2-Span Precast Prestressed I-Girder Bridge

This example illustrates the design of a two span precast prestressed I-Girder bridge. The bridge has two equal spans of 112.00 feet. An AASHTO modified Type VI girder will be used. The bridge has a 30 degree skew. Standard ADOT parapet and fence as shown in SD 1.04 and SD 1.05 will be used. A half section of the bridge consists of a 1'-0" parapet, a 6'-0" sidewalk, a 14'-0" outside shoulder, one 12'-0" lane and half a 12'-0" turning land. The overall out-to-out width of the bridge is 78'-0". A plan view and typical section of the bridge are shown in Figures 1 and 2.

The following legend is used for the references shown in the left-hand column:

[2.2.2]	LRFD Specification Article Number
[2.2.2-1]	LRFD Specification Table or Equation Number
[C2.2.2]	LRFD Specification Commentary
[A2.2.2]	LRFD Specification Appendix
[BPG]	ADOT LRFD Bridge Practice Guideline

Bridge Geometry

Span lengths	112.00, 112.00 ft
Bridge width	78.00 ft
Roadway width	64.00 ft
Superstructure depth	6.83 ft
Girder spacing	9.00 ft
Web thickness	6.00 in
Top slab thickness	8.00 in
Deck overhang	3.00 ft

Minimum Requirements

[2.5.2.6.3-1]

The minimum span to depth ratio for a simple span precast prestressed concrete I-girder bridge is 0.045 resulting in a minimum depth of (0.045)(110.75) = 4.98 feet. Since the girder depth of 6 feet exceeds the minimum, the criteria is satisfied.

Concrete Deck Slab Minimum Requirements

Slab thickness	8.00 in
Top concrete cover	2.50 in
Bottom concrete cover	1.00 in
Wearing surface	0.50 in

Future Configuration

The bridge will be evaluated for both the current configuration and a future configuration with additional lanes but without sidewalks.

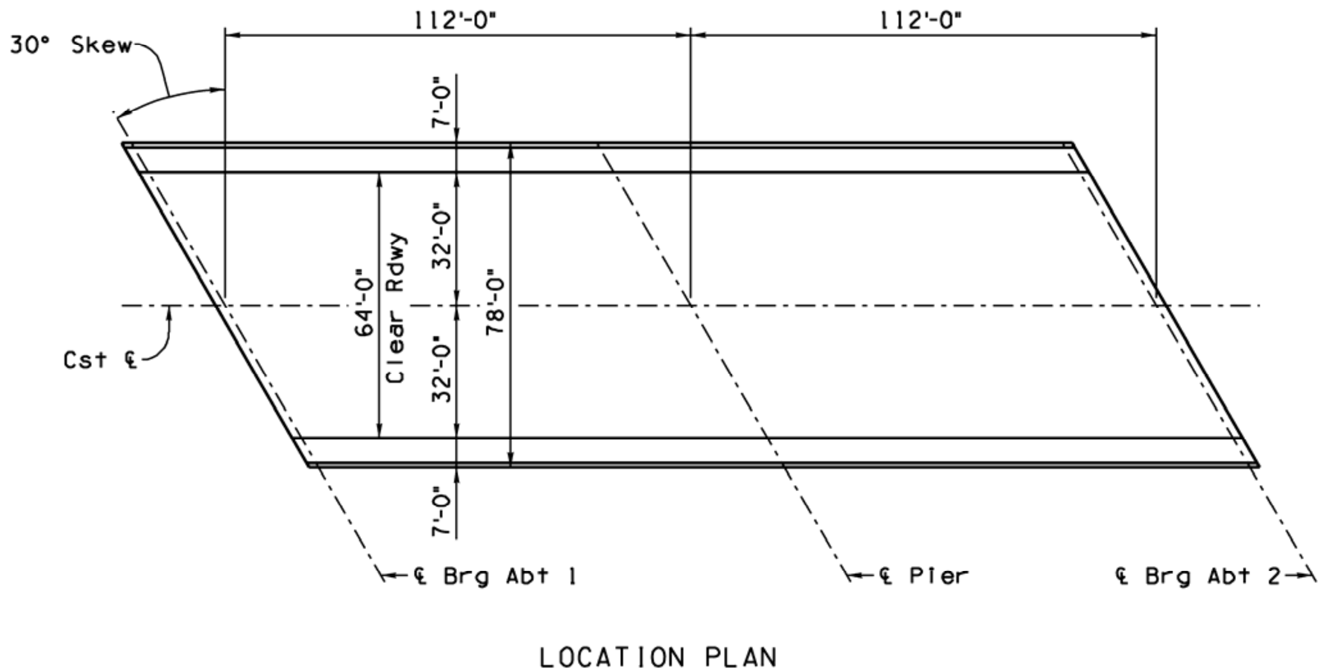


Figure 1

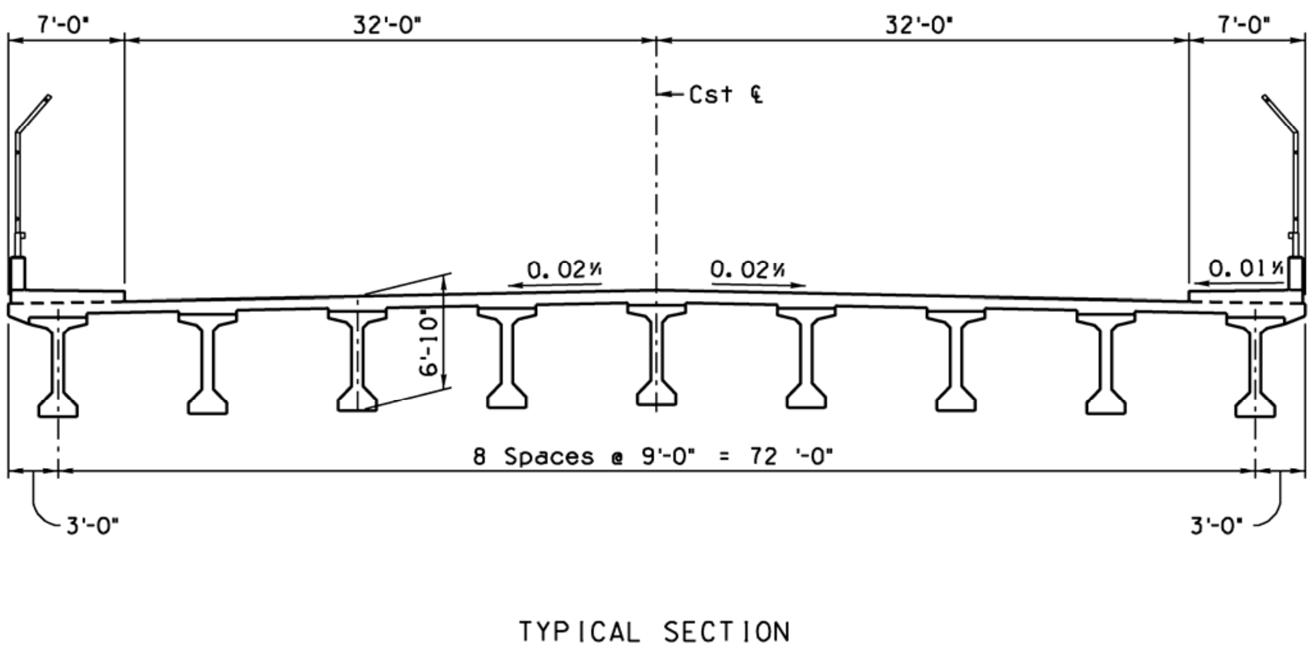


Figure 2

Material Properties

Reinforcing Steel

Yield Strength $f_y = 60$ ksi

[5.4.3.2] Modulus of Elasticity $E_s = 29{,}000$ ksi

Prestressing Strand

Low relaxation prestressing strands

½" diameter strand	A_{ps}	$= 0.153 \text{ in}^2$
[5.4.4.1-1] Tensile Strength	f_{pu}	= 270 ksi
[5.4.4.1-1] Yield Strength	f_{py}	= 243 ksi
[5.4.4.2] Modulus Elasticity	E_p	= 28500 ksi

Concrete

The final and release concrete strengths are specified below:

Precast I-Girder	Deck
$f'_c = 5.0$ ksi	$f'_c = 4.5$ ksi
$f'_{ci} = 4.7$ ksi	

[3.5.1-1] Unit weight for normal weight concrete is listed below:

Unit weight for computing E_c = 0.145 kcf

Unit weight for DL calculation = 0.150 kcf

[C5.4.2.4] The modulus of elasticity for normal weight concrete where the unit weight is 0.145 kcf may be taken as shown below:

Precast I-Girder

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{5.0} = 4070 \; ksi$$

$$E_{ci} = 1820\sqrt{f'_{ci}} = 1820\sqrt{4.7} = 3946 \; ksi$$

Deck Slab

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{4.5} = 3861 \; ksi$$

[5.7.1] The modular ratio of reinforcing to concrete should be rounded to the nearest whole number.

[5.7.2.2]

Precast I-Girder

$$n = \frac{28{,}500}{4070} = 7.00 \text{ Use n = 7 for Prestressing in Girder}$$

$$n = \frac{29{,}000}{4070} = 7.13 \text{ Use n = 7 for Reinforcing in Girder}$$

Deck Slab

$$n = \frac{29{,}000}{3861} = 7.51 \text{ Use n = 8 for Deck}$$

[5.7.2.2]

β_1 = The ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone stress block.

Precast I-Girder

$$\beta_1 = 0.85 - 0.05 \cdot \left[\frac{f'_c - 4.0}{1.0}\right] = 0.85 - 0.05 \cdot \left[\frac{5.0 - 4.0}{1.0}\right] = 0.800$$

Deck Slab

$$\beta_1 = 0.85 - 0.05 \cdot \left[\frac{f'_c - 4.0}{1.0}\right] = 0.85 - 0.05 \cdot \left[\frac{4.5 - 4.0}{1.0}\right] = 0.825$$

Modulus of Rupture [5.4.2.6]

The modulus of rupture for normal weight concrete has two values. When used to calculate service level cracking, as specified in Article 5.7.3.4 for side reinforcing or in Article 5.7.3.6.2 for determination of deflections, the following equation should be used:

Service Level Cracking

$$f_r = 0.24\sqrt{f'_c}$$

Precast I-Girder

$$f_r = 0.24\sqrt{5.0} = 0.537\ ksi$$

Deck Slab

$$f_r = 0.24\sqrt{4.5} = 0.509\ ksi$$

When the modulus of rupture is used to calculate the cracking moment of a member for determination of the minimum reinforcing requirement as specified in Article 5.7.3.3.2, the following equation should be used:

Minimum Reinforcing

$$f_r = 0.37\sqrt{f'_c}$$

Precast I-Girder

$$f_r = 0.37\sqrt{5.0} = 0.827\ ksi$$

Deck Slab

$$f_r = 0.37\sqrt{4.5} = 0.785\ ksi$$

Limit States
[1.3.2]

In the LRFD Specification, the general equation for design is shown below:

$$\sum \eta_i \gamma_i Q_i \le \varphi R_n = R_r$$

For loads for which a maximum value of γ_i is appropriate:

$$\eta_i = \eta_D \eta_R \eta_I \ge 0.95$$

For loads for which a minimum value of γ_i is appropriate:

$$\eta_i = \frac{1}{\eta_D \eta_R \eta_I} \le 1.0$$

[1.3.3]

Ductility
For strength limit state for conventional design and details complying with the LRFD Specifications and for all other limit states:

$\eta_D = 1.00$

[1.3.4]

Redundancy
For the strength limit state for conventional levels of redundancy and for all other limit states:

$\eta_R = 1.0$

[1.3.5]

Operational Importance
For the strength limit state for typical bridges and for all other limit states:

$\eta_I = 1.0$

For an ordinary structure with conventional design and details and conventional levels of ductility, redundancy, and operational importance, it can be seen that $\eta_i = 1.0$ for all cases. Since multiplying by 1.0 will not change any answers, the load modifier η_i has not been included in this example.

[BPG]

For actual designs, the importance factor may be a value other than one. The importance factor should be selected in accordance with the ADOT LRFD Bridge Practice Guidelines.

DECK DESIGN

[BPG]

As bridges age, decks are one of the first element to show signs of wear and tear. As such ADOT has modified some LRFD deck design criteria to reflect past performance of decks in Arizona. Section 9 of the Bridge Practice Guidelines provides a thorough background and guidance on deck design.

Effective Length
[9.7.2.3]

ADOT Bridge Practice Guidelines specify that deck design be based on the effective length rather than the centerline-to-centerline distance specified in the LRFD Specification. The effective length for Type VI modified precast girders is the clear spacing between flange tips plus the distance between the flange tip and the web. For this example with a centerline-to-centerline web spacing of 9.00 feet and a top flange width of 40 inches, clear spacing = 9.00 – 40/12 = 5.67 feet. The effective length is then 5.67 + (17/12) = 7.08 feet. The resulting minimum deck slab thickness per ADOT guidelines is 8.00 inches.

Method of Analysis

In-depth rigorous analysis for deck design is not warranted for ordinary bridges. The empirical design method specified in [9.7.2] is not allowed by ADOT Bridge Group. Therefore the approximate elastic methods specified in [4.6.2.1] will be used. Dead load analysis will be based on a strip analysis using the simplified moment equation of [$w S^2 / 10$] where “S” is the effective length. Metal stay-in-place forms with a weight of 0.012 ksf including additional concrete will be used.

Live Loads
[A4.1]

The unfactored live loads found in Appendix A4.1 will be used. Multiple presence and dynamic load allowance are included in the chart. Since ADOT bases deck design on the effective length, the chart should be entered under S equal to the effective length of 7.08 feet rather than the centerline-to-centerline distance of 9.00 feet. Since the effective length is used the correction for negative moment from centerline of the web to the design section should be zero. Entering the chart under S = 7.25 feet for simplicity yields:

Pos LL M = 5.32 ft-k/ft
Neg LL M = -6.13 ft-k/ft (0 inches from centerline)

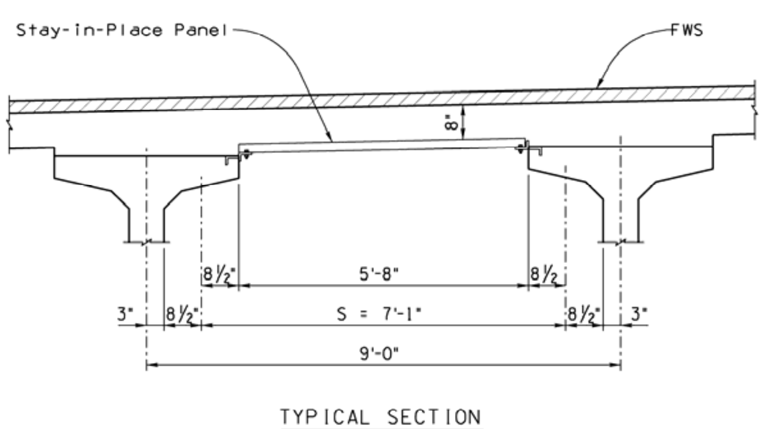


Figure 3

Positive Moment Design

A summary of positive moments follows:

DC Loads

Deck	$0.150(8.00/12)(7.08)^2 \div 10$		= 0.50
SIP Panel	$0.012(7.08)^2 \div 10$		= <u>0.06</u>
		DC	= 0.56 ft-k

DW Loads

FWS	$0.025(7.08)^2 \div 10$	= 0.13 ft-k

Vehicle

LL + IM	= 5.32 ft-k

Service I Limit State [3.4.1]

Deck design is normally controlled by the service limit state. The working stress in the deck is calculated by the standard methods used in the past. For this check Service I moments should be used.

$$M_s = 1.0 \cdot (M_{DC} + M_{DW}) + 1.0 \cdot (M_{LL+IM})$$

M_s = 1.0(0.56 + 0.13) + 1.0(5.32) = 6.01 ft-k

Try #5 reinforcing bars

d_s = 8.00 – 1 clr – 0.625/2 - 0.5 ws = 6.19 in

Determine approximate area reinforcing as follows:

$$A_s \approx \frac{M_s}{f_s jd_s} = \frac{(6.01)\cdot(12)}{(24.0)\cdot(0.9)\cdot(6.19)} = 0.539 \text{ in}^2$$

Try #5 @ 6 ½ inches

A_s = (0.31)(12 / 6.50) = 0.572 in^2

Allowable Stress [5.7.3.4-1]

[BPG]

The allowable stress for a deck under service loads is not limited by the LRFD Specifications. The 2006 Interim Revisions replaced the direct stress check with a maximum spacing requirement to control cracking. However, the maximum allowable stress in a deck is limited to 24 ksi per the LRFD Bridge Practice Guidelines.

Determine stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.572}{(12)\cdot(6.19)} = 0.007701$$

np = 8(0.007701) = 0.06161

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.06161) + (0.06161)^2} - 0.06161 = 0.295$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.295}{3} = 0.902$$

$$f_s = \frac{M_s}{A_s jd_s} = \frac{(6.01)\cdot(12)}{(0.572)\cdot(0.902)\cdot(6.19)} = 22.58 \text{ ksi} < 24 \text{ ksi}$$

Since the applied stress is less than 24 ksi, the LRFD Bridge Practice Guideline service limit state requirement is satisfied.

Control of Cracking [5.7.3.4]

For all concrete components in which the tension in the cross-section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in equation 5.7.3.4-1 shall be satisfied.

$f_{sa} = 0.80f_r = 0.80(0.24\sqrt{f'_c}) = 0.80(0.509) = 0.407$ ksi

$S_{cr} = (12.00)(7.50)^2 \div 6 = 112.5$ in^3

$$f_{cr} = \frac{M_s}{S_{cr}} = \frac{(6.01)\cdot(12)}{112.5} = 0.641\ ksi > f_{sa} = 0.407 \text{ ksi}$$

Since the service limit state cracking stress exceeds the allowable, the spacing, s, of mild steel reinforcing in the layer closest to the tension force shall satisfy the following:

[5.7.3.4-1]

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

where

$\gamma_e = 0.75$ for Class 2 exposure condition for decks

$d_c = 1.0$ clear $+ 0.625 \div 2 = 1.31$ inches

f_s = 22.58 ksi

h_{net} = 7.50 inches

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{1.31}{0.7 \cdot (7.50 - 1.31)} = 1.30$$

$$s \le \frac{(700)\cdot(0.75)}{(1.30)\cdot(22.58)} - (2)\cdot(1.31) = 15.27 \text{ in}$$

Since the spacing of 6.50 inches is less than 15.27 inches, the cracking criteria is satisfied.

Strength I Limit State [3.4.1]

Factored moment for Strength I is as follows:

$$M_u = \gamma_{DC}(M_{DC}) + \gamma_{DW}(M_{DW}) + 1.75(M_{LL+IM})$$

$$M_u = 1.25 \cdot (0.56) + 1.50 \cdot (0.13) + 1.75 \cdot (5.32) = 10.21 \text{ft-k}$$

Flexural Resistance [5.7.3] [5.7.3.2.2-1]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \phi M_n = \phi A_s f_y \left(d_s - \frac{a}{2} \right)$$

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.572)\cdot(60)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(12)} = 0.906 \text{ in}$$

[5.7.3.2.3]

a = β_1c = (0.825)(0.906) = 0.75 in

The tensile strain must be calculated as follows:

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{6.19}{0.906} - 1 \right) = 0.017$$

[5.5.4.2.1]

Since $\varepsilon_T > 0.005$, the member is tension controlled and φ = 0.90.

$$M_r = (0.90)\cdot(0.572)\cdot(60)\cdot\left(6.19 - \frac{0.75}{2} \right) \div 12 = 14.97 \text{ ft-k}$$

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the strength limit state is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated this limit. Below a net tensile strain in the extreme tension steel of 0.005, the factored resistance is reduced as the tension reinforcement quantity increases. This reduction compensates for the decreasing ductility with increasing overstrength.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u. A conservative simplification for positive moments is to ignore the 0.5 inch wearing surface for this calculation. If this check is satisfied there is no further calculation required. If the criteria is not satisfied one check should be made with the wearing surface subtracted and one with the full section to determine which of the two is more critical.

$S_c = (12.0)(8.00)^2 / 6 = 128 \text{ in}^3$

$f_r = 0.37\sqrt{f'_c} = 0.785\ ksi$

$1.2M_{cr} = 1.2 f_r S_c = (1.2)\cdot(0.785)\cdot(128) \div 12 = 10.05 \text{ ft-k}$

$1.2M_{cr} = 10.05 \le M_r = 14.97 \text{ ft-k}$

$\therefore$ The minimum reinforcement limit is satisfied.

Fatigue Limit State [9.5.3] & [5.5.3.1]

Fatigue need not be investigated for concrete deck slabs in multigirder applications.

The interior deck is adequately reinforced for positive moment using #5 @ 6 ½”.

Distribution Reinforcement [9.7.3.2]

Reinforcement shall be placed in the secondary direction in the bottom of slabs as a percentage of the primary reinforcement for positive moments as follows:

$$\frac{220}{\sqrt{S}} = \frac{220}{\sqrt{7.08}} = 83\ \textit{percent} < 67 \text{ percent maximum}$$

Use 67% Maximum.

$A_s = 0.67(0.572) = 0.383 \text{ in}^2$

Use #5 @ 9” $\Rightarrow A_s = 0.413 \text{ in}^2$

The LRFD Specification does not allow for a reduction of this reinforcing in the outer quarter of the span as was allowed in the Standard Specifications.

Skewed Decks [9.7.1.3] [BPG]

For bridges with skews greater than 25 degrees, the LRFD Specification states that the primary reinforcing shall be placed perpendicular to the girders. However, the Bridge Practice Guidelines has modified this limit to 20 degrees. For the 30 degree skew in this example, the transverse deck reinforcing is placed normal to the girders.

Negative Moment Design

A summary of negative moments follows:

DC Loads

Deck	$0.150(8.00 / 12)(7.08)^2 \div 10$	= -0.50 ft-k
SIP Panel	$0.012(7.08)^2 \div 10$	= -0.06
	DC	= -0.56 ft-k

DW Loads

FWS	$0.025(7.08)^2 \div 10$	= -0.13 ft-k

Vehicle

LL + IM	= -6.13 ft-k

Service I Limit State [3.4.1]

Deck design is normally controlled by the service limit state. The working stress in the deck is calculated by the standard methods used in the past. For this check Service I moments should be used.

$$M_s = 1.0(M_{DC} + M_{DW}) + 1.0(M_{LL+IM})$$

M_s = 1.0(0.56 + 0.13) + 1.0(6.13) = 6.82 ft-k

Try #5 reinforcing bars

d_s = 8.00 – 2.50 clear – 0.625 / 2 = 5.19 inches

Determine approximate area reinforcing as follows:

$$A_s \approx \frac{M_s}{f_s jd_s} = \frac{(6.82)\cdot(12)}{(24.0)\cdot(0.9)\cdot(5.19)} = 0.730 \text{ in}^2$$

Try #5 @ 5 inches

A_s = (0.31)(12 / 5) = 0.744 in^2

Allowable Stress

Determine stress due to service moment:

$$p = \frac{A_s}{bd_s} = \frac{0.744}{(12)\cdot(5.19)} = 0.01195$$

np = 8(0.01195) = 0.09557

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.09557) + (0.09557)^2} - 0.09557 = 0.352$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.352}{3} = 0.883$$

$$f_s = \frac{M_s}{A_s j d_s} = \frac{(6.82)\cdot(12)}{(0.744)\cdot(0.883)\cdot(5.19)} = 24.00\ ksi \leq 24.0\,\text{ksi}$$

Since the applied stress is less than the allowable specified in the LRFD Bridge Practice Guidelines, the service limit state stress requirement is satisfied.

