	$P_{eff} = 0.77\,P_i$
Assumption 11	$\Delta f_{pSS} = 0$ ksi	$\Delta f_{pSS} = 1.06$ ksi
Assumption 12	$\Delta f_r = 2.50$ ksi	$\Delta f_r = 2.74$ ksi

Summary

Assumption 2 is a function of the type of girder/beam. This assumption would be closer for an I-girder.

Assumption 4 appears to be questionable.

Assumptions 5 and 6 do not reflect current ADOT practice on concrete strengths.

Assumption 8 is not bad for this example but could differ considerably for some girder spacings.

APPENDIX E

ANALYSIS COMPARISON

1) Transformed Section

2) Net Section with Elastic Gain

3) Gross Section with Elastic Gain

4) Gross Section

1) Transformed Section

The concrete and prestress strand stresses will be calculated using transformed section properties.

Transfer Stresses P/S and Beam

The effective prestress force prior to transfer is:

P_i = [(0.75)(270) – 2.23](34)(0.153) = 1041.80 kips

The stress in the concrete using transformed section properties at transfer is:

$$f_t = (1041.80)\cdot\left(\frac{1}{799.36} - \frac{(12.695)\cdot(17.481)}{117{,}014}\right) + \frac{(738)\cdot(12)\cdot(17.481)}{117{,}014}$$

f_t = -0.673 + 1.323 = 0.650 ksi

$$f_{cgp} = (1041.80)\cdot\left(\frac{1}{799.36} + \frac{(12.695)\cdot(12.695)}{117{,}014}\right) - \frac{(738)\cdot(12)\cdot(12.695)}{117{,}014}$$

f_{cgp} = 2.738 – 0.961 = 1.777 ksi

$$f_b = (1041.80)\cdot\left(\frac{1}{799.36} + \frac{(12.695)\cdot(15.519)}{117{,}014}\right) - \frac{(738)\cdot(12)\cdot(15.519)}{117{,}014}$$

f_b = 3.057 – 1.175 = 1.882 ksi

The loss of stress in the prestress strand = (1.777)(28,500) / (3818) = 13.26 ksi

Deck Placement

The stress in the concrete from the deck pour using transformed section properties at service is:

$$f_t = \frac{(330)\cdot(12)\cdot(17.443)}{116{,}626} = 0.592$$

$$f_{cgp} = -\frac{(330)\cdot(12)\cdot(12.733)}{116{,}626} = -0.432$$

$$f_b = -\frac{(330)\cdot(12)\cdot(15.557)}{116{,}626} = -0.528$$

The gain in stress in the prestress strand = (-0.432)(28,500) / (4070) = -3.03 ksi

Composite DL

The stress in the concrete from the composite dead load is:

$$f_t = \frac{(89+88)\cdot(12)\cdot(13.414)}{180{,}203} = 0.158$$

$$f_{cgp} = -\frac{(89+88)\cdot(12)\cdot(16.762)}{180{,}203} = -0.198$$

$$f_b = -\frac{(89+88)\cdot(12)\cdot(19.586)}{180{,}203} = -0.231$$

The gain in stress in the prestress strand = (-0.198)(28,500) / (4070) = -1.39 ksi

Live Load +IM

The stress in the concrete from the live load plus dynamic load allowance is:

Service I

$$f_t = \frac{(645)\cdot(12)\cdot(13.414)}{180{,}203} = 0.576$$

Service III

$$f_{cgp} = -\frac{(0.8)\cdot(645)\cdot(12)\cdot(16.762)}{180{,}203} = -0.576$$

$$f_b = -\frac{(0.8)\cdot(645)\cdot(12)\cdot(19.586)}{180{,}203} = -0.673$$

The gain in stress in the prestress strand = (-0.576)(28,500) / (4070) = -4.03 ksi

Time-Dependent Loss

The 33.32 ksi prestress loss results in a loss in prestress force of:

P_i = (33.32)(34)(0.153) = 173.33 kips

The stress in the concrete using net section properties is:

$$f_t = (-173.33)\cdot\left(\frac{1}{760.55} - \frac{(13.342)\cdot(16.834)}{110{,}441}\right) = 0.125$$

$$f_b = (-173.33)\cdot\left(\frac{1}{760.55} + \frac{(13.342)\cdot(16.166)}{110{,}441}\right) = -0.566$$