Control of Cracking [5.7.3.4]

The deck must be checked for control of cracking. For all concrete components in which the tension in the cross section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in equation 5.7.3.4-1 shall be satisfied.

$f_{sa} = 0.80f_r = 0.80(0.24\sqrt{f'_c}\,) = 0.80(0.509) = 0.407$ ksi

$S_{cr} = (12.00)(7.50)^2 \div 6 = 112.5\ \text{in}^3$

$$f_{cr} = \frac{M_s}{S_{cr}} = \frac{(6.82)\cdot(12)}{112.5} = 0.727\ ksi\ > f_{sa} = 0.407\ \text{ksi}$$

Since the service limit state cracking stress exceeds the allowable, the spacing, s, of mild steel reinforcing in the layer closest to the tension force shall satisfy the following:

[5.7.3.4-1]

$$s \leq \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

$\gamma_e = 0.75$ for Class 2 exposure condition for decks

$d_c = 2.50$ clear $+ 0.625 \div 2 = 2.81$ inches

$f_s = 24.00$ ksi

h = 8.00 inches

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{2.81}{0.7\cdot(8.00 - 2.81)} = 1.77$$

$$s \leq \frac{(700)\cdot(0.75)}{(1.77)\cdot(24.00)} - (2)\cdot(2.81) = 6.74\ \text{in}$$

Since the 5 inch spacing is less than 6.74”, the cracking criteria is satisfied.

Strength I Limit State [3.4.1]

Factored moment for Strength I is as follows:

$$M_u = \gamma_{DC}(M_{DC}) + \gamma_{DW}(M_{DW}) + 1.75(M_{LL+IM})$$

$$M_u = 1.25 \cdot (0.56) + 1.50 \cdot (0.13) + 1.75 \cdot (6.13) = 11.62 \text{ ft-k}$$

Flexural Resistance [5.7.3]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right)$$

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.744) \cdot (60)}{(0.85) \cdot (4.5) \cdot (0.825) \cdot (12)} = 1.179 \text{ in}$$

[5.7.3.2.3]

a = β_1c = (0.825)(1.179) = 0.97 inches

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{5.19}{1.179} - 1 \right) = 0.010$$

[5.5.4.2.1]

Since ε_T > 0.005, the member is tension controlled and φ= 0.90.

$$M_r = (0.90) \cdot (0.744) \cdot (60) \cdot \left(5.19 - \frac{0.97}{2} \right) \div 12 = 15.75 \text{ ft-k}$$

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the strength limit state is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions has eliminated this requirement.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u. The most critical cracking load for negative moment will be caused by ignoring the 0.5 inch wearing surface and considering the full depth of the section.

$S_c = 12(8.00)^2 \div 6 = 128 \text{ in}^3$

$$1.2M_{cr} = 1.2 f_r S_c = (1.2) \cdot (0.785) \cdot (128) \div 12 = 10.05 \text{ ft-k}$$

$$1.2M_{cr} = 10.05 \le M_r = 15.75 \text{ ft-k}$$

$\therefore$ The minimum reinforcement limit is satisfied.

Fatigue Limit State [9.5.3] & [5.5.3.1]

Fatigue need not be investigated for concrete deck slabs in multigirder applications.

The interior deck is adequately reinforced for negative moment using #5 @ 5”.

Shear [C4.6.2.1.6]

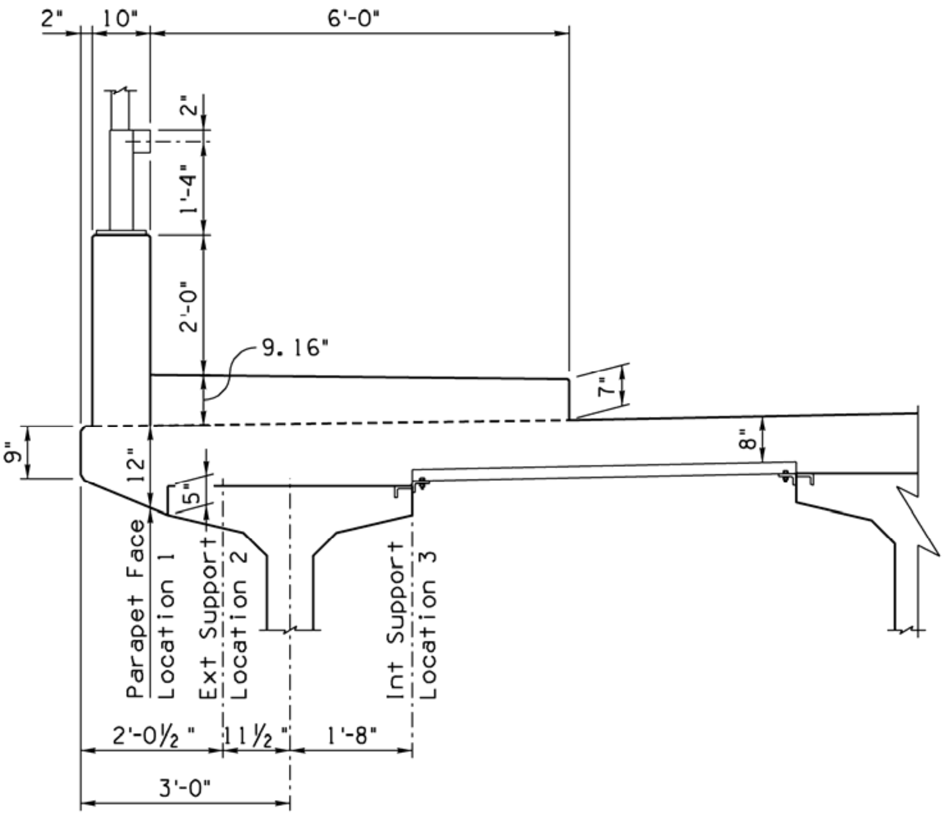
Past practice has been not to check shear in typical decks. For a standard concrete deck shear need not be investigated.

Overhang Design [A13.4.1]

The overhang shall be designed for three design cases described below:

Design Case 1

Design Case 1: Transverse forces specified in [A13.2]
Extreme Event Limit State



DESIGN CASE 1

Figure 4

The deck overhang must be designed to resist the forces from a railing collision using the forces given in Section 13, Appendix A. A TL-4 railing is generally acceptable for the majority of applications on major roadways and freeways. A TL-4 rail will be used. A summary of the design forces is shown below:

[A13.2-1]

Design Forces		Units
F_t, Transverse	54.0	kips
F_l, Longitudinal	18.0	kips
F_v, Vertical Down	18.0	kips
L_t and L_l	3.5	feet
L_v	18.0	feet
H_e Minimum	42.0	inch

Rail Design

The philosophy behind the overhang analysis is that the deck should be stronger than the barrier. This ensures that any damage will be done to the barrier which is easier to repair and that the assumptions made in the barrier analysis are valid. The forces in the barrier must be known to analyze the deck.

The resistance of each component of a combination bridge rail shall be determined as specified in Article A13.3.1 and A13.3.2.

A13.3.3

Concrete Railing

A13.3.1

R_w = total transverse resistance of the railing.
L_c = critical length of yield line failure. See Figures 5 and 6.

<u>For impacts within a wall segment:</u>

[A13.3.1-1]

$$R_w = \left(\frac{2}{2L_c - L_t}\right)\left(8M_b + 8M_w + \frac{M_c L_c^{\ 2}}{H}\right)$$

[A13.3.1-2]

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_w)}{M_c}}$$

The railing used on the bridge is the standard pedestrian rail and parapet as shown in ADOT SD 1.04 and SD 1.05 with a single rail. From previous analysis of the concrete parapet the following values have been obtained:

[BPG]

M_b = 0 ft-k
M_c = 12.04 ft-k
M_w = 30.15 ft-k

The height of the concrete parapet and rail are as follows:

Parapet H = 2.00 + 9.16 / 12 = 2.76 feet
Rail H = 2.76 + 1.33 = 4.09 feet

$$L_c = \frac{3.50}{2} + \sqrt{\left(\frac{3.50}{2}\right)^2 + \frac{8\cdot(2.76)\cdot(0+30.15)}{12.04}} = 9.39\,\text{ft}$$

$$R_w = \left(\frac{2}{2\cdot(9.39)-3.50}\right)\cdot\left(8\cdot(0)+8\cdot(30.15)+\frac{(12.04)\cdot(9.39)^2}{2.76}\right) = 81.92\ \text{k}$$

A13.3.2

Post-and Beam Railing

From previous analysis of the post-and-beam rail as shown on SD 1.04 and SD 1.05 with a single traffic rail, the following values have been obtained:

L = 6.67 feet max
Rail Q_p = 3.30 in^3 M_p = (3.30)(46)/12 = 12.65 ft-k per rail
Post Q_p = 3.91 in^3 M_p = (3.91)(46)/12 = 14.99 ft-k
P_p = (14.99) / (1.3333) = 11.24 kips

Inelastic analysis shall be used for design of post-and-beam railings under failure conditions. The critical rail nominal resistance shall be taken as the least value of the following:

For failures modes involving an odd number of railing spans, N:

[A13.3.2-1]

$$R = \frac{16M_p + (N-1)\cdot(N+1)P_pL}{2NL - L_t}$$

$$For\ N = 1:\ R = \frac{16\cdot(12.65) + 0}{2\cdot(1)\cdot(6.67) - 3.50} = 20.57\ \text{kips}$$

$$For\ N = 3: R = \frac{16\cdot(12.65) + (3-1)\cdot(3+1)\cdot(11.24)\cdot(6.67)}{2\cdot(3)\cdot(6.67) - 3.50} = 21.97\ \text{kips}$$

$$For\ N = 5: R = \frac{16\cdot(12.65) + (5-1)\cdot(5+1)\cdot(11.24)\cdot(6.67)}{2\cdot(5)\cdot(6.67) - 3.50} = 31.67\ \text{kips}$$

For failure modes involving an even number of railing spans, N:

[A13.3.2-2]

$$R = \frac{16M_p + N^2P_pL}{2NL - L_t}$$

$$For\ N = 2: R = \frac{16\cdot(12.65) + (2)^2\cdot(11.24)\cdot(6.67)}{2\cdot(2)\cdot(6.67) - 3.50} = 21.67\ \text{kips}$$

$$For\ N = 4: R = \frac{16\cdot(12.65) + (4)^2\cdot(11.24)\cdot(6.67)}{2\cdot(4)\cdot(6.67) - 3.50} = 28.12\ \text{kips}$$

$$For\ N = 6: R = \frac{16\cdot(12.65) + (6)^2\cdot(11.24)\cdot 6.67}{2\cdot(6)\cdot(6.67) - 3.50} = 37.91\ \text{kips}$$

[A13.3.3] **Concrete Parapet and Metal Rail**

The resistance of the combination parapet and rail shall be taken as the lesser of the resistances determined for the following two failure modes.

For impact at midspan of a rail (One Span Failure):

[A13.3.3-1] $R_{bar} = R_R + R_W$

$R_{bar} = 20.57 + 81.92 = 102.49$ kips

[A13.3.3-2]

$$Y_{bar} = \frac{R_R H_R + R_w H_W}{R_{bar}}$$

$$Y_{bar} = \frac{(20.57)\cdot(4.09)+(81.92)\cdot(2.76)}{102.49} = 3.027 \text{ feet}$$

When the impact is at a post (2 Span Failure):

[A13.3.3-3] $R_{bar} = P_P + R'_R + R'_W$

R'_R = Ultimate transverse resistance of rail over two spans = 21.67 k

[A13.3.3-5]

$$R'_W = \frac{R_W H_W - P_P H_R}{H_W} = \frac{(81.92)\cdot(2.76)-(11.24)\cdot(4.09)}{2.76} = 65.26 \text{ k}$$

$R_{bar} = 11.24 + 21.67 + 65.26 = 98.17$ k

$$Y_{bar} = \frac{P_P H_R + R'_R H_R + R'_W H_W}{R_{bar}}$$

[A13.3.3-4]

$$Y_{bar} = \frac{(11.24)\cdot(4.09)+(21.67)\cdot(4.09)+(65.26)\cdot(2.76)}{98.17} = 3.206 \text{ ft}$$

Since the resistance, the lesser of the above values, equals 98.17 kips which is greater than the applied load of $F_t = 54.00$ kips, the rail is adequately designed.

Barrier Connection To Deck

The strength of the attachment of the parapet to the deck must also be checked. The deck will only see the lesser of the strength of the rail or the strength of the connection. For the parapet, #4 at 8 inches connects the parapet to the deck.

$A_s = (0.20)(12) / (8) = 0.300 \text{ in}^2$

$d_s = 10.00 – 1 \frac{1}{2} \text{ clear} – 0.50 / 2 = 8.25 \text{ inches}$

Flexure

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.300)\cdot(60)}{(0.85)\cdot(4.0)\cdot(0.850)\cdot(12)} = 0.519 \text{ in}$$

$a = \beta_1 c = (0.850)(0.519) = 0.44 \text{ inches}$

$$M_n = A_s f_y \left(d_s - \frac{a}{2}\right)$$

$$M_n = (0.300)\cdot(60)\cdot\left(8.25 - \frac{0.44}{2}\right) \div 12 = 12.05 \text{ ft-k}$$

$\varphi M_n = (1.00)(12.05) = 12.05 \text{ ft-k}$

$\varphi P_u = (12.05) \div (3.206) = \underline{3.759 \text{ k/ft}}$

Shear

The barrier to deck interface must also resist the horizontal collision load. The strength is determined using shear friction analysis. For #4 @ 8”:

$A_{vf} = (0.20)(12) / (8) = 0.300 \text{ in}^2 / \text{ft}$

$V_n = cA_{cv} + \mu[A_{vf}f_y + P_c]$

$V_n = 0.100(120.0) +1.0[(0.300)(60) + 0] = 30.00 \text{ k/ft}$

$\varphi V_n = (1.0)(30.00) = 30.00 \text{ k/ft}$

The strength of the connection is limited by the lesser of the shear or flexural strength. In this case, the resistance of the connection is 3.759 k/ft. The distribution at the base of the parapet is 9.39 + 2(2.76) = 14.91 feet as shown in Figure 6. Thus the connection will transmit (3.759)(14.91) = 56.05 kips which is greater than the required force of 54 kips.

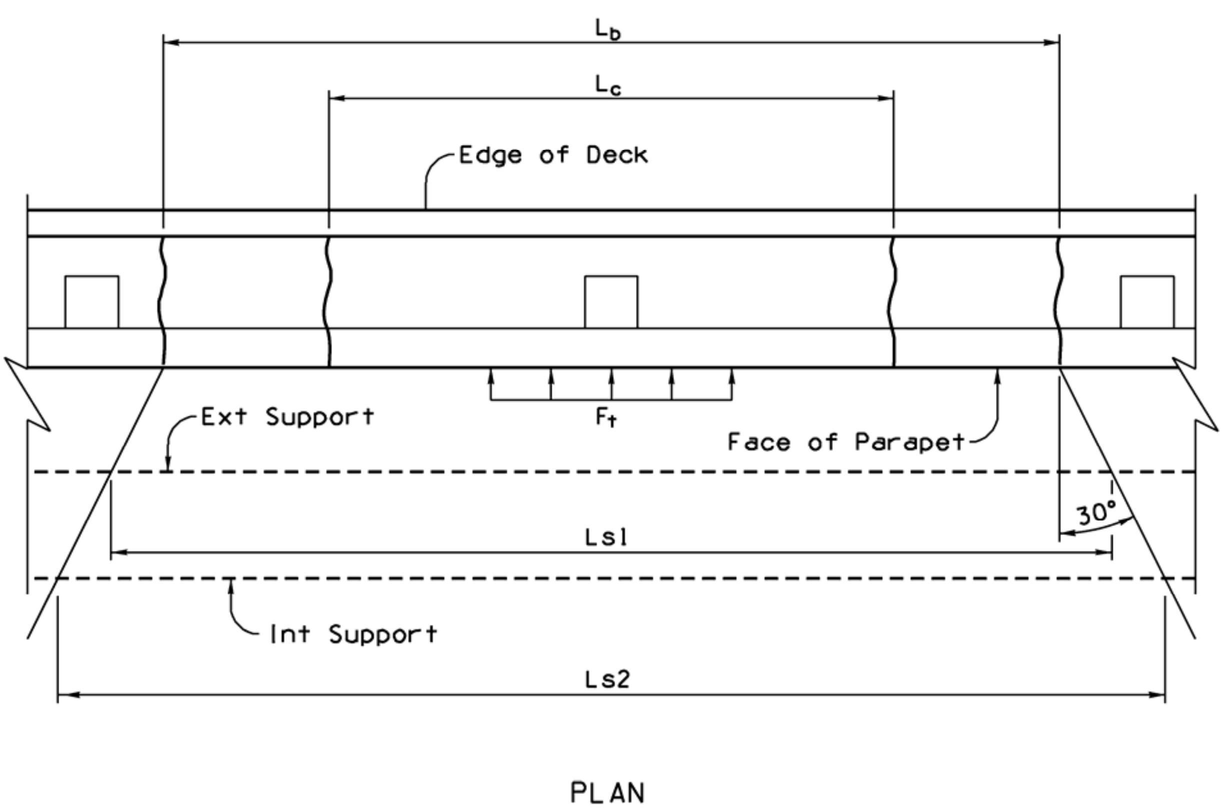


Figure 5

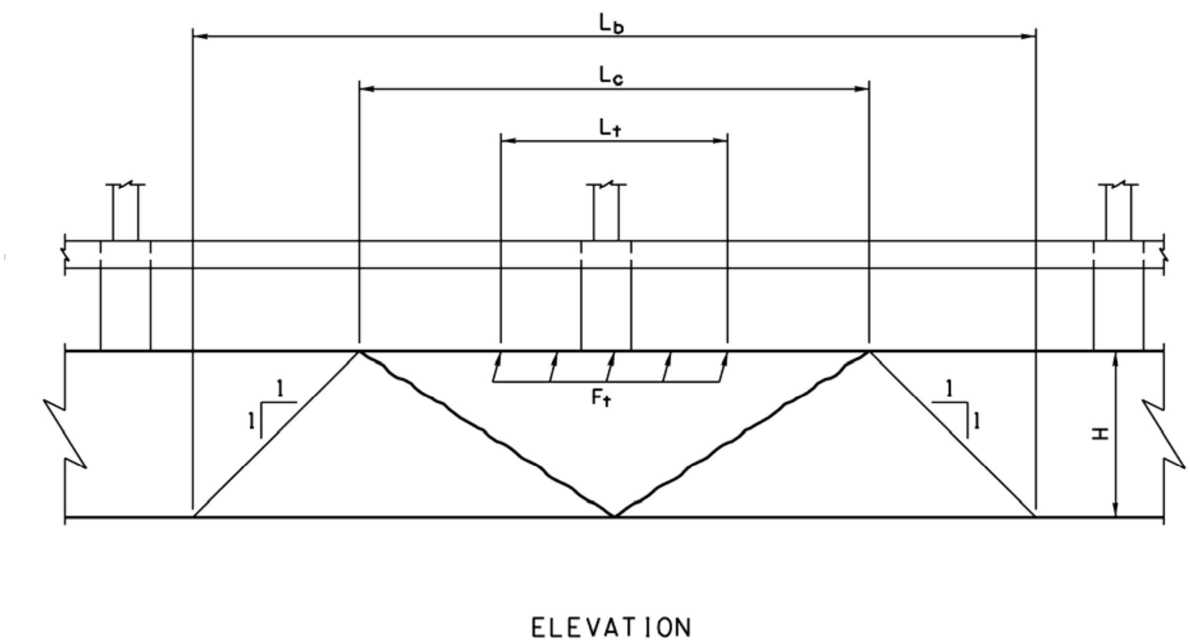


Figure 6

Face of Barrier Location 1 Figure 4

The design horizontal force in the barrier is distributed over the length L_b equal to L_c plus twice the height of the barrier. See Figures 5 and 6.

L_b = 9.39 + 2(2.76) = 14.91 ft
P_u = 98.17 / 14.91 = 6.584 k/ft ⇒ Use P_u = 3.759 k/ft per connection.

Dimensions

h = 9.00 + (4.00) (1.00) / (1.333) = 12.00 in
d_1 = 12.00 – 2.50 clr – 0.625 / 2 = 9.19 in

Moment at Face of Barrier

Deck = $0.150(9.00 / 12)(1.00)^2 \div 2$ = 0.06 ft-k
$0.150(3.00 / 12)(1.00)^2 \div 6$ = <u>0.01 ft-k</u>
= 0.07 ft-k

Fence, Rail & Parapet: w = 0.075 + 0.15(10/12)(2.76) = 0.420 k/ft
FR & P = 0.420(0.417) = 0.18 ft-k

Collision = 3.759[3.206 + (12.00/12) / 2] = 13.93 ft-k

[A13.4.1]

The load factor for dead load shall be taken as 1.0.

Extreme Event II [3.4.1]

M_u = 1.00(0.07 + 0.18) + 1.00(13.93) = 14.18 ft-k

e = M_u / P_u = (14.18)(12) / (3.759) = 45.27 in

Determine resulting forces in the top reinforcing (#5 @ 5"):

T_1 = (0.744)(60) = 44.64 k

A simplified method of analysis is available. If only the top layer of reinforcing is considered in determining strength, the assumption can be made that the reinforcing will yield. By assuming the safety factor for axial tension is 1.0 the strength equation can be solved directly. This method will determine whether the section has adequate strength. However, the method does not consider the bottom layer of reinforcing, does not maintain the required constant eccentricity and does not determine the maximum strain. For development of the equation and further discussion on the in-depth solution refer to Appendix A.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{44.64 - 3.759}{(0.85)\cdot(4.5)\cdot(12)} = 0.89 \text{ in}$

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(9.19 - \frac{0.89}{2}\right) - (3.759)\cdot\left(\frac{12.00}{2} - \frac{0.89}{2}\right)\right] \div 12$$

$\varphi\, M_n$ = 30.79 ft-k

Since φ M_n = 30.79 ft-k > M_u = 14.18 ft-k, the overhang has adequate strength at Location 1. Note that the resulting eccentricity equals (30.79)(12) ÷ 3.759 = 98.29 inches compared to the actual eccentricity of 45.27 inches that is fixed by the constant deck thickness, barrier height and dead load moment.

Development Length [5.11.2.1]

The reinforcing must be properly developed from the parapet face towards the edge of deck. The available embedment length equals 12 inches minus 2 inches clear or 10 inches.

[5.11.2.1.1]

For No. 11 bar and smaller $\dfrac{1.25 A_b f_y}{\sqrt{f'_c}} = \dfrac{(1.25)\cdot(0.31)\cdot(60)}{\sqrt{4.5}} = 10.96 \text{ in}$

But not less than 0.4 d_b f_y = 0.4(0.625)(60) = 15.00 in

Even with modification factors, the minimum required length is 12 inches. Since the available length is less than the required, the reinforcing is not adequately developed using straight bars.

Try developing the top bar with 180 degree standard hooks.

[5.11.2.4.1-1]

$$l_{hb} = \frac{38.0 d_b}{\sqrt{f'_c}} = \frac{(38.0)\cdot(0.625)}{\sqrt{4.5}} = 11.2 \text{ in}$$

Modify the basic development length with the modification factor of 0.7 for side cover of at least 2.50 inches for #11 bars and less.

l_{hb} = (11.2)(0.7) = 7.8 inches

Since the required development length of the hooked bar including modification factors is less than the available, the bars are adequately developed using hooked ends.

Exterior Support Location 2
Figure 4

The deck slab must also be evaluated at the exterior overhang support. At this location the design horizontal force is distributed over a length L_{s1} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the exterior support. See Figures 4, 5 and 6. Assume composite action between deck and girder for this extreme event. Using a distribution of 30 degrees from the face of barrier to the exterior support results in the following:

$L_{S1} = 9.39 + 2(2.76) + (2)[\tan(30)](1.04) = 16.11$ ft

$P_u = 98.17 / 16.11 = 6.094$ k/ft $\Rightarrow$ Use $P_u = 3.759$ k/ft per connection

Dimensions

$h = 8.00 + 5.00 + 3.00(8.5/17) = 14.50$ in

$d_1 = 14.50 – 2.50$ clr $– 0.625 / 2 = 11.69$ in

Moment at Exterior Support

.

DC Loads

Deck	$= 0.150(9.00 / 12)(2.04)^2 / 2$	= 0.23 ft-k
	$= 0.150(5.50 / 12)(2.04)^2 /6$	= 0.05 ft-k
Parapet	$= 0.420(0.417 + 1.042)$	= 0.61 ft-k
Sidewalk	$= 0.150(9.16 / 12)(1.04)^2 / 2$	= 0.06 ft-k
	DC	= 0.95 ft-k

DW Loads

FWS	= 0.00 ft-k

Collision $= 3.759[3.206 + (14.50 / 12) / 2]$ = 14.32 ft-k

[A13.4.1]

The load factor for dead load shall be taken as 1.0.

Extreme Event II [3.4.1]

$M_u = 1.00(0.95) + 1.00(0.00) + 1.00(14.32) = 15.27$ ft-k

$e = M_u / P_u = (15.27)(12) / (3.759) = 48.75$ in

Determine resulting force in the reinforcing:

$T_1 = (0.744)(60) = 44.64$ k

The simplified method of analysis is used based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{44.64 - 3.759}{(0.85)\cdot(4.5)\cdot(12)} = 0.89$ in

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(11.69 - \frac{0.89}{2}\right) - (3.759)\cdot\left(\frac{14.50}{2} - \frac{0.89}{2}\right)\right] \div 12$$

$\varphi M_n = 39.70$ ft-k

Since $\varphi M_n = 39.70$ ft-k > $M_u = 15.27$ ft-k, the overhang has adequate strength at Location 2.

Interior Support Location 3 Figure 4

The deck slab must also be evaluated at the interior point of support. The critical location will be at the edge of the girder flange where the deck will be the thinnest. Only the top layer of reinforcing will be considered. At this location the design horizontal force is distributed over a length L_{s2} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the interior support. See Figures 4, 5 and 6. Using a distribution of 30 degree from the face of the barrier to the interior support results in the following:

$L_{S2} = 9.39 + 2(2.76) + (2)[\tan(30)](3.67) = 19.15$ ft
$P_u = 98.17 / 19.15 = 5.126$ k/ft $\Rightarrow$ Use $P_u = 3.759$ k/ft per connection

Dimensions

$h = 8.00$ in
$d_1 = 8.00 – 2.50$ clr $– 0.625 / 2 = 5.19$ in

Moment at Interior Support

For dead loads use the maximum negative moments for the interior cells used in the interior deck analysis
.

DC = 0.56 ft-k
DW = 0.13 ft-k
Collision = 3.759[3.206 + (8.00 / 12) / 2] = 13.30 ft-k

[A13.4.1]

The load factor for dead load shall be taken as 1.0.

Extreme Event II [3.4.1]

$M_u = 1.00(0.56) + 1.00(0.13) + 1.00(13.30) = 13.99$ ft-k
$e = M_u / P_u = (13.99)(12) / (3.759) = 44.66$ in

Determine resulting force in the reinforcing:

$T_1 = (0.744)(60) = 44.64$ k

The simplified method of analysis is used based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{44.64 - 3.759}{(0.85)\cdot(4.5)\cdot(12)} = 0.89$ in

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(5.19 - \frac{0.89}{2}\right) - (3.759)\cdot\left(\frac{8.00}{2} - \frac{0.89}{2}\right)\right] \div 12$$

φM_n = 16.54 ft-k

Since φM_n = 16.54 ft-k > M_u = 13.99 ft-k, the deck has adequate strength at Location 3.

Design Case 1

Since the axial and flexural strength of the deck at the three locations investigated exceeds the factored applied loads, the deck is adequately reinforced for Design Case I.

Design Case 2 [A13.4.1]

Design Case 2: Vertical forces specified in [A13.2]
Extreme Event Limit State

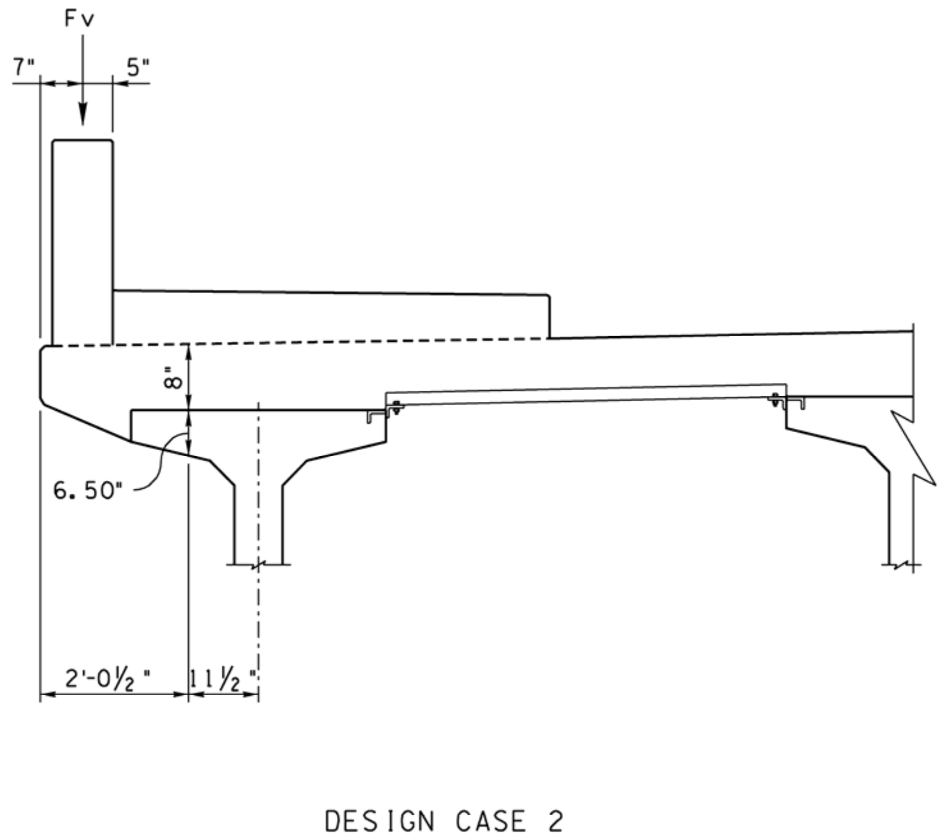


Figure 7

[A13.2-1]

This case represents a crashed vehicle on top of the parapet and is treated as an extreme event. The downward vertical force, F_v = l8.0 kips, is distributed over a length, F_l = 18.0 feet. The vehicle is assumed to be resting on top of the center of the barrier. See Figure 7.

At the exterior support:

DC Dead Loads = 1.03 ft-k
DW Dead Load = 0 ft-k

Vehicle
Collision = [18.0/18.0] [2.042 - (7/12)] = 1.46 ft-k

[3.6.1]

The load factor for dead load shall be taken as 1.0.

[A13.4.1]
Extreme Event II
[3.4.1]

$$M_u = 1.00(1.03) + 1.00(0) + 1.00(1.46) = 2.49 \text{ ft-k}$$

Flexural Resistance [5.7.3.2]

The flexural resistance of a reinforced concrete rectangular section is:

$$M_r = \varphi M_n = \varphi A_s f_y \left(d - \frac{a}{2} \right)$$

Try #5 reinforcing bars

d_s = 14.50 – 2.50 clr – 0.625 / 2 = 11.69 inches

Use #5 @ 5", the same reinforcing required for the interior span and overhang Design Case 1.

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(0.744)\cdot(60)}{(0.85)\cdot(4.5)\cdot(0.825)\cdot(12)} = 1.179 \text{ in}$$

[5.7.3.2.3]

$a = \beta_1 c$ = (0.825)(1.179) = 0.97 inches

$$\varepsilon_T = 0.003 \cdot \left(\frac{d_t}{c} - 1 \right) = 0.003 \cdot \left(\frac{11.69}{1.179} - 1 \right) = 0.027$$

[5.5.4.2.1]

Since ε_T > 0.005 the member is tension controlled.

$$M_n = (0.744)\cdot(60)\cdot\left(11.69 - \frac{0.97}{2} \right) \div 12 = 41.68 \text{ ft-k}$$

[1.3.2.1]

φ = 1.00

$M_r = \phi M_n$ = (1.00)(41.68) = 41.68 ft-k

Since the flexural resistance, M_r, is greater than the factored moment, M_u, the extreme limit state is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated this requirement.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification requires that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u.

$S_c = bh^2 / 6 = (12)(14.50)^2 / 6 = 420.5 \text{ in}^3$

$1.2M_{cr} = 1.2f_rS_c$ = 1.2(0.785)(420.5)/12 = 33.01 ft-k < M_r = 41.68 ft-k

Since the strength of the section exceeds 1.2 M_{cr}, the minimum reinforcing criteria is satisfied.

Design Case 3
[A13.4.1]

Design Case 3: The loads specified in [3.6.1] that occupy the overhang Strength and Service Limit State

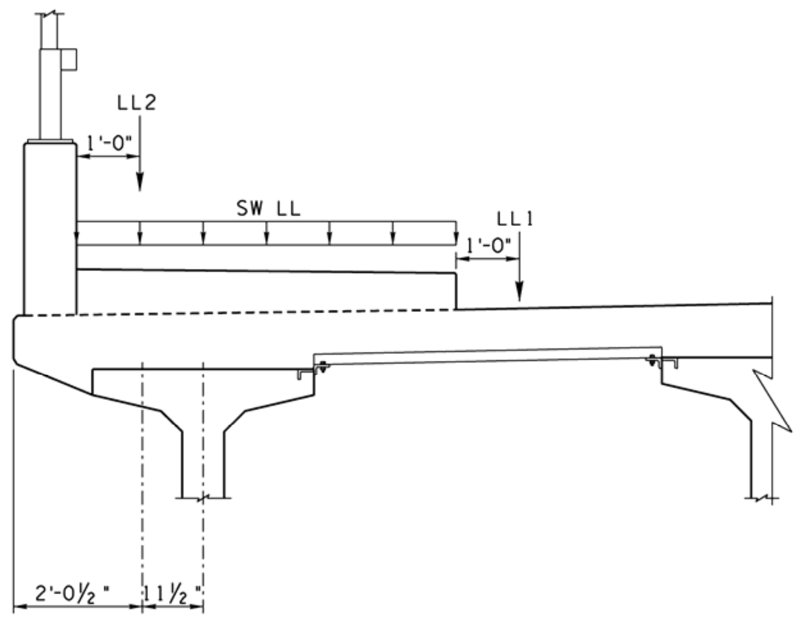


Figure 8

Due to the sidewalk, the normal vehicular live load (LL 1) does not act on the overhang. However, the overhang should be investigated for the situation where a vehicle is on the sidewalk (LL 2). Since this is not an ordinary event the Service Limit States should not be investigated. The occurrence of the event is not as rare as an Extreme Limit State. Therefore, this situation will be evaluated by the Strength I Limit State with a live load factor of 1.35.

[4.6.2.1.3-1]

For strength limit state, for a cast-in-place concrete deck overhang, the width of the primary strip is 45.00 + 10.0 X where X equals 0.04 feet, the distance from the point of load to the support.

Width Primary Strip(inches) = 45.0 + 10.0(0.04) = 45.40 in = 3.78 ft.

LL 2 + IM

(1.20)(16.00)(1.33)(0.04) / (3.78) = 0.27 ft-k

M_u = 1.25(1.03) + 1.50(0) + 1.35(0.27) = 1.65 ft-k

The flexural resistance will greatly exceed the factored applied load and will not be calculated here. The service and strength limit states should be checked for the sidewalk live load but this will not control by inspection. The overhang is adequately reinforced.

Figure 9 shows the required reinforcing in the deck slab.

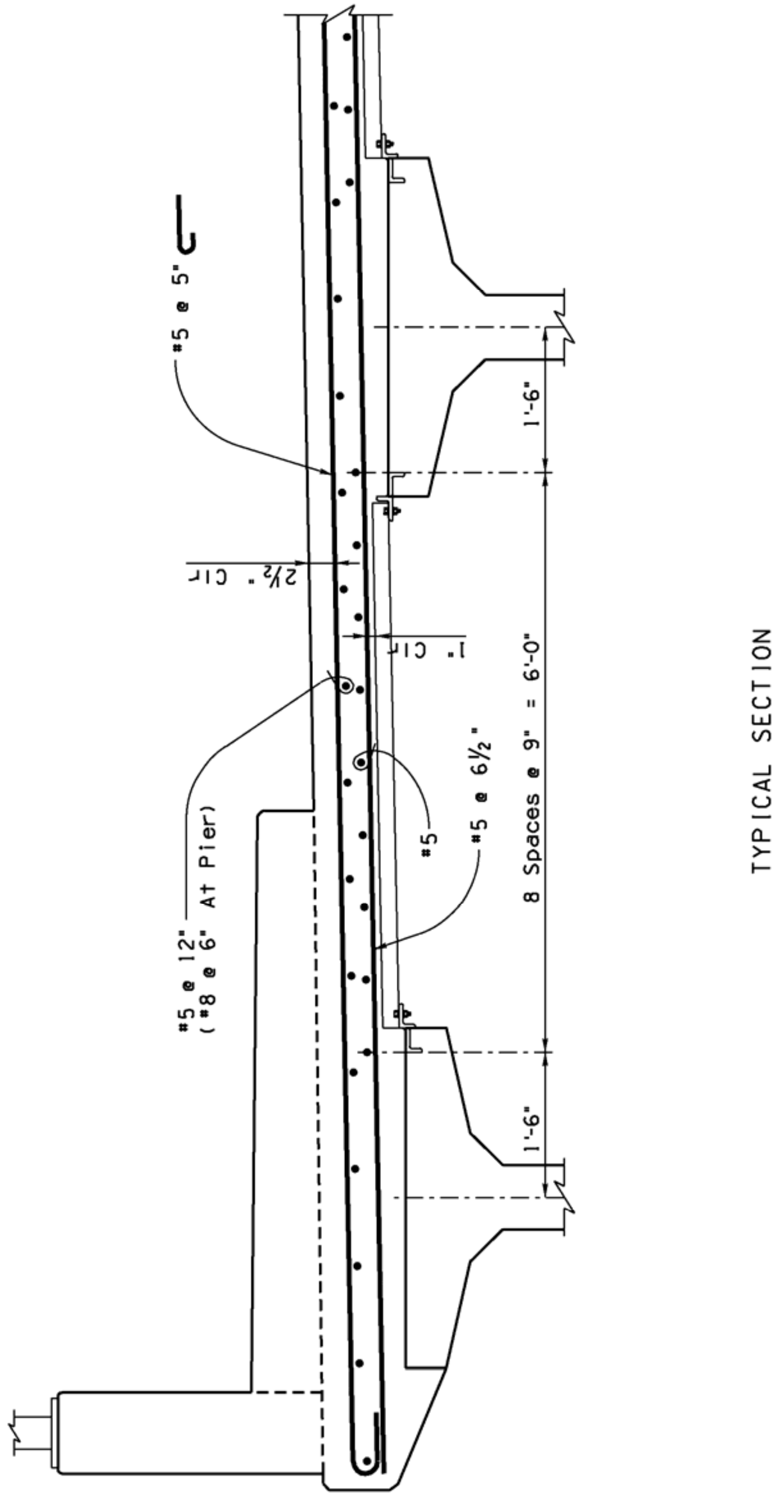


Figure 9

SUPERSTR DGN Precast Prestressed I-Girder

The section properties have been calculated subtracting the ½-inch wearing surface from the cast-in-place top slab thickness. However, this wearing surface has been included in weight calculations.

Step 1 – Determine Section Properties

Net and transformed section properties will be used for the structural design but gross section properties will be used for live load distribution and deflection calculations. The use of net section properties simplifies the prestress analysis while the use of transformed section properties simplifies the analysis for external loads.

For a precast prestressed concrete I-girder the net section properties are used for determination of stresses due to prestressing at release, self-weight and time-dependent losses. The transformed section properties are used for non-composite externally applied loads. The transformed composite section properties are used for the composite dead loads and live loads. The calculation of these properties is an iterative process since the required area of strands is a function of the number and location of the strands and is usually performed with computer software. These steps have been eliminated and the section properties will be shown for the final strand configuration.

For this problem the AASHTO modified Type VI girder section properties will be calculated.

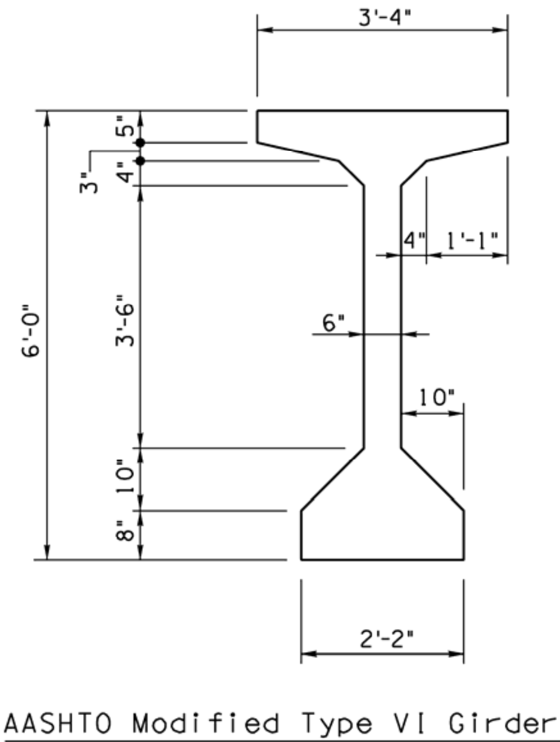


Figure 10

Effective Flange Width [4.6.2.6]

The effective flange width of the composite section must be checked to determine how much of the flange is effective.

Interior Girder

The effective flange width for an interior girder is the lesser of the following criteria:

(1) One quarter the effective span:
(1/4)(110.75)(12) = 332 inches

(2) The greater of the following:
Twelve times the effective slab depth plus the web width:
(12)(7.50) + 6 = 96 inches

or twelve times the effective slab depth plus one-half the top flange width:
(12)(7.50) + (40) / 2 = 110 inches

(3) The spacing between the girders:
(9.00)(12) = 108 inches ⇐ Critical

The effective flange width for an interior girder is 108 inches

Exterior Girder

The effective flange width for an exterior girder is one-half the girder spacing for an interior girder plus the lesser of the following criteria:

(4) One eight the effective span:
(1/8)(110.75)(12) = 166 inches

(5) The greater of the following:
Six times the effective slab depth plus one-half the web width:
(6)(7.50) + 3 = 48 inches

or six times the effective slab depth plus one-quarter the top flange width:
(6)(7.50) + (40) / 4 = 55 inches

(6) The overhang dimension:
(3.00)(12) = 36 inches ⇐ Critical

Therefore for an exterior girder the effective flange width equals 36 + 54 = 90 inches.

Modified Type VI Typical Section

The section properties for the typical section shown in Figure 10 are shown below:

Gross Section Properties

Gross Section – AASHTO Modified Type VI Girder

	No.	W	H	A	y	Ay	Io	A(y-yb)^2
1	1	26.00	8.00	208.00	4.00	832	1109	218876
2	½(2)	10.00	10.00	100.00	11.33	1133	556	63046
3	1	6.00	59.00	354.00	37.50	13275	102689	399
4	½(2)	4.00	4.00	16.00	62.67	1003	14	11009
5	½(2)	13.00	3.00	39.00	66.00	2574	20	34080
6	2	4.00	3.00	24.00	65.50	1572	18	20269
7	1	40.00	5.00	200.00	69.50	13900	417	218606
				941.00		34289	104823	566285

A_g = 941 in^2
y_b = 34289 / 941.00 = 36.439 in e_m = 36.439 – 5.50 = 30.939 in
y_t = 72.00 – 36.439 = 35.561 in
I_g = 104,823 + 566,285 = 671,108 in^4
r^2 = 671,108 / 941.00 = 713.19 in^2

Composite Properties

Composite Gross Section – Girder & Deck

n = 3861 / 4070 = 0.949

Interior Girder

n	W	H	A	y	Ay	Io	A(y-yb)^2
			941.00	36.439	34289	671108	293940
0.949	108.00	7.50	768.69	75.75	58228	3603	359870
			1709.69		92517	674711	653810

A_c = 1709.69 in^2
y_{cb} = 92517 / 1709.69 = 54.113 in e_m = 54.113 – 5.50 = 48.613 in
y_{ct} = 72.00 – 54.113 = 17.887 in
I_c = 674,711 + 653,810 = 1,328,521 in^4
r^2 = 1,328,521 / 1709.69 = 777.05 in^2

Exterior Girder

n	W	H	A	y	Ay	Io	$A(y-yb)^2$
			941.00	36.439	34289	671108	253833
0.949	90.00	7.50	640.58	75.75	48524	3003	335545
0.949	16.00	1.00	15.18	71.50	1086	1	5273
0.949	½*16.0	4.00	30.37	69.67	2116	27	8579
			1627.13		86015	674139	603230

$A_g = 1627.13 \text{ in}^2$

$y_b = 86015 / 1627.13 = 52.863$ in $\qquad e_m = 52.863 – 5.50 = 47.363$ in

$y_t = 72.00 – 52.863 = 19.137$ in

$I_g = 674{,}139 + 603{,}230 = 1{,}277{,}369 \text{ in}^4$

$r^2 = 1{,}277{,}369 / 1627.13 = 785.04 \text{ in}^2$

Stiffness Parameter

The longitudinal stiffness parameter, K_g, is required for determination of the live load distribution. This property is calculated based on gross section properties.