Stress Summary

A summary of stresses in the concrete follows:

Service I

$$f_t = 0.650 + 0.592 + 0.158 + 0.576 + 0.125 = 2.101 \text{ ksi}$$

Service III

$$f_b = 1.882 - 0.528 - 0.231 - 0.673 - 0.566 = -0.116 \text{ ksi}$$

A summary of stress in the strand is:

$$f_{ps} = (0.75)(270) - 2.23 - 13.26 + 3.03 + 1.39 + 4.03 - 33.32 = 162.14 \text{ ksi}$$

2) Net Section With Elastic Gain

The concrete and prestress strand stresses will be calculated using net section properties and including the elastic gain in prestress.

Transfer Stresses P/S and Beam

The effective prestress force after transfer including elastic shortening loss is:

P_i = [(0.75)(270) – 2.23 – 13.26](34)(0.153) = 972.83 kips

The stress in the concrete is:

$$f_t = (972.83)\cdot\left(\frac{1}{760.55} - \frac{(13.342)\cdot(16.834)}{110{,}441}\right) + \frac{(738)\cdot(12)\cdot(16.834)}{110{,}441}$$

f_t = -0.699 + 1.350 = 0.651 ksi

$$f_{cgp} = (972.83)\cdot\left(\frac{1}{760.55} + \frac{(13.342)\cdot(13.342)}{110{,}441}\right) - \frac{(738)\cdot(12)\cdot(13.342)}{110{,}441}$$

f_{cgp} = 2.847 - 1.070 = 1.777 ksi

$$f_b = (972.83)\cdot\left(\frac{1}{760.55} + \frac{(13.342)\cdot(16.166)}{110{,}441}\right) - \frac{(738)\cdot(12)\cdot(16.166)}{110{,}441}$$

f_b = 3.179 - 1.296 = 1.883 ksi

The loss of stress in the prestress strand = (1.777)(28,500) / (3818) = 13.26 ksi

Deck Placement

Since the strands are bonded to the concrete, the addition of external loads will add tension to the prestress strands. Solution of the problem of determining the concrete stress at the c.g. of the strands is similar to that for elastic shortening as shown in the following formula.

The elastic gain from the applied moment, M is shown below:

$$f_{cgp} = \frac{-Me}{I + \frac{E_p}{E_{ct}} A_{ps}\left(r^2 + e^2\right)}$$

$$f_{cgp} = \frac{-(330)\cdot(12)\cdot(13.342)}{110{,}441 + \frac{28500}{4070}\cdot(5.202)\cdot\left(145.21 + (13.342)^2\right)} = -0.432 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.432) = -3.03\,\text{ksi}$$

P_{gain} = (3.03)(5.202) = 15.74 k

The stress in the concrete from the deck pour is:

$$f_t = \frac{(330)\cdot(12)\cdot(16.834)}{110{,}441} + (15.74)\cdot\left(\frac{1}{760.55} - \frac{(13.342)\cdot(16.834)}{110{,}441}\right)$$

f_t = 0.604 - 0.011 = 0.593 ksi

$$f_b = -\frac{(330)\cdot(12)\cdot(16.166)}{110{,}441} + (15.74)\cdot\left(\frac{1}{760.55} + \frac{(13.342)\cdot(16.166)}{110{,}441}\right)$$

f_b = -0.580 + 0.051 = -0.529 ksi

Time-Dependent Loss

The time-dependent loss of 33.32 ksi will not cause any elastic gain. The loss of stress in the concrete will be:

$$f_t = -(33.32)\cdot(5.202)\cdot\left(\frac{1}{760.55} - \frac{(13.342)\cdot(16.834)}{110{,}441}\right)$$

f_t = 0.125 ksi

$$f_b = -(33.32)\cdot(5.202)\cdot\left(\frac{1}{760.55} + \frac{(13.342)\cdot(16.166)}{110{,}441}\right)$$

f_b = -0.566 ksi

Net Composite Section Properties

The net composite section properties must be calculated as follows:

A_c = 760.55 + (0.949)(48.00)(4.50) = 965.53 in^2

y_b = [(760.55)(16.166) + 0.949(4.50)(48.00)(35.25)] ÷ 965.53 = 20.218 in

e = 20.218 – 2.824 = 17.394 in

y_t = 33.00 –20.318 = 12.782 in

I_t = 110,441 + 760.55(20.218 – 16.166)^2 + 0.949(48.00)(4.50)^3 ÷12
+ 0.949(48.00)(4.50)(35.25 – 20.218)^2 = 169,593 in^4

r^2 = 169,593 / 965.53 = 175.65 in^2

Composite Dead Load

The elastic gain from the applied moment, M is shown below:

$$\text{Elastic Gain: } f_{cgp} = \frac{-Me}{I + \frac{E_p}{E_{ct}} A_{ps}\left(r^2 + e^2\right)}$$

$$f_{cgp} = \frac{-(89+88)\cdot(12)\cdot(17.394)}{169{,}593 + \frac{28500}{4070}\cdot(5.202)\cdot\left(175.65+(17.394)^2\right)} = -0.198 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.198) = 1.38 \text{ ksi}$$

P_{gain} = (1.38)(5.202) = 7.18 k

The stress in the concrete from the composite dead load is:

$$f_t = \frac{(89+88)\cdot(12)\cdot(12.782)}{169{,}593} + (7.18)\cdot\left(\frac{1}{965.53} - \frac{(17.394)\cdot(12.782)}{169{,}593}\right)$$

f_t = 0.160 - 0.002 = 0.158 ksi

$$f_b = -\frac{(89+88)\cdot(12)\cdot(20.218)}{169{,}593} + (7.18)\cdot\left(\frac{1}{965.53} + \frac{(17.394)\cdot(20.218)}{169{,}593}\right)$$

f_b = -0.253 + 0.022 = -0.231 ksi

Live Load + IM

The elastic gain from the applied moment, M is shown below:

$$f_{cgp} = \frac{-(645)\cdot(12)\cdot(17.394)}{169{,}593 + \frac{28500}{4070}\cdot(5.202)\cdot\left(175.65+(17.394)^2\right)} = -0.720 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.720) = -5.04 \text{ ksi}$$

P_{gain} = (5.04)(5.202) = 26.22 k

The stress in the concrete from the live load is:

Service I

$$f_t = \frac{(645)\cdot(12)\cdot(12.782)}{169{,}593} + (26.22)\cdot\left(\frac{1}{965.53} - \frac{(17.394)\cdot(12.782)}{169{,}593}\right)$$

$f_t = 0.583 - 0.007 = 0.576$ ksi

Service III

LL + IM = (0.8)(645) = 516 ft-k

$$f_b = -\frac{(516)\cdot(12)\cdot(20.218)}{169{,}593} + (0.8)\cdot(26.22)\cdot\left(\frac{1}{965.53} + \frac{(17.394)\cdot(20.218)}{169{,}593}\right)$$

$f_b = -0.738 + 0.065 = -0.673$ ksi

A summary of stresses in the concrete follows:

Service I

$f_t = 0.651 + 0.593 + 0.158 + 0.576 + 0.125 = 2.103$ ksi

Service III

$f_b = 1.883 - 0.529 - 0.231 - 0.673 - 0.566 = -0.116$ ksi

A summary of stress in the strand is:

$f_{ps} = (0.75)(270) - 2.23 - 13.26 + 3.03 + 1.38 + (0.8)5.04 - 33.32$
$= 162.13$ ksi

3) Gross Section With Elastic Gain

The concrete and prestress strand stresses will be calculated using gross section properties and including the elastic gain in prestress.

Elastic Shortening

The elastic shortening loss is determined using gross section properties as follows:

$$\Delta f_{pES} = \frac{f_{pbt} A_{ps}\left(r^2 + e_m{}^2\right) - e_m M_g}{A_{ps}\left(r^2 + e_m{}^2\right) + \dfrac{I \cdot E_{ci}}{E_p}}$$

$$\Delta f_{pES} = \frac{(200.27)\cdot(5.202)\cdot\left(145.43 + (13.252)^2\right) - (13.252)\cdot(738)\cdot(12)}{(5.202)\cdot\left(145.43 + (13.252)^2\right) + \dfrac{(111{,}361)\cdot(3818)}{28{,}500}}$$

Δf_{pES} = 13.09 ksi

Transfer Stresses P/S and Beam

The effective prestress force after transfer including elastic shortening loss is:

P_i = [(0.75)(270) – 2.23 – 13.09](34)(0.153) = 973.71 kips

The stress in the concrete is:

$$f_t = (973.71)\cdot\left(\frac{1}{765.75} - \frac{(13.252)\cdot(16.924)}{111{,}361}\right) + \frac{(738)\cdot(12)\cdot(16.924)}{111{,}361}$$

f_t = -0.689 + 1.346 = 0.657 ksi

$$f_{cgp} = (973.71)\cdot\left(\frac{1}{765.75} + \frac{(13.252)\cdot(13.252)}{111{,}361}\right) - \frac{(738)\cdot(12)\cdot(13.252)}{111{,}361}$$

f_{cgp} = 2.807 - 1.054 = 1.753 ksi

$$f_b = (973.71)\cdot\left(\frac{1}{765.75} + \frac{(13.252)\cdot(16.076)}{111{,}361}\right) - \frac{(738)\cdot(12)\cdot(16.076)}{111{,}361}$$

f_b = 3.134 - 1.278 = 1.856 ksi

The loss in stress in the prestress strand = (1.753)(28,500) / (3818) = 13.09 ksi

Deck Placement

Since the strands are bonded to the concrete, the addition of external loads will add tensile stress to the strands. Solution of the problem of determining the concrete stress at the c.g. of the strands is similar to that of elastic shortening but in this case the prestress component is a function of the applied load.

The elastic gain from the applied moment, M is shown below:

$$f_{cgp} = \frac{-Me}{I + \frac{E_p}{E_{ct}} A_{ps}\left(r^2 + e^2\right)}$$

$$f_{cgp} = \frac{-(330)\cdot(12)\cdot(13.252)}{111{,}361 + \frac{28500}{4070}\cdot(5.202)\cdot\left(145.43 + (13.252)^2\right)} = -0.426 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.426) = -2.99 \text{ ksi}$$

P_{gain} = (2.99)(5.202) = 15.53 k

The stress in the concrete from the deck pour is:

$$f_t = \frac{(330)\cdot(12)\cdot(16.924)}{111{,}361} + (15.53)\cdot\left(\frac{1}{765.75} - \frac{(13.252)\cdot(16.924)}{111{,}361}\right)$$

f_t = 0.602 - 0.011 = 0.591 ksi

$$f_b = -\frac{(330)\cdot(12)\cdot(16.076)}{111{,}361} + (15.53)\cdot\left(\frac{1}{765.75} + \frac{(13.252)\cdot(16.076)}{111{,}361}\right)$$

f_b = -0.572 + 0.050 = -0.522 ksi

Time-Dependent Loss

The time-dependent loss of 33.32 ksi will not cause any elastic gain. The stress in the concrete from the time-dependent loss is:

$$f_t = -(33.32)\cdot(5.202)\cdot\left(\frac{1}{765.75} - \frac{(13.252)\cdot(16.924)}{111{,}361}\right)$$

f_t = 0.123 ksi

$$f_b = -(33.32)\cdot(5.202)\cdot\left(\frac{1}{765.75} + \frac{(13.252)\cdot(16.076)}{111{,}361}\right)$$

f_b = -0.558 ksi

Composite Dead Load

The elastic gain from the applied moment, M is shown below:

$$f_{cgp} = \frac{-Me}{I + \frac{E_p}{E_{ct}} A_{ps}\left(r^2 + e^2\right)}$$

$$f_{cgp} = \frac{-(89+88)\cdot(12)\cdot(17.301)}{171{,}153 + \frac{28500}{4070}\cdot(5.202)\cdot\left(176.31 + (17.301)^2\right)} = -0.195 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.195) = -1.37 \text{ ksi}$$

P_{gain} = (1.37)(5.202) = 7.10 k

The stress in the concrete from the composite dead load is:

$$f_t = \frac{(177)\cdot(12)\cdot(12.875)}{171{,}153} + (7.10)\cdot\left(\frac{1}{970.73} - \frac{(17.301)\cdot(12.875)}{171{,}153}\right)$$

f_t = 0.160 - 0.002 = 0.158 ksi

$$f_b = -\frac{(177)\cdot(12)\cdot(20.125)}{171{,}153} + (7.10)\cdot\left(\frac{1}{970.73} + \frac{(17.301)\cdot(20.125)}{171{,}153}\right)$$

f_b = -0.250 + 0.022 = -0.228 ksi

Live Load + IM

The elastic gain from the applied moment, M is shown below:

$$f_{cgp} = \frac{-(645)\cdot(12)\cdot(17.301)}{171{,}153 + \frac{28500}{4070}\cdot(5.202)\cdot\left(176.31 + (17.301)^2\right)} = -0.710 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.710) = -4.98 \text{ ksi}$$