[4.6.2.2.1-1]

$K_g = n(I + A\, e_g^2)$ where n is the ratio of beam to slab modulus

$e_g = 72.00 – 36.439 + 7.50 / 2 = 39.311$ in

$K_g = (4070 / 3861)(671{,}108 + 941(39.311)^2) = 2{,}240{,}331 \text{ in}^4$

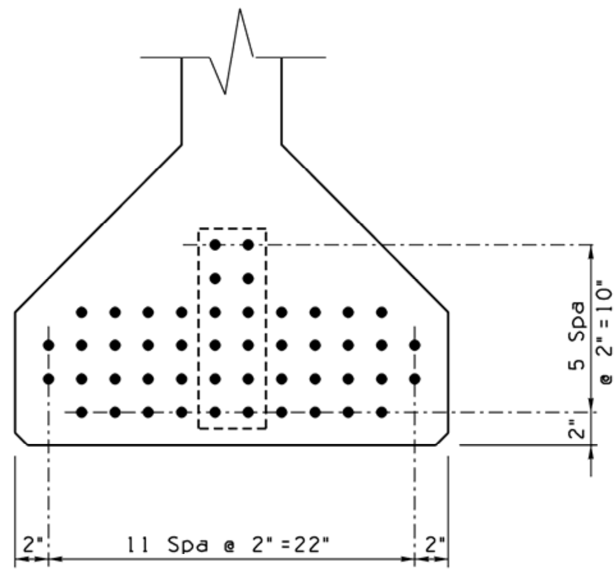
Volume/Surface Ratio

The surface area of the girder is:

Perimeter = 40 + 26 + 2(5 + 13.34 + 5.66 + 42 + 14.14 + 8) = 242 in

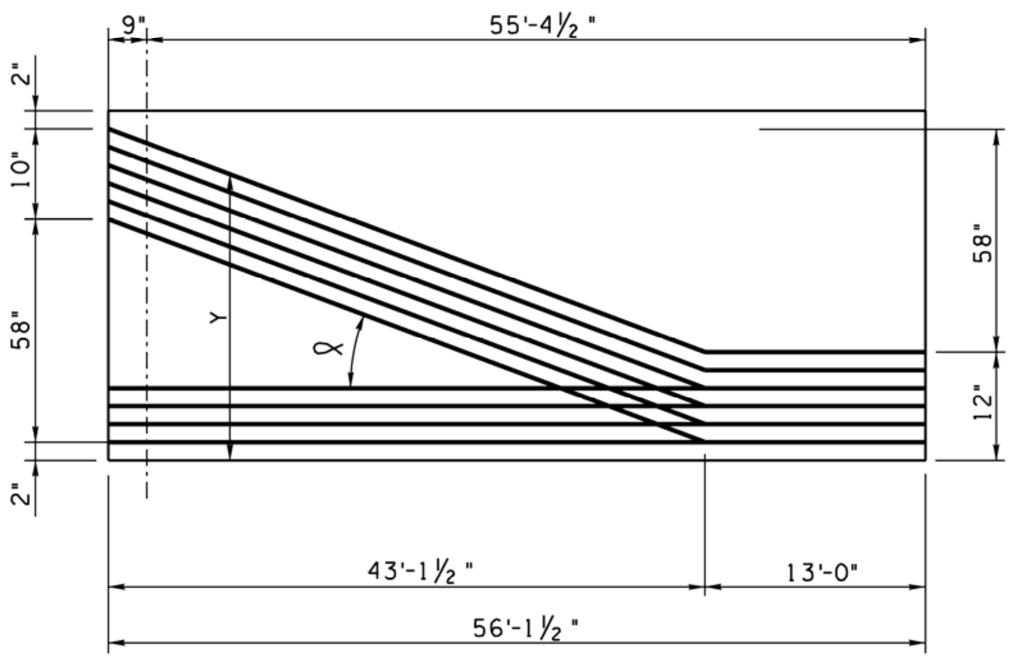
V/S = 941 ÷ 242 = 3.89 in

The final strand pattern to be used in determining transformed section properties is shown in Figures 11 and 12.



MIDSPAN STRAND PATTERN

Figure 11



HARPED STRANDS

Figure 12

Midspan Transformed Properties

Transformed section properties are calculated at the midspan based on the strand pattern shown in Figures 11 and 12. The area of prestress and c.g. of the strands are calculated as follows:

$A_s = (0.153)(48) = 7.344\ in^2$
c.g. = [10(2) + 12(4) + 12(6) + 10(8) + 2(10) + 2(12)] ÷ 48 = 5.50 in.

Net Section – I-Girder

$A_n = 941 – 7.344 = 933.66\ in^2$
$y_{nb} = [941(36.439) – 7.344(5.50)] ÷ 933.66 = 36.682$ in
$e_m = 36.682 – 5.50 = 31.182$ in
$y_{nt} = 72.00 – 36.682 = 35.318$ in
$I_n = 671{,}108 + 941(36.439 – 36.682)^2 – 7.344(36.682 – 5.50)^2$
$I_n = 664{,}023\ in^4$
$r^2 = I / A = 664{,}023 / 933.66 = 711.20\ in^2$

Transformed Section – I-Girder (n = 7.00) at Service (f'_c = 5.0 ksi)

$A_t = 933.66 + (7.00)(7.344) = 985.07\ in^2$
$y_{tb} = [933.66(36.682) + (7.00)(7.344)(5.50)] ÷ 985.07 = 35.055$ in
$e_m = 35.055 – 5.50 = 29.555$ in
$y_{tt} = 72.00 – 35.055 = 36.945$ in
$I_t = 664{,}023 + 933.66(36.682 – 35.055)^2 + 7.00(7.344)(35.055 – 5.50)^2$
$I_t = 711{,}399\ in^4$
$r^2 = I / A = 711{,}399 / 985.07 = 722.18\ in^2$

Composite Section – I-Girder & Deck

n = 3861 / 4070 = 0.949

Interior Girder

$A_{tc} = 985.07 + 0.949(7.50)(108) = 1753.76\ in^2$
$y_{tcb} = [985.07(35.055) + 0.949(7.50)(108)(75.75)] ÷ 1753.76 = 52.892$ in
$e_m = 52.892 – 5.50 = 47.392$ in
$y_{tct} = 72.00 – 52.892 = 19.108$ in
$I_{tc} = 711{,}399 + 985.07(35.055 – 52.892)^2 + 0.949(108)(7.50)^3 ÷ 12$
$+ 0.949(108)(7.50)(75.75 – 52.892)^2 = 1{,}430{,}042\ in^4$

Exterior Girder

A_{tc} = 985.07 + 0.949[(90.00)(7.50) + (16.00)(1.00) + 0.5(16.00)(4.00)]
= 1671.20 in^2
y_{tcb} = [985.07(35.055) + 0.949(90.00)(7.50)(75.75) +
0.949(16.00)(1.00)(71.50) + 0.949(0.5)(16.00)(4.00)(69.667)]
÷1671.20 = 51.614 in
e_m = 51.614 – 5.50 = 46.114 in
y_{tct} = 72.00 – 51.614 = 20.386 in
I_{tc} = 711,399 + 985.07(35.055 – 51.614)^2 + 0.949(90.00)(7.50)^3 ÷12 +
0.949(16.00)(1.00)^3 ÷12 + 0.949(16.00)(4.00)^3 ÷36 +
0.949(90.00)(7.50)(75.75 – 51.614)^2 + 0.949(16.00)(1.00)(71.50 –
51.614)^2 + 0.949(0.5)(16.00)(4.00)(69.667 – 51.614)^2 = 1,373,603 in^4

Transfer Length Properties

The section properties are also required near the ends of the beam at a distance equal to the transfer length from the end of the beam. Since transformed section properties are being used, the section properties will vary with the change in center of gravity of the strands. The transfer length of the bonded prestressing strands is 60 times the strand diameter. For 0.5 inch diameter strand the transfer length equals 30 inches. The centerline of bearing is 9 inches from the girder end resulting in a transfer length of 21 inches from the centerline of bearing.

See Figure 12 for a diagram of the harped strands. The rise in the top strand at the end of the transfer length is:

Y = 12.00 + (58.00)(43.125 – 30/12) / (43.125) = 66.638 in

$$cg = \frac{12\cdot(66.638-5.00)+8\cdot(2.0)+10\cdot(4.0)+10\cdot(6.0)+8\cdot(8.0)}{48} = 19.160$$

At transfer length from the girder end:

<u>Net Section</u> – I-Girder

A_n = 941 – 7.344 = 933.66 in^2
y_{nb} = [941(36.439) –7.344(19.160)] ÷ 933.66 = 36.575 in
e_t = 36.575 – 19.160 = 17.415 in
y_{nt} = 72.00 – 36.575 = 35.425 in
I_n = 671,108 + 941(36.439 – 36.575)^2 – 7.344(36.575 – 19.160)^2
I_n = 668,898 in^4
r^2 = 668,898 / 933.66 = 716.43 in^2

Transformed Section – I-Girder (n = 7.00) at Service (f'_c = 5.0 ksi)

$A_t = 933.66 + 7.00(7.344) = 985.07 \text{ in}^2$
$y_{tb} = [933.66(36.575) + 7.00(7.344)(19.160)] \div 985.07 = 35.666 \text{ in}$
$e_t = 35.666 – 19.160 = 16.506 \text{ in}$
$y_{tt} = 72.00 – 35.666 = 36.334 \text{ in}$
$I_t = 668{,}898 + 933.66(36.575 – 35.666)^2 + 7.00(7.344)(35.666 – 19.160)^2$
$I_t = 683{,}675 \text{ in}^4$

Composite Section – I-Girder & Deck

$n = 3861 / 4070 = 0.949$

Interior Girder

$A_{tc} = 985.07 + 0.949(7.50)(108.00) = 1753.76 \text{ in}^2$
$y_{tcb} = [985.07(35.666) + 0.949(7.50)(108.0)(75.75)] \div 1753.76 = 53.235 \text{ in}$
$y_{tct} = 72.00 – 53.235 = 18.765 \text{ in}$
$I_{tc} = 683{,}675 + 985.07(53.235 – 35.666)^2 + 0.949(108.0)(7.50)^3 \div 12$
$+ 0.949(108.0)(7.50)(75.75 – 53.235)^2 = 1{,}381{,}008 \text{ in}^4$

Exterior Girder

$A_{tc} = 985.07 + 0.949[(90.00)(7.50) + (16.00)(1.00) + 0.5(16.00)(4.00)]$
$= 1671.20 \text{ in}^2$
$y_{tcb} = [985.07(35.666) + 0.949(90.00)(7.50)(75.75) +$
$0.949(16.00)(1.00)(71.50) + 0.949(0.5)(16.00)(4.00)(69.667)]$
$\div 1671.20 = 51.974 \text{ in}$
$y_{tct} = 72.00 – 51.974 = 20.026 \text{ in}$
$I_{tc} = 683{,}675 + 985.07(51.974 – 35.666)^2 + 0.949(90.00)(7.50)^3 \div 12 +$
$0.949(16.00)(1.00)^3 \div 12 + 0.949(16.00)(4.00)^3 \div 36 +$
$0.949(90.00)(7.50)(75.75 – 51.974)^2 +$
$0.949(16.00)(1.00)(71.50 – 51.974)^2 +$
$0.949(0.5)(16.00)(4.00)(69.667 – 51.974)^2 = 1{,}326{,}098 \text{ in}^4$

A more precise method of determining the transformed section properties is shown in Appendix A. The method shown in Appendix A treats each row as a separate area rather than treating the entire area as lumped together at the center of gravity.

Design Span

Each girder line has a 1'-0" gap between the girder ends over the pier. The centerline of bearing is located 9 inches from the end of the girder. The design span is then 112.00 – 1.00 / 2 – 0.75 = 110.75 feet.

Step 2 – Determine Loads and Stresses

The flexural design of the precast prestressed I-girder is based on simple span positive moments. Normally moments, shears and stresses are calculated at tenth points using computer software programs. For this problem, only critical values will be determined.

Dead Load
[3.5.1]

In LFRD design, the dead load is separated between DC loads and DW loads since their load factors differ. For precast girders, each load is also separated by the section property used to determine the stresses. The DC loads that use the net section properties include moments from the self-weight of the precast beam. The DC loads that use the transformed section properties include the moments from externally applied loads including the build-up, stay-in-place forms, diaphragm and the cast-in-place deck. For this problem the SIP Forms will be assumed to weight 15 psf. The DC loads that use the composite transformed section properties include the parapet, rail, fence and sidewalk. The DW load that uses the composite transformed section properties includes the 0.025 ksf Future Wearing Surface and any utilities. The composite dead load and future wearing surface are distributed equally to all girders.

Loads

Non-Composite Dead Loads

Interior	Self Weight	0.150(941 /144)	= 0.980 k/ft
	Int Slab	0.150(8.00 / 12)(9.00)	= 0.900 k/ft
	Build-up	0.150[(2.00)(40.00) / 144]	= 0.083 k/ft
	SIP Panel	0.015(5.67)	= 0.085 k/ft
Exterior	Ext Slab	0.150(8.00 / 12)(7.50)	= 0.750 k/ft
	Overhang	0.150(1.00 / 12)(1.33)	= 0.017 k/ft
	OH Taper	0.150(4.00 / 12)(1.33)(1/2)	= 0.033 k/ft
			0.800 k/ft
	SIP Panel	0.015(5.67) / 2	= 0.043 k/ft

Intermediate Diaphragm

$0.150(0.75)[(9.00)(6.00) – (941/144) – (0.67)(6.83)]$ = 4.83 k
$0.150(4.00)(4.00)(1/2)(2)(5.67) / 144$ = 0.09 k
4.92 k

Composite Dead Load

Sidewalk	0.150(6.00)[(7.00 + 9.16) /2](2) / 12	= 1.212 k/ft
Parapet	0.150(0.8333)(2.77)(2)	= 0.692 k/ft
Rail & Fence	0.075(2)	= 0.150 k/ft
		2.054 k/ft

Composite DC = (2.054) / (9 girders) = 0.228 k/ft

FWS 0.025(64.00) / (9 girders) = 0.178 k/ft

Midspan Moments

Interior Girder

DC Loads – Net Section Properties

Self Weight	$0.980(110.75)^2 \div 8$	= 1503 ft-k

DC Loads – Transformed Section Properties

Slab	$0.900(110.75)^2 \div 8$	= 1380 ft-k
Build-up	$0.083(110.75)^2 \div 8$	= 127 ft-k
SIP Form	$0.085(110.75)^2 \div 8$	= 130 ft-k
Interm Diaph	$4.92(110.75) \div 4$	= 136 ft-k
		= 1773 ft-k

DC Loads – Composite Transformed Section Properties

Composite DL	$0.228(110.75)^2 \div 8$	= 350 ft-k

DW Loads – Composite Transformed Section Properties

FWS	$0.178(110.75)^2 \div 8$	= 273 ft-k

Exterior Girder

DC Loads – Net Section Properties

Self Weight	$0.980(110.75)^2 \div 8$	= 1503 ft-k

DC Loads – Transformed Section Properties

Slab	$0.800(110.75)^2 \div 8$	= 1227 ft-k
Build-up	$0.083(110.75)^2 \div 8$	= 127 ft-k
SIP Form	$0.043(110.75)^2 \div 8$	= 66 ft-k
Interm Diaph	$(4.92 \div 2)(110.75) \div 4$	= 68 ft-k
		= 1488 ft-k

DC Loads – Composite Transformed Section Properties

Composite DL	$0.228(110.75)^2 \div 8$	= 350 ft-k

DW Loads – Composite Transformed Section Properties

FWS	$0.178(110.75)^2 \div 8$	= 273 ft-k

Future Without Sidewalks

DC Loads – Composite Transformed Section Properties

Composite DL $[(0.692+0.150) \div 9](110.75)^2 \div 8 =$ 143 ft-k

DW Loads – Composite Transformed Section Properties

FWS $[(76.0)(0.025) \div 9](110.75)^2 \div 8 =$ 324 ft-k

Transfer Length Moments

At a distance x from the support, the moment from a uniform load is:

$M_x = (w)(x)(L – x) \div 2$

Interior Girder

DC Loads – Net Section Properties

Self Weight	0.980(1.75)(110.75 - 1.75)÷2	= 93 ft-k

DC Loads – Transformed Section Properties

Slab	0.900(1.75)(110.75 – 1.75) ÷2	= 86 ft-k
Build-up	0.083(1.75)(110.75 – 1.75) ÷2	= 8 ft-k
SIP Form	0.085(1.75)(110.75 - 1.75) ÷2	= 8 ft-k
Interm Diaph	4.92(1.75) ÷2	= 4 ft-k
		= 106 ft-k

DC Loads – Composite Transformed Section Properties

Composite DL	0.228(1.75)(110.75 - 1.75) ÷2	= 22 ft-k

DW Loads – Composite Transformed Section Properties

FWS	0.178(1.75)(110.75 - 1.75) ÷2	= 17 ft-k

Exterior Girder

DC Loads – Net Section Properties

Self Weight	0.980(1.75)(110.75 - 1.75)÷2	= 93 ft-k

DC Loads – Transformed Section Properties

Slab	0.800(1.75)(110.75 – 1.75) ÷2	= 76 ft-k
Build-up	0.083(1.75)(110.75 – 1.75) ÷2	= 8 ft-k
SIP Form	0.043(1.75)(110.75-1.75)÷2	= 4 ft-k
Interm Diaph	(4.92÷2)(1.75) ÷2	= 2 ft-k
		= 90 ft-k

DC Loads – Composite Transformed Section Properties

Composite DL	0.228(1.75)(110.75 - 1.75) ÷2	= 22 ft-k

DW Loads – Composite Transformed Section Properties

FWS	0.178(1.75)(110.75 - 1.75) ÷2	= 17 ft-k

Hold-Down Moments

DC Loads – Net Section Properties

Self Weight	0.980(42.375)(110.75 – 42.375) ÷ 2	= 1420 ft-k

Live Load
[3.6.1]
[BPG]

The HL-93 live load in the LRFD specification differs from the HS-20-44 load in the Standard Specifications. For design of the precast prestressed I-Girder, ADOT calculates the live load moments assuming a simple span.

Midspan Moments

The maximum moment at midspan from the design lane load is caused by loading the entire span. The force effects from the design lane load shall not be subject to a dynamic load allowance. At midspan the moment equals the following:

Design Lane

$$M_{lane} = w \cdot (l)^2 \div 8 = 0.640 \cdot (110.75)^2 \div 8 = 981 \text{ ft-k}$$

The maximum design truck moment results when the truck is located with the middle axle at midspan. The truck live load positioned for maximum moment at midspan is shown below:

Design Truck

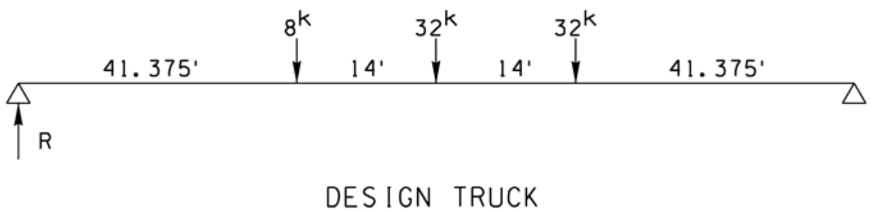


Figure 13

$$R = [8 \cdot (69.375) + 32 \cdot (55.375) + 32 \cdot (41.375)] \div 110.75 = 32.966 \text{ kips}$$
$$M_{truck} = 32.966 \cdot (55.375) - 8 \cdot (14) = 1714 \text{ ft-k}$$

The maximum design tandem moment results when the tandem is located with one of the axles at midspan. The tandem live load positioned for maximum moment is shown below:

Design Tandem

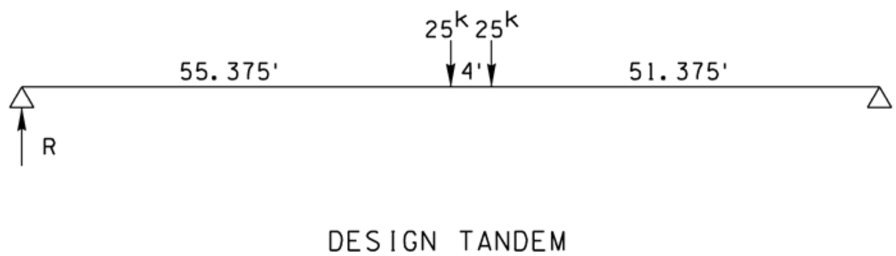


Figure 14

$$R = [25 \cdot (55.375) + 25 \cdot (51.375)] \div 110.75 = 24.097 \text{ kips}$$
$$M_{\tan dem} = 24.097 \cdot (55.375) = 1334 \text{ ft-k}$$

By inspection the moment from the combination of design truck and design lane load is higher than the combination of design tandem and design lane load.

Sidewalk LL [3.6.1.6]

The sidewalk live load is as follows:

w_{sw} = 0.075(6.00) = 0.450 k/ft

M_{sw} = 0.450(110.75)^2 ÷ 8 = 690 ft-k per side

LL Distribution

[4.6.2.2.1-1]

The LRFD Specification has made major changes to the live load distribution factors. The first step is to determine the superstructure type from Table 4.6.2.2.1-1. For precast prestressed concrete girders with a cast-in-place concrete deck the typical cross section is identified as Type (k).

Interior Girder

Since the range of applicability of all variables is within the allowable, the live load distribution factor for moment for an interior girder with one lane loaded may be taken as:

	Applicable Range
S = spacing of girders = 9.00 ft	$3.5 \le S \le 16.0$
L = span length of girder = 110.75 ft.	$20 \le L \le 240$
K_g = long stiffness parameter = 2,240,331 in^4	$10{,}000 \le K_g \le 7{,}000{,}000$
t_s = deck slab thickness = 7.50 in	$4.5 \le t_s \le 12.0$
N_b = Number Girders = 9	$N_b \ge 4$

[4.6.2.2.2b-1]

$$LL\ Distribution = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^{\ 3}}\right)^{0.1}$$

$$LL\ Distribution = 0.06 + \left(\frac{9.0}{14}\right)^{0.4}\left(\frac{9.00}{110.75}\right)^{0.3}\left(\frac{2{,}240{,}331}{(12.0)\cdot(110.75)\cdot(7.50)^3}\right)^{0.1} = 0.513$$

The live load distribution factor for moment for an interior girder with two or more lanes loaded is:

$$LL\ Distribution = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^{\ 3}}\right)^{0.1}$$

$$LL\ Distribution = 0.075 + \left(\frac{9.0}{9.5}\right)^{0.6}\left(\frac{9.00}{110.75}\right)^{0.2}\left(\frac{2{,}240{,}331}{(12.0)\cdot(110.75)\cdot(7.50)^3}\right)^{0.1}$$

LL Distribution = 0.748 ⇒ Critical

Exterior Girder [4.6.2.2.2d-1]

The live load distribution factor for one design lane loaded for moment for an exterior girder requires use of the lever rule. See Figure 15.

$$R = P_{wheel} (3.00) / (9.00) = 0.333 P_{wheel}$$

[4.3]

When the lever rule is used to determine live load distribution, the multiple presence factor is applied. For one vehicle, m = 1.20.

$$\text{LL Distribution} = 0.333(1.20) = 0.400 P_{wheel} = 0.200 P_{axle}$$

Use the lever rule to determine the portion of the pedestrian live load taken by the exterior girder.

$$R = P_{sw} (5.00 + 3.00) / 9.00 = 0.889 P_{sw}$$

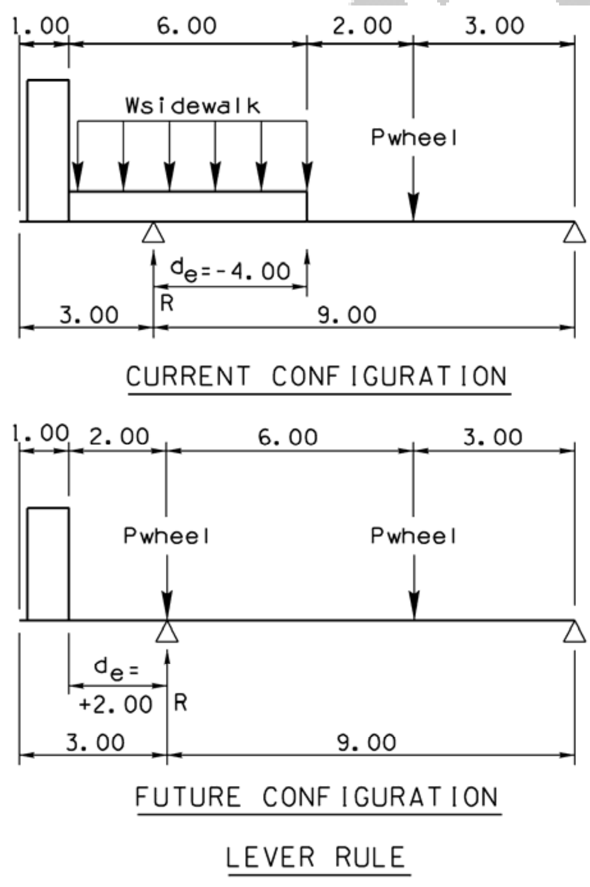


Figure 15

The live load distribution factor for two or more design lanes loaded for moment for an exterior girder is:

$$\text{LL Distribution} = e\, g_{interior}$$

$$e = 0.77 + d_e / 9.1$$

[4.6.2.2.1]

d_e = distance from the exterior web to the inside face of curb in feet. For this problem d_e equals -4.00 feet. Since this value is outside the range of applicability, this criteria is not evaluated and the distribution is based on the Lever Rule.

Range of Applicability: $-1.0 \le d_e \le 5.5$

[4.6.2.2.2d]

For girder bridges with diaphragms, an additional distribution check must be made. The distribution factor for exterior girders shall not be taken as less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section with the multiple presence factor applied.

[C4.6.2.2.2d-1]

$$R = \frac{N_L}{N_b} + \frac{X_{ext}\sum^{N_L} e}{\sum^{N_b} x^2}$$

$$\sum^{N_b} x^2 = 2\left[(9.0)^2 + (18.0)^2 + (27.0)^2 + (36.0)^2\right] = 4860$$

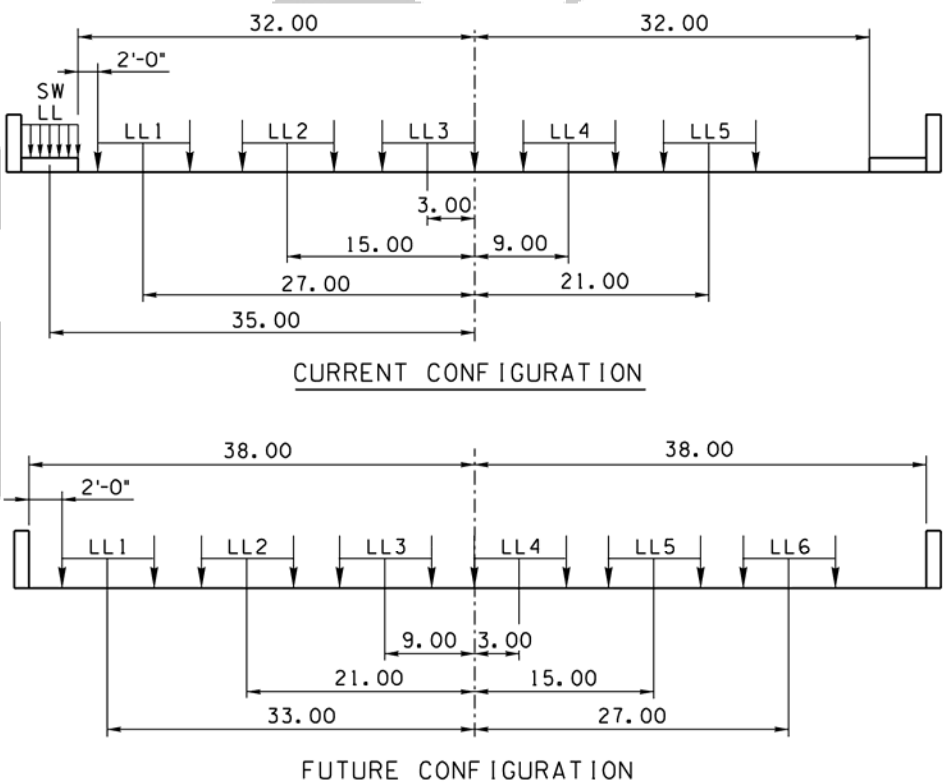


Figure 16

Current Configuration

For one vehicle:

$$R = \frac{1}{9} + \frac{(36.00)\cdot(27.00)}{4860} = 0.311$$

For two vehicles:

$$R = \frac{2}{9} + \frac{(36.00)\cdot(27.00 + 15.00)}{4860} = 0.533$$

For three vehicles:

$$R = \frac{3}{9} + \frac{(36.00)\cdot(27.00 + 15.00 + 3.00)}{4860} = 0.667$$

For four vehicles:

$$R = \frac{4}{9} + \frac{(36.00)\cdot(27.00 + 15.00 + 3.00 - 9.00)}{4860} = 0.711$$

For five vehicles:

$$R = \frac{5}{9} + \frac{(36.00)\cdot(27.00 + 15.00 + 3.00 - 9.00 - 21.00)}{4860} = 0.667$$

[3.6.1.1.2]

The pedestrian live load is treated as a vehicular load as far as the multiple presence factor is concerned. The vehicular distribution factors when the pedestrian load is present are:

$R_1 = 0.311(1.00) = 0.311$
$R_2 = 0.533(0.85) = 0.453$
$R_3 = 0.667(0.65) = 0.434$
$R_4 = 0.711(0.65) = 0.462 \Rightarrow$ Critical
$R_5 = 0.667(0.65) = 0.434$

When the pedestrian live load is not present the vehicular distribution factors are as follows:

$R_1 = 0.311(1.20) = 0.373$
$R_2 = 0.533(1.00) = 0.533$
$R_3 = 0.667(0.85) = 0.567 \Rightarrow$ Critical
$R_4 = 0.711(0.65) = 0.462$
$R_5 = 0.667(0.65) = 0.434$

For the current configuration, consideration must be given to the possibility of a vehicle on the sidewalk since there is not a separation barrier between the roadway and sidewalk. Only the design truck located two feet from the parapet face will be applied as a design lane load on the sidewalk is unlikely. For this situation use the Lever Rule with the following distribution:

LL Distribution = (1.20)[(9.00 + 3.00) / 9.00]÷2 = 0.800 ⇒ Critical

Using the second criteria for girder bridges with diaphragms:

$$R = [1.20] \cdot \left[\frac{1}{9} + \frac{(36.00) \cdot (33.00)}{4860} \right] = 0.427$$

Skew Effect
[4.6.2.2.2e-1]

Since the bridge is skewed 30 degrees, the live load skew reduction factor is applied. All appropriate variables are within the allowable range.

Skew Reduction = 1 – c_1 (tanθ)$^{1.5}$

$$c_1 = 0.25 \left(\frac{K_g}{12.0 L t_s^{\,3}} \right)^{0.25} \left(\frac{S}{L} \right)^{0.5}$$

$$c_1 = 0.25 \left(\frac{2{,}240{,}331}{(12.0) \cdot (110.75) \cdot (7.50)^3} \right)^{0.25} \left(\frac{9.00}{110.75} \right)^{0.5} = 0.1008$$

Skew Reduction = 1 – (0.1008) [tan(30)]$^{1.5}$ = 0.956

Dynamic Load Allowance
[3.6.2]

The dynamic load allowance IM equals 33%.

Dynamic load allowance applies to the truck or tandem but not to the design lane load. The dynamic load allowance has been included in the summation of live loads for one vehicle.

Live Load Summary

A summary of the critical live load moments follows:

Interior Girder
LL + IM = [981 + 1.33(1714)](0.748)(0.956) = 2332 ft-k

Exterior Girder
LL + IM = [981 + 1.33(1714)](0.567)(0.956) = 1767 ft-k
LL + IM + PL = [981 + 1.33(1714)](0.462)(0.956) + 690(0.889)
= 2054 ft-k ⇐ Critical
LL + IM (Sidewalk) = [1.33(1714)](0.800)(0.956) = 1743 ft-k

Future Configuration [3.6.1.1.1]

In addition to the current configuration, consideration should be made for a future configuration consisting of an additional lane without the sidewalks. See Figures 15 and 16. For this configuration, the live load distribution for an exterior girder for one lane loaded is:

[4.6.2.2.2d-1]

$R = P_{wheel}\ (9.00 + 3.00) / (9.00) = 1.333\ P_{wheel}$

When the lever rule is used to determine live load distribution, the multiple presence factor is applied. For one vehicle, m = 1.20.

LL Distribution = $1.333\ P_{wheel}\ (1.20) = 1.600\ P_{wheel} = 0.800\ P_{axle}$ ⇐Use

The live load distribution factor for two or more design lanes loaded for moment for an exterior girder is:

LL Distribution = e $g_{interior}$

$e = 0.77 + d_e / 9.1$

d_e = distance from the exterior web to the inside face of barrier in feet. For this problem d_e equals 2.00 feet. See Figure 15. The value of d_e is inside the range of applicability of the formula ($-1.0 \le d_e \le 5.5$).

$e = 0.77 + 2.00 / 9.1 = 0.990$

LL Distribution = 0.990(0.748) = 0.741

For girder bridges with diaphragms, an additional distribution check must be made. The distribution factor for exterior girders shall not be taken as less than that which would be obtained by assuming that the cross-section deflects and rotates as a rigid cross-section with the multiple presence factors applied. See Figure 16.

[C4.6.2.2.2d-1]

$$R = \frac{N_L}{N_b} + \frac{X_{ext} \sum^{N_L} e}{\sum^{N_b} x^2}$$

For one vehicle:

$$R = \frac{1}{9} + \frac{(36.00)\cdot(33.00)}{4860} = 0.356$$

For two vehicles:

$$R = \frac{2}{9} + \frac{(36.00)\cdot(33.00 + 21.00)}{4860} = 0.622$$

For three vehicles:

$$R = \frac{3}{9} + \frac{(36.00)\cdot(33.00 + 21.00 + 9.00)}{4860} = 0.800$$

For four vehicles:

$$R = \frac{4}{9} + \frac{(36.00)\cdot(33.00 + 21.00 + 9.00 - 3.00)}{4860} = 0.889$$

For five vehicles:

$$R = \frac{5}{9} + \frac{(36.00)\cdot(33.00 + 21.00 + 9.00 - 3.00 - 15.00)}{4860} = 0.889$$

For six vehicles:

$$R = \frac{6}{9} + \frac{(36.00)\cdot(33.00 + 21.00 + 9.00 - 3.00 - 15.00 - 27.00)}{4860} = 0.800$$

The multiple presence factors must be added to the above distribution factors as follows:

R_1 = 0.356(1.20) = 0.427
R_2 = 0.622(1.00) = 0.622
R_3 = 0.800(0.85) = 0.680 ⇐ Critical
R_4 = 0.889(0.65) = 0.578
R_5 = 0.889(0.65) = 0.578
R_6 = 0.800(0.65) = 0.520

Live Load Summary

Exterior Girder
LL + IM = [981 + 1.33(1714)](0.800)(0.956) = 2494 ft-k

This moment is higher than the live load for the current configuration and may control the design. For this problem, the exterior girder will not be investigated but in a real problem an investigation for this situation would be required. Due to the reconfiguration, the dead load of the sidewalk will be gone but there will be some additional future wearing surface. It is not obvious whether the exterior girder for the future configuration would control.

Transfer Length Moment

At the transfer length the live load moment is:

Lane $M = 0.640(1.75)(110.75 – 1.75) \div 2 = 61$ ft-k
Truck $M = 1.75[(32)(109.00) + (32)(95.00) + 8(81.00)] \div 110.75 = 113$ ft-k
Tandem $M = 1.75[(25)(109.00) + 25(105.00)] \div 110.75 = 85$ ft-k
Ped LL $M = 0.450(1.75)(110.75 – 1.75) \div 2 = 43$ ft-k

Interior $M_{LL+IM} = [61 + 1.33(113)](0.748)(0.956) = 151$ ft-k
Exterior $M_{LL+IM+Ped} = [61 + 1.33(113)](0.462)(0.956) +[43](0.889) = 132$ ft-k
$M_{LL+IM\ sdwlk} = [1.33(113)](0.800)(0.956) = 115$ ft-k

[3.4.1-1]

The LRFD Specification has made major changes to the group load combinations contained in [T3.4.1-1]. There are several limit states that must be considered in design of the superstructure. Limit states for this problem are as follows:

Midspan Moment

STRENGTH I – Basic load combination relating to the normal vehicular use of the bridge without wind.

$M_u = 1.25(DC) + 1.50(DW) + 1.75(LL + IM)$

Interior
$M_u = 1.25(1503 + 1773 + 350) + 1.50(273) + 1.75(2332) = 9023$ ft-k

Exterior
$M_u = 1.25(1503 + 1488 + 350) + 1.50(273) + 1.75(2054) = 8180$ ft-k

SERVICE LIMIT STATES – For the service limit states, the stresses are calculated using the appropriate section property type as follows:

Midspan Stresses

Net Non-Composite
$M = 1503$ ft-k

$$f_t = \frac{(1503)\cdot(12)\cdot(35.318)}{664{,}023} = 0.959\ \text{ksi}$$

$$f_b = \frac{(1503)\cdot(12)\cdot(36.682)}{664{,}023} = -0.996\ \text{ksi}$$

Transformed Non-Composite

Interior Girder

M = 1773 ft-k

$$f_t = \frac{(1773)\cdot(12)\cdot(36.945)}{711{,}399} = 1.105 \text{ ksi}$$

$$f_b = \frac{(1773)\cdot(12)\cdot(35.055)}{711{,}399} = -1.048 \text{ ksi}$$

Exterior Girder

M = 1488 ft-k

$$f_t = \frac{(1488)\cdot(12)\cdot(36.945)}{711{,}399} = 0.927 \text{ ksi}$$

$$f_b = \frac{(1488)\cdot(12)\cdot(35.055)}{711{,}399} = -0.880 \text{ ksi}$$

Transformed Composite

Interior Girder

Dead Load

M = 350 + 273 = 623 ft-k

$$f_t = \frac{(623)\cdot(12)\cdot(19.108)}{1{,}430{,}042} = 0.100 \text{ ksi}$$

$$f_b = \frac{(623)\cdot(12)\cdot(52.892)}{1{,}430{,}042} = -0.277 \text{ ksi}$$

LL + IM

M = 2332 ft-k

$$f_t = \frac{(2332)\cdot(12)\cdot(19.108)}{1{,}430{,}042} = 0.374 \text{ ksi}$$

$$f_b = \frac{(2332)\cdot(12)\cdot(52.892)}{1{,}430{,}042} = -1.035 \text{ ksi}$$

<u>Exterior Girder</u>

<u>Dead Load</u>

M = 350 + 273 = 623 ft-k

$$f_t = \frac{(623)\cdot(12)\cdot(20.386)}{1{,}373{,}603} = 0.111 \text{ ksi}$$

$$f_b = \frac{(623)\cdot(12)\cdot(51.614)}{1{,}373{,}603} = -0.281 \text{ ksi}$$

<u>LL + IM</u>

M = 2054 ft-k

$$f_t = \frac{(2054)\cdot(12)\cdot(20.386)}{1{,}373{,}603} = 0.366 \text{ ksi}$$

$$f_b = \frac{(2054)\cdot(12)\cdot(51.614)}{1{,}373{,}603} = -0.926 \text{ ksi}$$

<u>SERVICE I</u> – Load combination relating to normal operational use of the bridge including wind loads to control crack width in reinforced concrete structures. For a precast member with a cast-in-place deck where transformed section properties are used each service state must be broken into subgroups depending upon the section properties used to determine the stress.

M_s = 1.0(DC + DW) + 1.0(LL + IM)

Interior

$\sum f_t$ = 1.0(0.959 + 1.105 + 0.100) + 1.0(0.374) = 2.538 ksi

Exterior

$\sum f_t$ = 1.0(0.959 + 0.927 + 0.111) + 1.0(0.366) = 2.363 ksi

<u>SERVICE III</u> – Load combination relating only to tension in prestressed concrete superstructures with the objective of crack control.

M_s = 1.0(DC + DW) + 0.80(LL + IM)

Interior

$\sum f_b$ = 1.0(-0.996 – 1.048 – 0.277) + 0.8(-1.035) = -3.149 ksi ⇐ Critical

Exterior

$\sum f_b$ = 1.0(-0.996 – 0.880 – 0.281) + 0.8(-0.926) = -2.898 ksi

Transfer Length Stresses

SERVICE LIMIT STATES

Net Non-Composite
M = 93 ft-k

$$f_t = \frac{(93)\cdot(12)\cdot(35.425)}{668{,}898} = 0.059 \text{ ksi}$$

$$f_b = \frac{(93)\cdot(12)\cdot(36.575)}{668{,}898} = -0.061 \text{ ksi}$$

Interior Girder

Interior Girder
Transformed Non-Composite
M = 106 ft-k

$$f_t = \frac{(106)\cdot(12)\cdot(36.334)}{683{,}675} = 0.068 \text{ ksi}$$

$$f_b = \frac{(106)\cdot(12)\cdot(35.666)}{683{,}675} = -0.066 \text{ ksi}$$

Transformed Composite
Dead Load
M = 22 + 17 = 39 ft-k

$$f_t = \frac{(39)\cdot(12)\cdot(18.765)}{1{,}381{,}008} = 0.006 \text{ ksi}$$

$$f_b = \frac{(39)\cdot(12)\cdot(53.235)}{1{,}381{,}008} = -0.018 \text{ ksi}$$

Live Load
M = 151 ft-k

$$f_t = \frac{(151)\cdot(12)\cdot(18.765)}{1{,}381{,}008} = 0.025 \text{ ksi}$$

$$f_b = \frac{(151)\cdot(12)\cdot(52.235)}{1{,}381{,}008} = -0.069 \text{ ksi}$$

SERVICE I
$\sum f_t$ = 1.0(0.059 + 0.068 + 0.006) + 1.0(0.025) = 0.158 ksi
SERVICE III
$\sum f_b$ = 1.0(-0.061 – 0.066 – 0.018) + (0.8)(-0.069) = -0.200 ksi

Exterior Girder

Exterior Girder

Transformed Non-Composite

M = 90 ft-k

$$f_t = \frac{(90)\cdot(12)\cdot(36.334)}{683{,}675} = 0.057 \text{ ksi}$$

$$f_b = \frac{(90)\cdot(12)\cdot(35.666)}{683{,}675} = -0.056 \text{ ksi}$$

Transformed Composite

Dead Load

M = 22 + 17 = 39 ft-k

$$f_t = \frac{(39)\cdot(12)\cdot(20.026)}{1{,}326{,}098} = 0.007 \text{ ksi}$$

$$f_b = \frac{(39)\cdot(12)\cdot(51.974)}{1{,}326{,}098} = -0.018 \text{ ksi}$$

Live Load

M = 132 ft-k

$$f_t = \frac{(132)\cdot(12)\cdot(20.026)}{1{,}326{,}098} = 0.024 \text{ ksi}$$

$$f_b = \frac{(132)\cdot(12)\cdot(51.974)}{1{,}326{,}098} = -0.062 \text{ ksi}$$

SERVICE I

$\sum f_t =$ 1.0(0.059 + 0.057 + 0.007) + 1.0(0.024) = 0.147 ksi

SERVICE III

$\sum f_b =$ 1.0(-0.061 – 0.056 – 0.018) + 0.8(-0.062) = -0.185 ksi

Prestress Design [5.9]

Step 3 – Determine Number of Strands

The design of a precast prestressed concrete girder involves making assumptions, calculating results, comparing the results to the assumptions and repeating the process until convergence occurs. The iterative process will not be shown in this example. Rather the final iteration with valid assumptions and calculations will be shown. Since the interior girder controls the design of this bridge, only the interior girder prestress design will be shown. For a real project both the interior and exterior girders would be checked.

The required number of strands, the associated center of gravity, and long-term losses, the release concrete strength and final concrete strength must be assumed. For this problem assume the following:

No. ½” Diameter Strands = 48
Time-Dependent Losses = 34.00 ksi
Total Prestress Losses = 54.58 ksi
f'_{ci} = 4.7 ksi
f'_{c} = 5.0 ksi

The first calculation is to determine a strand pattern and associated center of gravity. For this problem 48 strands are required with the pattern shown in Figure 11. The center of gravity from the bottom equals:

$$c.g. = \frac{(10)\cdot(2)+(12)\cdot(4)+(12)\cdot(6)+(10)\cdot(8)+(2)\cdot(10)+(2)\cdot(12)}{48} = 5.500$$

In this pattern 2 strands in each row are harped with the top row 2 inches from the top of the girder and the hold-down point 13’-0” from the center of the girder. See Figure 12.

[BPG]

With the above values the number of strands can be determined from the basic equation for stress for a prestressed member. The number of strands is based on an allowable tensile limit of $0.0948\sqrt{f'_c}$ in the bottom fiber at midspan after all losses.

$$\frac{f_{se}A_{str}NS}{A_{net}}\left(1+\frac{e_m y_b}{r^2}\right)+\sum f_b \geq -0.0948\sqrt{f'_c}$$

Solving for the number of strands, NS for an interior girder, results in:

$$NS \geq \left[-\sum f_b - 0.0948\sqrt{f'_c}\right]\cdot\frac{A_{net}}{f_{se}A_{str}}\cdot\left(\frac{r^2}{r^2+e_m y_b}\right)$$

Where:

f_{se} = 0.75(270) – 54.58 = 147.92 ksi

e_m = 36.682 – 5.500 = 31.182 in

Allowable Tension = $0.0948\sqrt{5.0} = -0.212$ ksi

$$NS \geq \left[-\left(-3.149\right)-0.212\right]\cdot\frac{933.66}{\left(147.92\right)\cdot\left(0.153\right)}\cdot\left(\frac{711.20}{711.20+\left(31.182\right)\cdot\left(36.682\right)}\right)$$

NS = 46.5

Use 48 strands

Step 4 – Determine Losses

Loss of Prestress [5.9.5]

Total losses in a prestressed precast member are due to relaxation before transfer, elastic shortening, and the time-dependent losses consisting of shrinkage, creep and relaxation losses.

Relaxation Loss At Transfer [5.9.5.4.4b][2004]

The relaxation loss is broken up into two parts: the relaxation before transfer and the relaxation after transfer. The relaxation before transfer is the loss in stress from the time the strands are pulled until they are released. The equation for relaxation before transfer in the 2004 Specification is slightly different than the equation shown in the commentary of the Standard Specifications. The denominator has been changed from 45 to 40. For concrete release strengths less than 4.5 ksi, 18 hours may be assumed between time of concrete pour and time of strand release. Typically the strands will be pulled the day before the concrete is poured. Conservatively assume a total of 36 hours exists between time of stressing and time of release. The 2006 Interim Revisions deleted this loss without any explanation. However, when the strands are tensioned relaxation will occur until the strands are cut and the force transferred to the concrete. In some regions of the country fabricators overstress the strands initially to compensate for these losses but the fabricators in Arizona do not.