P_{gain} = (4.98)(5.202) = 25.88 k

The stress in the concrete from the live load is:

Service I

$$f_t = \frac{(645)\cdot(12)\cdot(12.875)}{171{,}153} + (25.88)\cdot\left(\frac{1}{970.73} - \frac{(17.301)\cdot(12.875)}{171{,}153}\right)$$

f_t = 0.582 - 0.007 = 0.575 ksi

Service III

LL + IM = (0.8)(645) = 516 ft-k

$$f_b = -\frac{(516)\cdot(12)\cdot(20.125)}{171{,}153} + (0.8)\cdot(25.88)\cdot\left(\frac{1}{970.73} + \frac{(17.301)\cdot(20.125)}{171{,}153}\right)$$

f_b = -0.728 + 0.063 = -0.665 ksi

A summary of stresses in the concrete follows:

Service I

f_t = 0.657 + 0.591 + 0.158 + 0.575 + 0.123 = 2.106 ksi

Service III

f_b = 1.856 – 0.522 – 0.228 – 0.665 – 0.558 = -0.117 ksi

A summary of stress in the strand is:

f_{ps} = (0.75)(270) – 2.23 – 13.09 + 2.99 + 1.37 + (0.8)(4.98) – 33.32
= 162.20 ksi

Summary
Top Fiber Stress

	Method 1	Method 2	Method 3	Method 4
Girder	1.323	1.350	1.346	1.346
P/S	-0.673	-0.699	-0.689	-0.689
Subtotal	0.650	0.651	0.657	0.657
Deck	0.592	0.604	0.602	0.602
Elastic Gain		-0.011	-0.011	
Subtotal	0.592	0.593	0.591	0.602
Comp DL	0.158	0.160	0.160	0.160
Elastic Gain		-0.002	-0.002	
Subtotal	0.158	0.158	0.158	0.160
LL + IM	0.576	0.583	0.582	0.582
Elastic Gain		-0.007	-0.007	
Subtotal	0.576	0.576	0.575	0.582
P/S Losses	0.125	0.125	0.123	0.123
Total	**2.101**	**2.103**	**2.104**	**2.126**

Bottom Fiber Stress

	Method 1	Method 2	Method 3	Method 4
Girder	-1.175	-1.296	-1.278	-1.278
P/S	3.057	3.179	3.134	3.134
Subtotal	1.882	1.883	1.856	1.856
Deck	-0.528	-0.580	-0.572	-0.572
Elastic Gain		0.051	0.050	
Subtotal	-0.528	-0.529	-0.522	-0.572
Comp DL	-0.231	-0.253	-0.250	-0.250
Elastic Gain		0.022	0.022	
Subtotal	-0.231	-0.231	-0.228	-0.250
LL + IM	-0.673	-0.738	-0.728	-0.728
Elastic Gain		0.065	0.063	
Subtotal	-0.673	-0.673	-0.665	-0.728
P/S Losses	-0.566	-0.566	-0.558	-0.558
Total	**-0.116**	**-0.116**	**-0.117**	**-0.252**

Summary

As can be seen in the above table, use of transformed section properties (Method 1) and use of net section properties considering elastic gain (Method 2) produce nearly identical results except for minor differences due to rounding. Use of gross section properties considering elastic gain (Method 3) produces stresses close to the first two methods with the final sum almost identical. Any of these three methods should be acceptable.

Use of gross section properties without considering elastic gain (Method 4) produces higher stresses than the other three methods. While this method is the simpliest to use it is overly conservative and should not be used.

Method 1 requires maximum effort to determine the section properties but once determined the calculation of concrete stresses is simple. Method 2 requires some effort to determine the section properties and also requires consideration of elastic gain. Use of gross section properties is simpliest for calculation of section properties but does require use of the elastic gain to produce reliable results.