The relaxation loss before transfer for low-relaxation strands is:

[5.9.5.4.4b-1][2004]

$$\Delta f_{pRbt} = \frac{\log\left(24.0t\right)}{40.0}\left[\frac{f_{pj}}{f_{py}}-0.55\right]f_{pj}$$

$$\Delta f_{pRbt} = \frac{\log\left(36\right)}{40.0}\left[\frac{0.75}{0.90}-0.55\right]\cdot\left(0.75\right)\cdot\left(270\right)=2.23\text{ ksi}$$

The stress before transfer = (0.75)(270) – 2.23 = 200.27 ksi

Elastic Shortening [5.9.5.2.3a]

Elastic shortening losses can be calculated directly with a rather lengthy equation in lieu of a trial and error method. The equation for calculation of elastic shortening in the LRFD Commentary [C5.9.5.2.3a-1] is correct as long as the variable f_{pbt} includes the relaxation before transfer. The equation shown in the Commentary has been modified by dividing both the numerator and denominator by the area of the girder. This modification eliminates the need to work with large numbers improving the accuracy of the calculations. The net section properties are used in this calculation.

[C5.19.5.3.3a-1] Modified

$$\Delta f_{pES} = \frac{f_{pbt} A_{ps}\left(r^2 + e_m{}^2\right) - e_m M_g}{A_{ps}\left(r^2 + e_m{}^2\right) + \dfrac{I \cdot E_{ci}}{E_p}}$$

The elastic shortening loss will be calculated at the midspan.

Midspan Losses

<u>0.5 Span</u>

e_m = 36.682 – 5.500 = 31.182 in

A_{ps} = (48)(0.153) = 7.344 in^2

$$A_{ps}\left(r^2 + e_m{}^2\right) = (7.344)\cdot\left(711.20 + (31.182)^2\right) = 12{,}364$$

$$\frac{I \cdot E_{ci}}{E_p} = \frac{(664{,}023)\cdot(3946)}{28{,}500} = 91{,}938$$

$$\Delta f_{pES} = \frac{(0.75 \cdot 270 - 2.23)\cdot(12364) - 31.182\cdot(1503)\cdot(12)}{12{,}364 + 91{,}938}$$

Δf_{pES} = 18.35 ksi

Calculate f_{cgp} and verify the elastic shortening loss.

$$f_{cgp} = 7.344\cdot\left[(0.75)\cdot(270) - 2.23 - 18.35\right]\cdot\left(\frac{1}{933.66} + \frac{(31.182)^2}{664{,}023}\right) - \frac{(1503)\cdot(12)\cdot(31.182)}{664{,}023} = 2.540 \text{ ksi}$$

[5.9.5.2.3b-1]

$$\Delta f_{pES} = \frac{E_p}{E_{ci}}\cdot f_{cgp} = \left[\frac{28500}{3946}\right]\cdot(2.540) = 18.35 \text{ ksi OK}$$

Elastic Shortening Losses Transformed Section Properties

An alternate method of determining the elastic shortening losses is to apply the self-weight of the member plus the prestress before transfer to the transformed section. The Commentary in Article C5.9.5.2.3a states that when calculating concrete stresses using transformed section properties, the effects of losses and gains due to elastic deformations are implicitly accounted for and the elastic shortening loss, $\Delta f_{pES,}$ should not be included in the prestressing force applied to the transformed section at transfer.

The transformed section properties were previously calculated using a modular ratio of 7.00 at Service. Section properties will be calculated with the modular ratio of 7.22 based on the concrete strength at release.

$$n - 1 = 28500 / 3946 - 1 = 6.22$$

$$A_t = 941 + (6.22)(7.344) = 986.68 \text{ in}^2$$

$$y_{tb} = [(941)(36.439) + (6.22)(7.344)(5.500)] / 986.68 = 35.007 \text{ in}$$

$$e_t = 35.007 – 5.500 = 29.507 \text{ in}$$

$$I_t = 671{,}108 + (941)(36.439 – 35.007)^2 + (6.22)(7.344)(35.007-5.50)^2 = 712{,}809 \text{ in}^4$$

The effective prestress force is the jacking stress minus the relaxation loss from time of stressing till time of transfer.

$$P_t = [(0.75)(270) – 2.23](0.153)(48) = 1470.78 \text{ k}$$

The concrete stress at the centroid of the prestress steel is:

$$f_{cgp} = (1470.78)\cdot\left[\frac{1}{986.68} + \frac{(29.507)^2}{712{,}809}\right] - \frac{(1503)\cdot(12)\cdot(29.507)}{712{,}809}$$

$$f_{cgp} = 2.540 \text{ ksi}$$

The elastic shortening loss is then determined as follows:

$$\Delta f_{pES} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{3946}\cdot(2.540) = 18.35 \text{ ksi}$$

This method of determining the elastic shortening loss eliminates the need to estimate losses or use a trial and error approach. The method is direct, simple and produces the same elastic shortening loss as the Commentary equation using the net section properties.

Approximate Time-Dependent Losses - 2006 [5.9.5.3]

For standard precast pretensioned members the long-term prestress losses due to creep of concrete, shrinkage of concrete and relaxation of prestress steel may be estimated as follows:

[5.9.5.3-1]

$$\Delta f_{pLT} = 10.0\frac{f_{pi}A_{ps}}{A_g}\gamma_h\gamma_{st} + 12.0\gamma_h\gamma_{st} + \Delta f_{pR}$$

in which:

[5.9.5.3-2]

$$\gamma_h = 1.7 - 0.01H = 1.7 - 0.01(40) = 1.30$$

[5.9.5.3-3]

$$\gamma_{st} = \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 4.7} = 0.877$$

f_{pi} = prestressing steel stress immediately prior to transfer.

f_{pi} = (0.75)(270) – 2.23 = 200.27 ksi

Δf_{pR} = an estimate of relaxation loss taken as 2.5 ksi for low relaxation strand.

$$\Delta f_{pLT} = 10.0\cdot\frac{(200.27)\cdot(7.344)}{941}\cdot(1.30)\cdot(0.877) + 12.0\cdot(1.30)\cdot(0.877) + 2.50$$

Δf_{pLT} = 34.00 ksi

The final loss is shown below:

[5.9.5.1-1]

$\Delta f_{pT} = \Delta f_{pRbt} + \Delta f_{pES} + \Delta f_{pLT}$

Δf_{pT} = 2.23 + 18.35 + 34.00 = 54.58 ksi

The refined method of determining time-dependent losses is shown in Appendix C. At the time this problem was developed, Bridge Group was in the process of evaluating this method of loss calculation. For the purpose of this example, the approximate losses will be used. For actual designs, the method of determining the time-dependent losses should be in accordance with the ADOT LRFD Bridge Practice Guidelines.

[BPG]

Transfer Length Losses

The elastic shortening loss will be calculated at the transfer length.

Transfer Length

Elastic Shortening [5.9.5.2.3a]

c.g. = 19.160 in

e_{tl} = 36.575 – 19.160 = 17.415 in

$$A_{ps}\left(r^2 + e_m{}^2\right) = (7.344)\cdot\left(716.43 + (17.415)^2\right) = 7489$$

$$\frac{I \cdot E_{ci}}{E_p} = \frac{(668{,}898)\cdot(3946)}{28{,}500} = 92{,}613$$

$$\Delta f_{pES} = \frac{(0.75\cdot 270 - 2.23)\cdot(7489) - 17.415\cdot(93)\cdot(12)}{7489 + 92{,}613}$$

Δf_{pES} = 14.79 ksi

Calculate f_{cgp} and verify the elastic shortening loss.

$$f_{cgp} = 7.344\cdot\left[(0.75)\cdot(270) - 2.23 - 14.79\right]\cdot\left(\frac{1}{933.66} + \frac{(17.415)^2}{668{,}898}\right) - \frac{(93)\cdot(12)\cdot(17.415)}{668{,}898} = 2.048 \text{ ksi}$$

$$\Delta f_{pES} = \frac{E_p}{E_{ci}}\cdot f_{cgp} = \left[\frac{28500}{3946}\right]\cdot(2.048) = 14.79 \text{ ksi OK}$$

Time-Dependent Losses

The time-dependent losses will the same value at all locations when using the approximate equation.

Δf_{pLT} = 34.00 ksi

Total Losses

The final loss is shown below:

$\Delta f_{pT} = \Delta f_{pRbt} + \Delta f_{pES} + \Delta f_{pLT}$

Δf_{pT} = 2.23 + 14.79 + 34.00 = 51.02 ksi

Hold-Down Point

The elastic shortening loss must also be calculated at the hold-down point to determine the concrete release stresses.

Hold-Down Point

e_{hd} = 36.682 – 5.500 = 31.182 in

Elastic Shortening [5.9.5.2.3a]

$$A_{ps}\left(r^2 + e_m{}^2\right) = (7.344)\cdot\left(711.20 + (31.182)^2\right) = 12{,}364$$

$$\frac{I \cdot E_{ci}}{E_p} = \frac{(664{,}023)\cdot(3946)}{28{,}500} = 91{,}938$$

$$\Delta f_{pES} = \frac{(0.75\cdot 270 - 2.23)\cdot(12364) - 31.182\cdot(1420)\cdot(12)}{12{,}364 + 91{,}938}$$

Δf_{pES} = 18.65 ksi

Calculate f_{cgp} and verify the elastic shortening loss.

$$f_{cgp} = 7.344\cdot\left[(0.75)\cdot(270) - 2.23 - 18.65\right]\cdot\left(\frac{1}{933.66} + \frac{(31.182)^2}{664{,}023}\right) - \frac{(1420)\cdot(12)\cdot(31.182)}{664{,}023} = 2.581 \text{ ksi}$$

$$\Delta f_{pES} = \frac{E_p}{E_{ci}}\cdot f_{cgp} = \left[\frac{28500}{3946}\right]\cdot(2.581) = 18.64 \text{ ksi OK}$$

The time-dependent losses will the same value at all locations when using the approximate equation.

Δf_{pLT} = 34.00 ksi

The final loss is shown below:

$\Delta f_{pT} = \Delta f_{pRbt} + \Delta f_{pES} + \Delta f_{pLT}$

Δf_{pT} = 2.23 + 18.65 + 34.00 = 54.88 ksi

Prestress Strand Stress [5.9.3-1]

Step 5 – Check Allowable Stress in Strands

There are two limits for the stress in prestress strands for pretensioned members. The first allowable limit is immediately prior to transfer. The elastic shortening loss should not be included. This check is on the stress in the strands immediately prior to transfer. At this time there is no elastic shortening only relaxation before transfer since the stress has not been transferred to the concrete. The strands are usually pulled to a stress equal to 0.75 f_{pu}.

(1) $f_{pbt} = 0.75 - 2.23 / 270 = 0.742\ f_{pu} < 0.75\ f_{pu}$ OK.

The second stress limit is a service limit state after all losses. The dead load, excluding self-weight and live load plus dynamic load allowance is considered.

$f_{pe} = 0.75 f_{pu} - (54.58) / (270) = 0.548\ f_{pu}$ after all losses

At service limit state added dead load and live load plus dynamic allowance stresses are added to the strand stress since the strands are bonded.

$$f_{service} = \left[\frac{(1773) \cdot 12 \cdot (29.555)}{711{,}399} + \frac{(350 + 273 + 2332) \cdot 12 \cdot (47.392)}{1{,}430{,}042} \right] \cdot \frac{28{,}500}{4070} = 14.419 \text{ ksi}$$

Strand stress = $0.548\ f_{pu} + (14.419) / (270 f_{pu}) = 0.601\ f_{pu}$

(2) Strand stress = $0.601\ f_{pu} < 0.80\ f_{py} = 0.80(0.90)\ f_{pu} = 0.720\ f_{pu}$

Since the two criteria for stress in the strand are met, the jacking coefficient of 0.75 is satisfactory.

Step 6 – Verify Initial Concrete Strength

Once the amount of prestressing steel is determined from tension criteria, the resulting concrete stress and required concrete strength can be determined. Service I Limit State is used to determine the initial concrete compressive stress. The concrete stress in compression before time dependent losses is

[5.9.4.1.1] limited to:

$$\textit{Allowable Compression} = 0.60\, f'_{ci} = (0.60)\cdot(4.7) = 2.820\ ksi$$

The basic equation for stress in concrete follows:

$$f_s = A_{ps}\, f_{si}\left(\frac{1}{A} + \frac{e_m y}{I}\right) + \frac{\sum(\gamma M)y}{I}$$

<u>Transfer Length (1.75 feet from CL Brg)</u>

f_{si} = (0.75)(270) – 2.23 – 14.79 = 185.48 ksi

$$f_b = \frac{(93)\cdot(12)\cdot(36.575)}{668{,}898} = -0.061\ \text{ksi}$$

$$f_b = (7.344)\cdot(185.48)\cdot\left[\frac{1}{933.66} + \frac{(17.415)\cdot(36.575)}{668{,}898}\right] - 0.061 = 2.695\ \text{ksi}$$

<u>Hold-Down Point (42.375 feet from CL Brg)</u>

f_{si} = (0.75)(270) – 2.23 – 18.65 = 181.62 ksi

$$f_b = \frac{(1420)\cdot(12)\cdot(36.682)}{664{,}023} = -0.941\ \text{ksi}$$

$$f_b = (7.344)\cdot(181.62)\cdot\left[\frac{1}{933.66} + \frac{(31.182)\cdot(36.682)}{664{,}023}\right] - 0.941 = 2.785\ \text{ksi}$$

Normally the release strength should be less than 4.5 ksi. Higher values of up

[BPG] to 5.0 ksi may be allowed with permission of ADOT Bridge Group. For this problem the release strength is close enough. Therefore f'_{ci} = 4.7 ksi is acceptable.

[5.9.4.1.2-1]

Step 7 – Temporary Tension at Ends

The ends and hold-down points of the precast girders must be checked to ensure that the eccentricity is limited to keep any tension within the allowable limits. As with the compressive check, the end critical location will be at the end of the transfer length.

The allowable tension in the top of the precast girder without additional mild reinforcement equals:

$$\text{Allowable Tension} = 0.0948\sqrt{f'_{ci}} = 0.0948\sqrt{4.7} = 0.206 \text{ ksi} < 0.200 \text{ ksi}$$

<u>Transfer Length (1.75 feet from CL Brg)</u>

$$f_{si} = (0.75)(270) - 2.23 - 14.79 = 185.48 \text{ ksi}$$

$$f_t = \frac{(93)\cdot(12)\cdot(35.425)}{668{,}898} = 0.059 \text{ ksi}$$

$$f_t = (7.344)\cdot(185.48)\cdot\left[\frac{1}{933.66} - \frac{(17.415)\cdot(35.425)}{668{,}898}\right] + 0.059 = 0.262 \text{ ksi}$$

<u>Hold-Down Point (42.375 feet from CL Brg)</u>

$$f_{si} = (0.75)(270) - 2.23 - 18.65 = 181.62 \text{ ksi}$$

$$f_t = \frac{(1420)\cdot(12)\cdot(35.318)}{664{,}023} = 0.906 \text{ ksi}$$

$$f_t = (7.344)\cdot(181.62)\cdot\left[\frac{1}{933.66} - \frac{(31.182)\cdot(35.318)}{664{,}023}\right] + 0.906 = 0.122 \text{ ksi}$$

Since there is no tension in the top of the girder, the criteria is satisfied without adding mild reinforcing in the top of the girder.

Step 8 – Determine Final Concrete Strength

[5.9.4.2-1]

The required final concrete strength is determined after all prestress losses at the midspan and the transfer length. Service load combinations are used for the three compressive load cases.

Midspan

Midspan

f_{pe} = 0.75(270) – 54.58 = 147.92 ksi

F_{pe} = (7.344)(147.92) = 1086.32 k

Case I – Permanent Loads plus Effective Prestress

Allowable Compression = 0.45 f'$_c$ = (0.45)(5.0) = 2.250 ksi

$$f_t = (1086.32)\cdot\left[\frac{1}{933.66} - \frac{(31.182)\cdot(35.318)}{664{,}023}\right] + 0.959 + 1.105 + 0.100$$

f_t = 1.526 ksi < 2.250 ksi Allowable

Case II – One-half the Case I loads plus LL + IM

Allowable Compression = 0.40 f'$_c$ = (0.40)(5.0) = 2.000 ksi

$$f_t = \frac{1}{2}\cdot[1.526] + 0.374 = 1.137\ ksi$$

f_t = 1.137 ksi < 2.000 ksi Allowable

Case III – Effective Prestress, Permanent Loads and Transient Loads

Allowable Compression = 0.60 φ_w f'$_c$ = 0.60(1.00)(5.0) = 3.000 ksi

Since the I-girder is not a hollow section φ_w shall be taken equal to 1.0.

$$f_t = (1086.32)\cdot\left[\frac{1}{933.66} - \frac{(31.182)\cdot(35.318)}{664{,}023}\right] + 2.538 = 1.900\ \text{ksi}$$

f_t = 1.900 ksi < 3.000 ksi Allowable

Transfer Length

Transfer Length

f_{pe} = 0.75(270) – 51.02 = 151.48 ksi

F_{pe} = (7.344)(151.48) = 1112.47 k

Case I – Permanent Loads plus Effective Prestress

$$f_b = (1112.47)\cdot\left[\frac{1}{933.66} + \frac{(17.415)\cdot(36.575)}{668{,}898}\right] - 0.061 - 0.066 - 0.018$$
$$= 2.106 \text{ ksi}$$

f_b = 2.106 ksi < 2.250 ksi Allowable

Case II – One-half the Case I loads plus LL + IM

Allowable Compression = 0.40 f'_c = (0.40)(5.0) = 2.000 ksi

$$f_b = \frac{1}{2}\cdot[2.106] - 0.069 = 0.984\ ksi$$

f_b = 0.984 ksi < 2.000 ksi Allowable

Case III – Effective Prestress, Permanent Loads and Transient Loads

Allowable Compression = 0.60 φ_w f'_c = 0.60(1.00)(5.0) = 3.000 ksi

Since the I-girder is not a hollow section φ_w shall be taken equal to 1.0.

$$f_b = (1112.47)\cdot\left[\frac{1}{933.66} + \frac{(17.415)\cdot(36.575)}{668{,}898}\right] - 0.061 - 0.066 - 0.018 - 0.069$$

$$= 2.037 \text{ ksi}$$

f_b = 2.037 ksi < 3.000 ksi Allowable

Step 9 – Determine Final Concrete Tension

Determination of the tension in the concrete is a Service III Limit State. The allowable tension after all losses is limited to:

[BPG]

$$Allowable\ Tension = 0.0948\sqrt{f'_c} = 0.0948\sqrt{5.0} = 0.212\ ksi$$

The basic equation for stress in concrete is:

$$f = A_{ps}f_{se}\left[\frac{1}{A} + \frac{e_m y}{I}\right] - \frac{\sum(\gamma M)y}{I}$$

<u>Bottom fiber at Midspan</u>

$$f_b = (1086.32)\cdot\left[\frac{1}{933.66} + \frac{(31.182)\cdot(36.682)}{664{,}023}\right] - 3.149 = -0.114\ \text{ksi}$$

Since the tension is less than the allowable tension the criteria is satisfied.

Fatigue Limit State [5.5.3.1]

Fatigue of the reinforcement need not be checked for fully prestressed components designed to have extreme fiber tensile stress due to Service III Limit State within the tensile stress limit specified in Table 5.9.4.2.2-1.

Flexural Resistance [5.7.3]

Step 10 – Flexural Resistance

The flexural resistance of the I-girder must exceed the applied factored loads. Strength I is used.

$M_r = \varphi M_n < \sum \gamma M = M_u$

Midspan

STRENGTH I: M_u = 9023 ft-k

$A_{ps} = (0.153)(48) = 7.344 \text{ in}^2$

[5.7.3.1.1-1]

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$

[5.7.3.1.1-2]

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 2\left(1.04 - \frac{243}{270}\right) = 0.28 \text{ for low relaxation strand}$$

For a rectangular section without mild reinforcing steel:

d_p = 72.00 + 8.00 – 0.5 w.s. – 5.50 = 74.00 inches

When the depth of the stress block is contained in the top deck slab only:

[5.7.3.1.1-4]

$$c = \frac{A_{ps} f_{pu}}{0.85 f'_c \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}}$$

$$c = \frac{(7.344)\cdot(270)}{0.85\cdot(4.5)\cdot(0.825)\cdot(108.00) + 0.28\cdot(7.344)\cdot\dfrac{270}{74.00}} = 5.69 < t_{slab}$$

Since the stress block depth is less than the slab thickness, the section is treated as a rectangular section:

$$a = c\beta_1 = (5.69)\cdot(0.825) = 4.69 \ in$$

$$f_{ps} = (270)\cdot\left(1 - (0.28)\cdot\frac{5.69}{74.00}\right) = 264.19 \ ksi$$

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$

[5.7.3.2.2-1]

$$M_n = (7.344)\cdot(264.19)\cdot\left(74.00 - \frac{4.69}{2}\right) \div 12 = 11{,}585 \text{ ft-k}$$

The tensile strain must be calculated as follows:

$$\varepsilon_T = 0.003\left(\frac{d_T}{c} - 1\right) = 0.003\cdot\left(\frac{74.00}{5.69} - 1\right) = 0.036$$

[5.5.4.2.1]

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 1.00$.

$\varphi M_n = 1.0(11{,}585) = 11{,}585$ ft-k > 9023 ft-k

$\therefore$ Section is adequate for flexural strength

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions has eliminated this requirement. The tensile strain check and any adjustment of phi as required will satisfy this requirement.

Minimum Reinforcing [5.7.3.3.2]

There is also a minimum amount of reinforcement that must be provided in a section. The amount of prestressed and nonprestressed tensile reinforcement shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

$1.2\ M_{cr}$
or
$1.33\ M_u$

The cracking moment is determined on the basis of elastic stress distribution and the modulus of rupture of the concrete.

[5.7.3.3.2-1]

$$M_{cr} = S_c\left(f_r + f_{cpe}\right) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c f_r$$

The formula does not apply well to using transformed section properties since there are two non-composite dead load section moduli. Therefore gross section properties will be used.

$$S_c = \frac{I}{y_b} = \frac{1{,}328{,}521}{54.113} = 24{,}551 \text{ in}^3$$

$$S_{nc} = \frac{I}{y_b} = \frac{671{,}108}{36.439} = 18{,}417 \text{ in}^3$$

The net section properties with the inclusion of the elastic shortening loss in used to determine the effective stress due to prestress.

$$f_{pe} = (1086.32)\cdot\left[\frac{1}{933.66} + \frac{(31.182)\cdot(36.682)}{664{,}023}\right] = 3.035 \text{ ksi}$$

$$M_{cr} = (24{,}551)\cdot(0.827 + 3.035) \div 12 - (1773)\cdot\left(\frac{24{,}551}{18{,}417} - 1\right) = 7311 \text{ ft-k}$$

$$S_c f_r = (24{,}551)\cdot(0.827) \div 12 = 1692 \text{ ft-k}$$

$1.2\ M_{cr} = (1.2)(7311) = 8773 < \varphi M_n = 11{,}585$ ft-k

$\therefore$ The minimum reinforcing requirement is satisfied.

Positive Moment Continuity Connection

The girders must be connected at the bottom to resist any positive moment at the supports. This moment is caused by the restraint of the girders, as the ends tend to rotate due to creep and shrinkage. Usually strands are extended and hooked up into the cast-in-place diaphragm to resist this moment.

The LRFD Specification does not contain any direction as to design for these forces. Therefore, the design for the positive moment connection will follow the procedure outlined in the PCA publication "Design of Continuous Highway Bridges with Precast, Prestressed Concrete Girders", August 1969. Figures referenced PCA in this section refer to Figures in that publication. The end rotations from dead load, prestress and differential shrinkage from the cast-in-place slab are restrained by the continuity connection with appropriate creep factors considered. In addition negative moments will result from the composite dead load such as the parapet and sidewalk. For a two span bridge, positive moments will not result from live loads.

Creep Factor

Since creep is time dependent, the amount of positive restraint moment induced depends on the time when the continuity connection is made. The sooner the connection is made the higher the restraint moments will be. For design purposes 30 days will be used. For design, the ultimate creep value of 0.35×10^{-6} may be taken from PCA Figure 5 for a concrete with an initial modulus of 3946 ksi using the 20-year creep curve as the ultimate creep. This value must be modified to adjust for the effect of age when the girders are prestressed and for the volume/surface ratio.

Assuming the prestress is transferred to the concrete at day one, the creep adjustment factor is 1.80 from PCA Figure 6. The girder volume/surface ratio is 3.89. The creep adjustment factor for volume/surface ratio is 1.21 from PCA Figure 7.

The amount of creep that has occurred before the connection is made at an assumed time of 30 days is 40 percent from PCA Figure 8. This means that 60 percent of the creep occurs after the connection is made contributing to the restraint moment.

The adjusted creep strain is:

$$\varepsilon_s = (0.35 \times 10^{-6})(1.80)(1.21)(0.60) = 0.457 \times 10^{-6}$$

The effects of creep under prestress and dead load can be evaluated by standard elastic analysis methods by assuming the elements were cast and prestressed as a monolithic continuous girder. The variable φ is the ratio of creep strain to elastic strain. This value can be determined by multiplying the creep strain by the modulus of concrete as follows:

$$\varphi = (0.457 \times 10^{-6})(3946 \times 10^{3}) = 1.803$$

The continuity moments are then multiplied by the following factor to account for creep:

$$Creep\ Factor = \left(1 - e^{-\varphi}\right) = \left(1 - e^{-1.803}\right) = 0.835$$

Non-Composite DL and P/S

Once the girders are restrained, additional creep rotation from the non-composite dead load of the girder, diaphragms and deck slab will cause a restraint moment. This restraint moment is the moment at the support resulting from the analysis of a continuous beam with the weight of the girder, diaphragms and slab adjusted by the dead load creep factor. From the continuous beam analysis the resulting dead loads follow:

Girder	-1537 ft-k
Diaphragms	-103 ft-k
Slab & NC DL	-1655 ft-k
	-3295 ft-k

CR: Adjusted DL = (-3295)(0.835) = -2751 ft-k

The final prestress force is applied to the continuous beam resulting in a positive support moment of 5779 ft-k.

CR: Adjusted P/S = (5779)(0.835) = 4825 ft-k

Composite DL

The parapet and sidewalk will cause negative moments at the piers. However, since the loads are applied after the bridge has been made continuous and are not the result of creep restraint, there is no creep modification factor for these loads. The following pier moments result:

DC: Parapet & Sidewalk -358 ft-k

Differential Shrinkage

The remaining force is the differential shrinkage caused by the time delay between casting the girders and placing the deck. During this time, the girder shortens due to shrinkage. When the deck is cured the deck and girder will shorten together. However, the deck must undergo all its shrinkage while the girder has already seen much of its shortening. The deck will shorten more relative to the girder causing a positive moment along the span. This results in a negative restraint moment at the support.

When test data is not available, the ultimate shrinkage of concrete at a relative humidity of 50 percent can be estimated as 0.600 x 10^{-3}. This value must be corrected for humidity variances. For a relative humidity of 40 percent the correction factor is 1.09 from PCA Figure 10.

Assuming a 30 day lapse between casting the girders and placing the deck, the girder will have undergone 40 percent of its shrinkage as seen from PCA Figure 8. This means that the girder/deck system will see a differential shrinkage equal to 40 percent of the total shrinkage. The differential shrinkage strain is:

$\varepsilon_s = (0.600 \times 10^{-3})(1.09)(0.40) = 0.262 \times 10^{-3}$

The equation for the differential shrinkage moment applied to the girder along its entire length is:

$$M_{ds} = \varepsilon_s E_b A_b \left(y_t + \frac{t}{2} \right)$$

ε_s = differential shrinkage strain
E_b = elastic modulus for the deck slab concrete = 3861 ksi
A_b = area of deck slab = (7.50)(108) = 810 in^2
y_t = distance to the top of beam from the centroid of the gross composite section = 17.887 in

$$M_{ds} = \left(0.262x10^{-3}\right)\cdot\left(3861\right)\cdot\left(810\right)\cdot\left(17.887 + \frac{7.50}{2}\right) \div 12$$

M_{ds} = 1477 ft-k

For a 2 span continuous girder with equal spans, the support moment will equal 1.50 times the uniformly applied moment. Therefore the support moment equals –(1.50)(1477) = -2216 ft-k.

The negative support moment due to differential shrinkage is adjusted for creep by the following factor:

$$Creep\ Factor = \frac{\left(1 - e^{-\varphi}\right)}{\varphi} = \frac{\left(1 - e^{-1.803}\right)}{1.803} = 0.463$$

The support moment for differential shrinkage must be adjusted by the above creep factor resulting in:

CR: Adjusted Differential Shrinkage = (0.463)(-2216) = -1026 ft-k

Limit States [3.4.1]

Combining the above loads results in the following:

<u>Service I Limit State</u>

M_s = 1.0(DC) + 1.0(SH + CR)
M_s = 1.0(– 358) + 1.0(-2751 + 4825 - 1026) = 690 ft-k

<u>Strength I Limit State</u>

M_u = 0.90(DC) + 0.5(SH + CR)
M_u = 0.90(– 358) + 0.5(-2751 + 4825 -1026) = 202 ft-k

The prestress strands will be extended to resist the positive moment. The strands will be designed based on the criteria in Report No. FHWA-RD-77-14, "End Connections of Pretensioned I-Beam Bridge", November 1974.

Service I Limit State

Try extending 8 strands (4 in the bottom row and 4 in the second row). For a service limit check:

A_s = 8(0.153) = 1.224 in^2
c.g. = [4(2.0) + 4(4.0)] / 8 = 3.00 in
d = 80.00 – 0.5 ws – 3.00 = 76.50 inch

$$p = \frac{A_s}{bd} = \frac{1.224}{(108.00)\cdot(76.50)} = 0.000148$$

np = (7) (0.000148) = 0.00104

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.00104) + (0.00104)^2} - 0.00104 = 0.045$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.045}{3} = 0.985$$

$$f_s = \frac{M_s}{A_s jd} = \frac{(690)\cdot(12)}{(1.224)\cdot(0.985)\cdot(76.50)} = 89.77 \text{ ksi}$$

From test data in the research report, the recommended development length due to service loads for strands bent 90° over a reinforcing bar is:

$L_e = 0.228 f_s + 8.25$" when $L_{pb} \le 8.25$"

$$L_e = 0.225\left[f_s - \frac{L_{pb} - 8.25}{0.472}\right] + L_{pb} \text{ when } L_{pb} > 8.25"$$

Normally the gap between girders is 12 inches. With two rows of strands extended, the lower one is extended 10 inches, while the upper is extended 8 inches. To simplify the design assume that both rows are extended only 8 inches. Therefore L_{pb}, the length to the bend, is ≤ 8.25 inches.

$$L_e = (0.228)(89.77) + 8.25 = 28.7 \text{ inches}$$

Strength I Limit State

The strength limit state must also be checked. From the research report an upper limit of 150 ksi is placed on the stress in the strand with the required development length as follows:

$$L_e = 0.163 f_{ps} + 8.25 \text{ when } L_{pb} \le 8.25"$$

$$L_e = 0.163\left[f_{ps} - \frac{L_{pb} - 8.25}{0.337} \right] + L_{pb} \text{ when } L_{pb} > 8.25"$$

Try a 29 inch extension and rearrange the equation to solve for f_{ps}.

$$f_{ps} = \frac{L_e - 8.25}{0.163} = \frac{29 - 8.25}{0.163} = 127.30 \text{ ksi} \le 150 \text{ ksi}$$

$$a = \frac{A_s f_{ps}}{0.85 f'_c b} = \frac{(1.224)\cdot(127.30)}{0.85\cdot(4.5)\cdot(108.00)} = 0.38 \text{ in}$$

$$\varphi M_n = \phi A_s f_{ps}\left(d - \frac{a}{2}\right) = (0.90)\cdot(1.224)\cdot(127.30)\cdot\left(76.50 - \frac{0.38}{2}\right) \div 12$$

$$\varphi M_n = 892 \text{ ft-k} > M_u = 202 \text{ ft-k}$$

Therefore, the service and strength limit states are satisfied by extending 8 strands a total of 29 inches.

Negative Moment Continuity Reinforcement

The precast prestressed I-girder behaves as a simple span under self-weight and the non-composite dead loads. However, this bridge type is made continuous to eliminate the expansion joints and improve the riding surface of the deck. Continuity is provided by designing an adequate amount of mild reinforcing steel in the top slab of the deck to resist the negative moments from the composite dead loads, live load plus dynamic load allowance and any creep or shrinkage restraint moment.

To maximize the negative moment, the restraint moment should be determined at a time of 120 days. This longer time will produce a greater negative shrinkage restraint moment than the normal 60 days assumed for the deck pour and the 30 days assumed for positive connection design. The method of determining the shrinkage restraint forces was shown in the previous section on positive moment continuity connection.

Creep Factor

The only variable that changes from the previous calculation is the time used for creep. The amount of creep that has occurred before the connection is made, at an assumed time of 120 days, is 65 percent from PCA Figure 8. This means that 35 percent of the creep occurs after the connection is made contributing to the restraint moment.

The adjusted creep strain is:

$$\varepsilon_s = \left(0.35\ x\,10^{-6}\right)\cdot\left(1.80\right)\cdot\left(1.21\right)\cdot\left(0.35\right) = 0.267\ x\,10^{-6}$$

The variable φ is determined as follows:

$$\varphi = (0.267 \text{ x } 10^{-6})\ (3946 \text{ x } 10^{3}) = 1.054$$

The continuity moments are then multiplied by the following factor to account for creep:

$$Creep\ Factor = \left(1 - e^{-\varphi}\right) = \left(1 - e^{-1.054}\right) = 0.651$$

CR: Adjusted DL = (-3295)(0.651) = -2145 ft-k

CR: Adjusted P/S = (5779)(0.651) = 3762 ft-k

Differential Shrinkage

Assuming a 120 day lapse between casting the girders and placing the deck, the girder will have undergone 65 percent of its shrinkage as seen from PCA Figure 8. This means that the girder/deck system will see a differential shrinkage equal to 65 percent of the total shrinkage. The differential shrinkage strain is:

$$\varepsilon_s = (0.600 \text{ x } 10^{-3})(1.09)(0.65) = 0.425 \text{ x } 10^{-3}$$

The differential shrinkage moment equals:

$M_{ds} = (0.425 \times 10^{-3})(3861)(810)(17.887 + 7.50 / 2) \div 12$
$M_{ds} = 2397$ ft-k

The negative support moment due to differential shrinkage is adjusted for creep by the following factor:

$$Creep\ Factor = \frac{(1 - e^{-\varphi})}{\varphi} = \frac{(1 - e^{-1.054})}{1.054} = 0.618$$

From a continuous beam analysis of uniformly applied moment from differential shrinkage, the support moment equals –(1.50)(2397) = –3596 ft-k. This value must be adjusted by the creep factor of 0.618 resulting in a moment equal to (0.618)(-3596) = -2222 ft-k.

A continuous beam analysis is made for DC, DW and LL+IM for two 112 foot spans. For an interior girder the live load from one vehicle is modified by the distribution factor of 0.748 and the skew reduction factor of 0.956. The sum of the creep and shrinkage moments equal -2145 + 3762 –2222 = -605 ft-k

Strength I Limit State

For the Strength I Limit State where DC and DW moments are positive the FWS is ignored and the following equation applies:

$M_u = 0.90(DC) + 1.75(LL + IM) + 0.5(CR + SH)$

When both DC and DW are negative the following equation applies:

$M_u = 1.25(DC) + 1.50(DW) + 1.75(LL + IM) + 0.5(CR + SH)$

A summary of negative moments follows:

Span 1

	0.4	0.5	0.6	0.7	0.8	0.9	1.0
DC (Parapet)	200	179	129	50	-57	-193	-358
DW (FWS)	156	140	100	39	-45	-151	-279
One Vehicle	-602	-752	-903	-1053	-1203	-1493	-2648
LL + IM	-430	-538	-646	-753	-860	-1068	-1894
SH + CR	-242	-303	-363	-424	-484	-545	-605
Service I	-472	-662	-880	-1127	-1446	-1957	-3136
Strength I	-694	-932	-1196	-1485	-1886	-2609	4483

Negative Moment Design

Even though the top surface serves as the deck, the allowable stress in the longitudinal direction is not limited to 24 ksi as is the case for transverse reinforcing. Therefore the service limit state may not control the design.

Service I Limit State [3.4.1]

$$M_s = 1.0(M_{DC} + M_{DW}) + 1.0(M_{LL+IM}) + 1.0(M_{SH+CR})$$

Composite Loads:

M_{LL+IM} = (-2648)(0.748)(0.956) = -1894 ft-k

M_s = 1.0(-358 - 279) + 1.0(-1894) +1.0(-605) = -3136 ft-k

Allowable Stress

Try #8 reinforcing bars

d_s = 80.00 – 2.50 clear – 0.625 – 1.00 / 2 = 76.38 inches

Since there is no direct check for the allowable stress, assume a stress of 36 ksi maximum with the understanding that the resulting area of steel may have to be adjusted.

$$A_s \approx \frac{M_s}{f_s jd_s} = \frac{(3136)\cdot(12)}{(36.0)\cdot(0.9)\cdot(76.38)} = 15.21 \text{ in}^2$$

Try #8 @ 6 inches

A_s = (18)(0.79) = 14.22 in^2 per girder

Determine stress block depth assuming a rectangular section.

$$p = \frac{A_s}{bd_s} = \frac{14.22}{(26.00)\cdot(76.38)} = 0.00716$$

np = 7(0.00716) = 0.05012

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.05012) + (0.05012)^2} - 0.05012 = 0.270$$

kd = (0.270)(76.38) = 20.62 in > 8.00 inch deep flange of girder

Since the depth of the stress block exceeds the depth of the flange, the section must be treated as T-section. To simplify the analysis a conservative assumption is made to ignore the 10 inch triangular flanges and treat the section as a T-section with a 26 inch wide by 8 inch deep flange and a 6 inch web as shown in Figure 17.

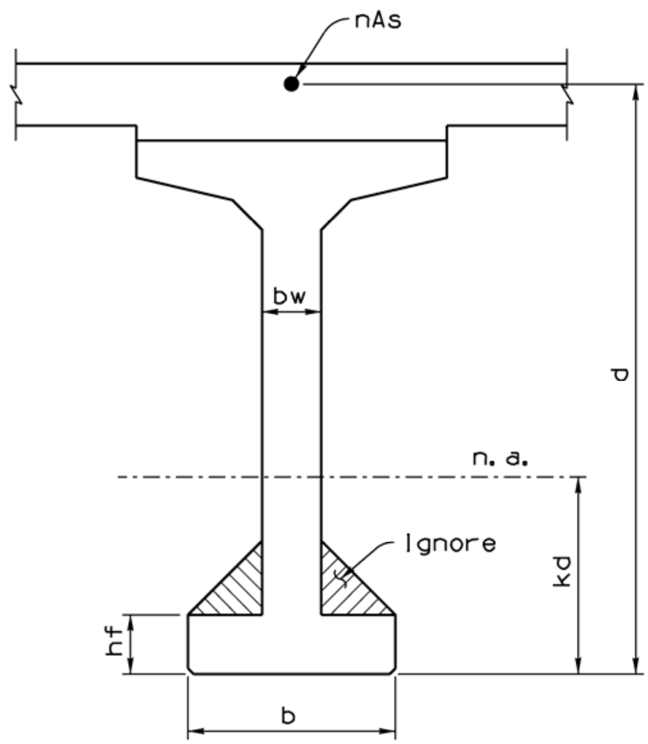


Figure 17

Transform the area of reinforcing into an equivalent area of concrete and take moments about the neutral axis resulting in the following equation:

$nA_s(d - kd) = (b - b_w)h_f(kd - h_f \div 2) + b_w(kd)^2 \div 2$

$(b_w \div 2)(kd)^2 + [(b - b_w)h_f + nA_s](kd) - (b - b_w)h_f^2 \div 2 - nA_s d = 0$

Solving the quadratic equation for kd results in the following coefficients:

$A = b_w \div 2 = 6.00 \div 2 = 3.00$

$B = (b - b_w)h_f + nA_s = (26 - 6)(8.00) + 7(14.22) = 259.54$

$C = -(b - b_w)h_f^2 \div 2 - nA_s d = -(26 - 6)(8.00)^2 \div 2 - 7(14.22)(76.38)$
$= -8243$

$$kd = \frac{-259.54 + \sqrt{(259.54)^2 - 4 \cdot (3.00) \cdot (-8243)}}{2 \cdot (3.00)} = 24.71 \text{ in}$$

Determine the moment of inertia of the cracked section about the neutral axis as follows:

$$I_{cr} = nA_s\left(d-kd\right)^2 + \frac{b_w\left(kd\right)^3}{3} + \frac{\left(b-b_w\right)h_f^{\,3}}{12} + \left(b-b_w\right)h_f\left(kd-\frac{h_f}{2}\right)^2$$

I_{cr} = (7)(14.22)(76.38 – 24.71)^2 + (6.00)(24.71)^3÷3 + (26 – 6)(8.00)^3÷12 + (26 – 6)(8.00)(24.71 – 8.00 ÷2)^2 = 365,404 in^4

y_t = 76.38 – 24.71 = 51.67 in

$$f_s = \frac{(7)\cdot(3136)\cdot(12)\cdot(51.67)}{365{,}404} = 37.25\ \text{ksi}$$

There is no direct allowable stress limit in the LRFD Specification. The Bridge Group limit of 24 ksi for decks applies to the transverse reinforcing not the longitudinal reinforcing. However, the reinforcing stress is required in determining the maximum allowable reinforcing spacing.

Control of Cracking [5.7.3.4]

For all concrete components in which the tension in the cross-section exceeds 80 percent of the modulus of rupture at the service limit state load combination the maximum spacing requirement in equation 5.7.3.4-1 shall be satisfied.

f_{sa} = 0.80f_r = 0.80($0.24\sqrt{f'_c}$) = 0.80(0.509) = 0.407 ksi

$$f_{cr} = \frac{M_s y_t}{I_g} = \frac{(3136)\cdot(12)\cdot(17.887+7.5)}{1{,}328{,}521} = 0.719\ ksi\ > f_{sa} = 0.407\text{ ksi}$$

Since the service limit state cracking stress exceeds the allowable, the spacing, s, of mild steel reinforcing in the layer closest to the tension force shall satisfy the following:

[5.7.3.4-1]

$$s \le \frac{700\gamma_e}{\beta_s f_s} - 2d_c$$

where

γ_e = 0.75 for Class 2 exposure condition for decks

d_c = 2.5 clear + 0.625 + 1.00 ÷ 2 = 3.63 inches

f_s = 37.25 ksi

h = 80.00 inches

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)} = 1 + \frac{3.63}{0.7 \cdot (80.00 - 3.63)} = 1.07$$

$$s \le \frac{(700)\cdot(0.75)}{(1.07)\cdot(37.25)} - (2)\cdot(3.63) = 5.91 \text{ in}$$

Since the spacing of 6.0 inches is approximately equal to 5.91 inches (1.5% overstress), the cracking criteria is satisfied.

Strength I Limit State [3.4.1]

Factored moment for Strength I is as follows:

$$M_u = \gamma_{DC}(M_{DC}) + \gamma_{DW}(M_{DW}) + 1.75(M_{LL+IM}) + 0.5(M_{SH+CR})$$

$$M_u = 1.25 \cdot (-358) + 1.50 \cdot (-279) + 1.75 \cdot (-1894) + 0.5 \cdot (-605)$$
M_u = -4483 ft-k

Verify section type by calculating depth of rectangular stress block.

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b_w}$$

$$c = \frac{(14.22)\cdot(60)}{0.85 \cdot (5.0)\cdot(0.800)\cdot(26.00)} = 9.65 \text{ in} > 8.00 \text{ in flange depth}$$

Since the depth is greater than the flange depth the section must be treated as a T-section. For a T-section the neutral axis is located at the depth c as follows:

[5.7.3.1.1-3]

$$c = \frac{A_s f_y - 0.85 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w}$$

$$c = \frac{(14.22)\cdot(60) - 0.85 \cdot (5.0)\cdot(26-6)\cdot(8.00)}{0.85 \cdot (5.0)\cdot(0.800)\cdot(6.00)} = 8.49 \text{ in}$$

The flexural resistance of a reinforced concrete T-section is:

$$M_r = \phi M_n$$

[5.7.3.2.2-1]

$$M_n = A_s f_y \left(d_s - \frac{a}{2}\right) + 0.85 f'_c (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

[5.7.3.2.3]

$$a = \beta_1 c = (0.800)\cdot(8.49) = 6.79 \text{ in}$$

$$M_n = (14.22)\cdot(60)\cdot\left(76.38 - \frac{6.79}{2}\right)\div 12$$

$$+0.85\cdot(5.0)\cdot(26-6)\cdot(8.00)\cdot\left(\frac{6.79}{2} - \frac{8.00}{2}\right)\div 12 = 5155 \text{ ft-k}$$

The neutral axis is close to the flange depth and the β_1 factor was eliminated from the numerator of the equation for 'c' in the 2006 Interims. This results in the unusual situation where the depth of the stress block in the web is less than the flange thickness producing a negative number for the second term above.

The tensile strain must be calculated as follows:

$$\varepsilon_T = 0.003\left(\frac{d_t}{c} - 1\right) = 0.003\cdot\left(\frac{76.38}{8.49} - 1\right) = 0.024$$

[5.5.4.2.1]

Since $\varepsilon_T > 0.005$, the member is tension controlled and $\varphi = 0.90$ for the reinforced member.

$$M_r = (0.90)\cdot(5155) = 4640 \text{ ft-k}$$

Since the flexural resistance, M_r = 4640 ft-k, is greater than the factored moment, M_u = 4483 ft-k, the strength limit state is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The 2006 Interim Revisions eliminated this requirement.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 M_u. Use the composite gross section properties for this calculation.

S_c = (1,328,521) / (17.887 + 7.50) = 52,331 in^3

$$1.2M_{cr} = 1.2 f_r S_c = (1.2)\cdot(0.785)\cdot(52{,}331)\div 12 = 4108 \text{ ft-k}$$
$$1.2M_{cr} = 4108 \le M_r = 4640 \text{ ft-k}$$

$\therefore$ The minimum reinforcement limit is satisfied.

Fatigue

The stress range in the continuous reinforcing over the pier must be checked for fatigue.

Fatigue Load [3.6.1.4]

The fatigue load shall be one design truck but with a constant 30.0 feet between the 32.0 kip axles. The dynamic load allowance of 15% shall be applied to the fatigue load. From the live load generator, the maximum fatigue truck moment including dynamic load allowance at the support is -813 ft-k. The minimum fatigue truck moment is zero since there can be no positive live load moment at the support in a 2 span bridge.

The live load distribution for an interior girder with one design lane loaded is 0.513.

Multiple Presence Factor [3.6.1.1.2]

The multiple presence factor is already included in the distribution factor. Therefore when investigating fatigue, the force effect is divided by 1.20.

Fatigue Limit State [3.4.1]

$$M_f = 0.75M_{LL+IM} = (0.75)(813)(0.513)(0.956) / 1.20 = 249 \text{ ft-k}$$

$$f_{range} = \frac{(7)\cdot(249)\cdot(12)\cdot(51.67)}{365{,}404} = 2.96 \text{ ksi}$$

[5.5.3.2-1]

$$f_f = 21 - 0.33 f_{\min} + 8\left(\frac{r}{h}\right)$$

When the actual value of r / h is not known use a value of 0.30.

$$f_{\min} = 2.96 + \frac{(7)\cdot(358+279)\cdot(12)\cdot(51.67)}{365{,}404} = 10.53 \text{ ksi}$$

$$f_f = 21 - (0.33)\cdot(10.53) + 8\cdot(0.30) = 19.93 \text{ ksi}$$

Since the stress range of 2.96 ksi is less than the allowable fatigue stress of 19.93 ksi, the fatigue criteria for the reinforcing is satisfied.

The superstructure is adequately reinforced for negative moment using #8 at 6 inches over the pier. This reinforcing can be reduced along the span based on the negative moment diagram considering the service, strength and fatigue limit states. The reinforcing anchorage requirements in 5.14.1.2.7b shall be satisfied.

Shear
[5.8]

The LRFD method of shear design is a complete change from the methods specified in the Standard Specifications and that used by ADOT. For this example an in-depth shear design will be performed at the critical locations in Span 1 near the abutment and pier.

Critical Section
[5.8.3.2]

d_v
[5.8.2.9]

The critical location is located a distance d_v from the face of the support. This creates a problem in that d_v is largest of three values, two of which are a function of distance from the support. To eliminate the iterative process in determining the critical shear location, a simplification is required. It is recommended that the equation, $d_v = 0.72$ h be used to determine the critical shear location. Since d_v is the larger of the three values determined in Step 3, using $d_v = 0.72h = (0.72)(72.0 + 7.50) / 12 = 4.77$ feet will be conservative.

Step 1 – Determine Shear

Shears and moments from a simple span analysis will be slightly higher near the abutment than from a continuous beam analysis and will be used in this analysis. Shears and moments from a continuous beam analysis will be used near the pier. The more critical design will be used in the final solution to obtain a symmetrical shear reinforcing pattern.

Abutment Shear

For a uniform load, w, distributed along the entire span, L, the shear at a distance x from the support is: $V_x = (w)(L / 2 – x)$.

Simple span shears 4.77 feet from the centerline bearing of abutment for an interior girder are determined as follows:

Girder	$V_{Crit} = 0.980\ [(110.75) / 2 – 4.77]$	= 49.6 kips
Diaphragm	$V_{Crit} = 4.92(1 / 2)$	= 2.5 kips
Non-Comp	$V_{Crit} = 1.068\ [(110.75) / 2 – 4.77]$	= 54.0 kips
Comp DL	$V_{Crit} = 0.228\ [(110.75) / 2 – 4.77]$	= 11.5 kips
DC	$V_{Crit} = 49.6 + 2.5 + 54.0 + 11.5$	= 117.6 kips
DW	$V_{Crit} = 0.178\ [(110.75) / 2 – 4.77]$	= 9.0 kips

For a partially distributed uniform load starting at a distance x from the support the shear at a distance x from the support is: $V_x = (w)(L – x)^2 \div (2L)$

Design Lane	$V_{Crit} = [(0.640)(110.75 - 4.77)^2\ ] \div[2(110.75)]$	= 32.5 k
Design Truck	$V_{Crit} = [32(105.98) + 32(91.98) + 8(77.98)] \div 110.75$	= 62.8 k
Design Tandm	$V_{Crit} = [25(105.98) + 25(101.98)] \div 110.75$	= 46.9 k
Vehicle	$V_{Crit} = 32.5 + 1.33(62.8)$	= 116.0 k
Ped LL	$V_{Crit} = [(0.450)(110.75 – 4.77)^2\ ] \div[2(110.75)]$	= 22.8 k

The LRFD Specification has made major changes to the live load distribution factors. There are different factors for flexure and shear. For precast prestressed concrete girders with a cast-in-place concrete deck, the typical cross section is identified as Type (k).

The range of applicability of all variables is within the allowable as shown below:

	Applicable Range
S = spacing of girders = 9.00 ft	$3.5 \le S \le 16.0$
L = span length of girder = 110.75 ft.	$20 \le L \le 240$
t_s = deck slab thickness = 7.50 in	$4.5 \le t_s \le 12.0$
N_b = Number Girders = 9	$N_b \ge 4$

The live load distribution factor for shear will be determined based on the requirements for both an interior and exterior girder. The more critical of the two values will be used in the design. The distribution of live load per lane for shear for an interior beam with one design lane loaded is:

Interior LL Distribution [4.6.2.2.3a-1]

$$LL\ Distribution = 0.36 + \frac{S}{25.0} = 0.36 + \frac{9.00}{25.0} = 0.720$$

The distribution for two or more design lanes loaded is:

[4.6.2.2.3a-1]

$$LL\ Distribution = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0} = 0.2 + \frac{9.00}{12} - \left(\frac{9.00}{35}\right)^{2.0} = 0.884$$

Exterior LL Distribution [4.6.2.2.3b-1]

The distribution of live load per lane for shear for an exterior beam with one design lane loaded is determined by the Lever Rule. The multiple presence factor must also be applied. See Figure 15. For the current configuration of the bridge the live load distribution factor is:

g = [(3.00) / (9.00)][1.20] = 0.400

The distribution of live load per lane for shear for an exterior beam with two or more design lanes loaded is:

$$g = e\, g_{interior}$$

$$e = 0.6 + \frac{d_e}{10} \text{ where } -1.0 \le d_e \le 5.5$$

Since d_e equals –4.00 and is outside the range of applicability for the distribution formula the formula cannot be used.

For girder bridges with diaphragms see the additional criteria for live load distribution factors for moments where R = 0.462 with the pedestrian load and R = 0.567 without the pedestrian load.

Skew Effect [4.6.2.2.3c-1]

For skewed bridges, the shear shall be adjusted to account for the effects of shear. The range of applicability of all variables is within the allowable as shown below:

	Applicable Range
θ = skew = 30 degrees	$0° \leq \theta \leq 60°$
S = spacing of girders = 9.00 ft	$3.5 \leq S \leq 16.0$
L = span length of girder = 110.75 ft.	$20 \leq L \leq 240$
N_b = Number Girders = 9	$N_b \geq 4$

For a 30 degree skew, the correction factor equals:

$$Skew\ Effect = 1.0 + 0.20\left(\frac{12.0 L t_s{}^3}{K_g}\right)^{0.3} \tan\theta$$

$$Skew\ Effect = 1.0 + 0.20\left(\frac{(12.0)\cdot(110.75)\cdot(7.50)^3}{2{,}240{,}331}\right)^{0.3} \tan(30) = 1.076$$

Live Load Shear

LL+IM

Interior Girder

V_{Crit} = (116.0)(0.884)(1.076) = 110.3 kips

Exterior Girder

V_{Crit} = (116.0)(0.462)(1.076) + (22.8)(0.889)(1.076) = 79.5 kips

V_{Crit} = (116.0)(0.567)(1.076) = 70.8 kips

By inspection the interior girder will control.

Strength I Limit State [3.4.1]

V_u = 1.25(117.6) + 1.50(9.0) + 1.75(110.3) = 353.5 kips

Sectional Model [5.8.3]

Step 2 – Determine Analysis Model

The sectional model of analysis is appropriate for the design of typical bridge girders where the assumptions of traditional beam theory are valid. Where the distance from the point of zero shear to the face of the support is greater than 2d the sectional model may be used. Otherwise, the strut-and-tie model should be used. Assume a 12 inch long bearing pad.

For Simple spans:

Point of Zero Shear to Face of Support = 110.75 / 2 – 0.50 = 54.88 ft

2d = 2(79.50 / 12) = 13.25 ft < 54.88 ft

$\therefore$ Sectional model may be used.

Step 3 – Shear Depth, d_v

[5.8.2.9]

The shear depth is the maximum of the following criteria:

1) $d_v = 0.9\, d_e$ where $d_e = \dfrac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = d_p$ when $A_s = 0$

For the determination of the shear depth ignore the harped strands since they are located in a compressive zone at the critical section.

$$c.g. = \frac{(8)\cdot(8.0)+(10)\cdot(6.0)+(10)\cdot(4.0)+(8)\cdot(2.0)}{36} = 5.00 \text{ in}$$

$$d_p = 79.50 - 5.00 = 74.50\ in$$

$$d_v = 0.9 d_p = 0.9\cdot(74.50) = 67.05 \text{ in}$$

2) 0.72h = 0.72(79.50) = 57.24 in

[C5.8.2.9-1]

3) $d_v = \dfrac{M_n}{A_s f_y + A_{ps} f_{pu}}$

At the critical location, 4.77 feet from the abutment, the 12 harped strands are in the compression zone and will be ignored for strength calculations.

A_s = (0.153)(36) = 5.508 in^2

[5.7.3.1.1-4]

$$c = \frac{(5.508)\cdot(270)}{0.85\cdot(4.5)\cdot(0.825)\cdot(108.00)+0.28\cdot(5.508)\cdot\dfrac{270}{74.50}} = 4.29\ in$$

Since c = 4.29 inches is less than the depth of the top slab = 7.50 inches, the section can be treated as a rectangular section.

$$a = c\beta_1 = (4.29)(0.825) = 3.54 \text{ in}$$

[5.7.3.1.1-1]

$$f_{ps} = (270)\cdot\left[1-(0.28)\cdot\frac{4.29}{74.50}\right] = 265.65 \text{ ksi}$$

The effect of the bond on the strand stress at the end of the girder must be considered. The embedment length, l_{px} = 9.00 + 4.77(12) = 66.24 inches is greater than the transfer length of 30 inches. The development length must also be satisfied as follows:

[5.11.4.2-1]

$$l_d \geq k\left(f_{ps} - \frac{2}{3} f_{pe}\right)d_b$$

k = 1.6 for pretensioned members with a depth greater than 24.0 inches.

f_{pe} = effective stress in the prestress steel after losses.
f_{pe} = (0.75)(270) – 51.02 = 151.48 ksi

f_{ps} = average stress in prestress steel at the time for which the nominal resistance of the member is required = 265.65 ksi.

$$l_d \geq (1.6)\cdot\left(265.65 - \frac{2}{3}\cdot 151.48\right)\cdot(0.5) = 131.7 \text{ inches} = 10.98 \text{ feet}$$

Since the strand is not fully developed, the flexural resistance is reduced. This is a complex problem to solve. AASHTO has an equation to determine the stress in the strand as a function of embedment length as shown below:

[5.11.4.2-4]

$$f_{px} = f_{pe} + \frac{(l_{px} - 60d_b)}{(l_d - 60d_b)}(f_{ps} - f_{pe})$$

$$f_{px} = 151.48 + \frac{(66.24 - 60\cdot 0.5)}{(131.7 - 60\cdot 0.5)}\cdot(265.65 - 151.48) = 192.16 \text{ ksi}$$

This reduced strand stress will result in a change in the neutral axis and resulting stress block depth.

$$c = \frac{(5.508)\cdot(192.16)}{0.85\cdot(4.5)\cdot(0.825)\cdot(108)+0.28\cdot(5.508)\cdot\frac{270}{74.50}} = 3.06 \text{ in}$$

a = (0.825)(3.06) = 2.52 in

For a rectangular section:

$$d_v = \frac{M_n}{A_{ps} f_{ps}} = \frac{A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)}{A_{ps} f_{ps}} = d_p - \frac{a}{2} = 74.50 - \frac{2.52}{2} = 73.24 \text{ in}$$

A lot of effort is required to determine d_v by this third option. It is recommended that only the first two methods be used for sections investigated within the development length of the girder ends unless shear is the controlling element in the design of the member.

Based on the above, the shear depth, $d_{v,}$ controlled by criteria 1, equals 67.05 inches.

Step 4 – Calculate, V_p

Due to the strand upturn at the hold-down points, some of the prestress force is in the upward vertical direction and directly resists the applied shear. Since the critical section for shear is located beyond the transfer length, the effective prestress force is used. Since the critical section for shear is near the transfer length, the transfer length losses are used. See Figure 12 for the angle of the cable path.

$$\alpha = \frac{(58.00)}{12\cdot(43.125)} = 0.11208 \text{ rads}$$

V_p = (12)(0.153) [(0.75)(270) – 51.02] (0.11208) = 31.2 kips

Step 5 – Check Shear Width, b_v

The LRFD Specification requires that web width be checked for minimum width to protect against crushing.

[5.8.3.3-2]

$$V_u <= \varphi V_n = \varphi(0.25f'_c b_v d_v + V_p)$$

$$\text{Required } b_v = \frac{\frac{353.5}{0.9} - 31.2}{(0.25)\cdot(5.0)\cdot(67.05)} = 4.31 \text{ inches}$$

Available b_v = 6.00 inches, ok

Step 6 – Evaluate Shear Stress

The equation for shear stress follows:

[5.8.2.9-1]

$$v_u = \frac{V_u - \varphi V_p}{\varphi b_v d_v} = \frac{353.5 - 0.90\cdot(31.2)}{0.90\cdot(6.00)\cdot(67.05)} = 0.899 \text{ ksi}$$

$$\frac{v_u}{f'_c} = \frac{0.899}{5.0} = 0.180$$

Step 7 – Estimate Crack Angle θ

The LRFD method of shear design usually involves several cycles of iteration. The first step is to estimate a value of θ, the angle of inclination of diagonal compressive stress. Since the formula is not very sensitive to this estimate assume that θ = 26.5 degrees. This simplifies the equation for the first iteration by setting the coefficient 0.5 cotθ = 1.0.

Step 8 – Calculate strain, ε_x

There are two formulae for the calculation of strain for sections containing at least the minimum amount of transverse reinforcing. The first formula is used for positive values of strain indicating tensile stresses, while the second formula is used for negative values of strain indicating compressive stresses.

Formula for ε_x for positive values:

[5.8.3.4.2-1]

$$\varepsilon_x = \left[\frac{\dfrac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}}{2(E_s A_s + E_p A_{ps})} \right]$$

Formula for e_x for negative values:

[5.8.3.4.2-2]

$$\varepsilon_x = \left[\frac{\dfrac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps} f_{po}}{2(E_c A_c + E_s A_s + E_p A_{ps})} \right]$$

General Procedure [5.8.3.4.2]

where:

A_c = area of concrete on the flexural tension side of the member. The tension side of the member is half the total depth = 0.5(79.50) = 39.75 in.
A_c = (26)(8) + 2(1/2)(10.0)(10.0) + (6.0)(31.75) = 498.5 in^2

A_{ps} = area of prestressing steel on the flexural tension side of the member.
A_{ps} = 0.153(36) = 5.508 in^2

A_s = area of nonprestressed steel on the flexural tension side of the member. A_s = 0.

f_{po} = a parameter taken as the modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete. For the usual levels of prestressing, a value of 0.7 f_{pu} will be appropriate where the section investigated is beyond the transfer length of 2.50 feet.

f_{po} = 0.70(270) = 189 ksi

N_u = factored axial force taken as positive if tensile.
N_u = 0 kips

V_u = factored shear force.
V_u = 353.5 kips

M_u = factored moment, not to be taken less than $V_u d_v$.

For a uniformly distributed load, the moment at a distance x from the support is:

$M_x = (w)\,(x)(L - x) \div 2$

The moments at the critical shear location are calculated below:

Girder	$M_{Crit} = 0.980(4.77)(110.75 - 4.77) / 2$	= 248 ft-k
Diaphragm	$M_{Crit} = 4.92(1 / 2)(4.77)$	= 12 ft-k
Non-Comp	$M_{Crit} = 1.068(4.77)(110.75 - 4.77) / 2$	= 270 ft-k
Comp DL	$M_{Crit} = 0.228(4.77)(110.75 - 4.77) / 2$	= 58 ft-k

DC $M_{Crit} = 248 + 12 + 270 + 58 = 588$ ft-k

DW $M_{Crit} = 0.178(4.77)(110.75 - 4.77) / 2 = 45$ ft-k

Design Lane $M_{Crit} = (32.5)(4.77) = 155$ ft-k
Design Truck $M_{Crit} = (62.8)(4.77) = 300$ ft-k
Design Tandm $M_{Crit} = (46.9)(4.77) = 224$ ft-k

LL+IM $M_{Crit} = [155 + 1.33(300)](0.748)(0.956) = 396$ ft-k

$M_{Crit} = 1.25(588) + 1.50(45) + 1.75(396) = 1496$ ft-k

$M_u = 1496 \text{ ft-k} < (353.5)(67.05) / 12 = 1975 \text{ ft-k}$

[5.8.3.4.2-1]

$$\varepsilon_x = \left[\frac{\dfrac{|1975\cdot 12|}{67.05} + 0 + 1.0\cdot|353.5 - 31.2| - (5.508)\cdot(189)}{2(29000\cdot 0 + 28500\cdot 5.508)}\right]$$

$\varepsilon_x = -0.00116 = -1.16 \text{ x } 1000$

Since the value is negative the second formula must be used. Note that for a prestressed member the second formula will usually be used.

[5.8.3.4.2-2]

$$\varepsilon_x = \left[\frac{\dfrac{|1975\cdot 12|}{67.05} + 0 + 1.0|353.5 - 31.2| - (5.508)\cdot(189)}{2(4070\cdot 498.5 + 28500\cdot 5.508)}\right]$$

$\varepsilon_x = -0.000084 = -0.084 \text{ x } 1000$

[5.8.3.4.2-1]

Now go into [T5.8.3.4.2-1] to read the values for θ and β. From the previously calculated value of $v_u / f'_c = 0.180$, enter the ≤ 0.200 row and from the calculated value of $\varepsilon_x = -0.084 \times 1000$, enter the ≤ -0.05 column. The new estimate for values is shown below:

θ = 26.7 degrees
β = 2.52

With the new value of θ, the strain must be recalculated.

$$\varepsilon_x = \left[\frac{\frac{|1975 \cdot 12|}{67.05} + 0 + 0.5 \cdot |353.5 - 31.2| \cot(26.7) - (5.508) \cdot (189)}{4{,}371{,}746} \right]$$

$\varepsilon_x = -0.000084 = -0.084 \times 1000$

With this new estimate for strain, reenter the table and determine new values for θ and β. Since our new values are the same as assumed, our iterative portion of the design is complete.

Step 9 - Calculate Concrete Shear Strength, V_c

The nominal shear resistance from concrete, V_c, is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316\beta\sqrt{f'_c}\, b_v d_v$$

$$V_c = 0.0316 \cdot (2.52) \cdot \sqrt{5.0} \cdot (6.00) \cdot (67.05) = 71.6 \text{ kips}$$

Step 10 - Determine Required Vertical Reinforcement, V_s

[5.8.3.3-4]

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s} = \frac{A_v f_y d_v \cot\theta}{s} \; where\ \alpha = 90°$$

$$V_u \le V_R = \phi V_n = \phi(V_c + V_s + V_p)$$

$$V_s = \frac{V_u}{\phi} - V_c - V_p = \frac{A_v f_y d_v \cot\theta}{s}$$

$$s = \frac{A_v f_y d_v \cot\theta}{\frac{V_u}{\phi} - V_c - V_p} = \frac{(0.62)\cdot(60)\cdot(67.05)\cdot\cot(26.7)}{\frac{353.5}{0.90} - 71.6 - 31.2} = 17.1 \text{ in}$$

Minimum Reinforcing

For normal girder design the reinforcing area will be fixed as a two-legged stirrup. The minimum transverse reinforcing requirement is satisfied by limiting the maximum allowable spacing to the following:

[5.8.2.5-1]

$$s_{\max} \le \frac{A_v f_y}{0.0316\sqrt{f'_c}\, b_v}$$

$$s_{\max} \le \frac{(0.62)\cdot(60)}{0.0316\sqrt{5.0}\,(6.00)} = 87.7 \text{ in}$$

Maximum Spacing
[5.8.2.7]

The maximum spacing of transverse reinforcing is limited by the following:

[5.8.2.7-1]

For $v_u < 0.125 f'_c \Rightarrow s_{\max} = 0.8 d_v \le 24.0$

[5.8.2.7-2]

For $v_u \ge 0.125 f'_c \Rightarrow s_{\max} = 0.4 d_v \le 12.0$

v_u = 0.899 ksi ≥ 0.125(5.0) = 0.625 ksi

s_{max} = 0.4(67.05) = 26.8 inches but not greater than 12 in

Use #5 stirrups at 12 inch spacing

$$V_s = \frac{(0.62)\cdot(60)\cdot(67.05)\cot(26.7)}{12} = 413.3 \text{ kips}$$

The shear strength is the lesser of:

[5.8.3.3-1]

$V_n = V_c + V_s + V_p$ = [71.6 + 413.3 + 31.2] = 516.1 kips

[5.8.3.3-2]

$V_n = 0.25 f'_c b_v d_v + V_p$ = [0.25(5.0)(6.00)(67.05) + 31.2] = 534.1 kips

φV_n = (0.90)(516.1) = 464.5 k > V_u = 353.5 k

$\therefore$ The transverse shear reinforcing is adequate.

Step 11 - Longitudinal Reinforcement

In addition to transverse reinforcement, shear requires a minimum amount of longitudinal reinforcement. The requirement for longitudinal reinforcement follows:

[5.8.3.5-1]

$$A_s f_y + A_{ps} f_{ps} \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

Where V_s is limited to V_u / φ = 353.5 / 0.90 = 392.8 kips

Bonded Strand
[5.11.4.2]

The effect of the bond on the strand at the end of the girder must be considered. The development length equals the following:

[5.11.4.2-1]

$$l_d \geq k\left(f_{ps} - \frac{2}{3} f_{pe}\right)d_b$$

k = 1.6 for pretensioned members with a depth greater than 24.0 inches.

f_{pe} = effective stress in the prestress steel after losses.
f_{pe} = (0.75)(270) – 51.02 = 151.48 ksi

f_{ps} = average stress in prestress steel at the time for which the nominal resistance of the member is required = 265.65 ksi.

$$l_d \geq (1.6)\cdot\left(265.65 - \frac{2}{3}\cdot 151.48\right)\cdot(0.5) = 131.7\text{ inches}$$

[5.11.4.2-4]

$$f_{px} = f_{pe} + \frac{(l_{px} - 60d_b)}{(l_d - 60d_b)}(f_{ps} - f_{pe})$$

$$f_{px} = 151.48 + \frac{(66.24 - 60\cdot 0.5)}{(131.7 - 60\cdot 0.5)}\cdot(265.65 - 151.48) = 192.16\text{ ksi}$$

Considering only the prestressing steel in the tension side of the member yields the following:

$$(5.508)\cdot(192.16) \geq \frac{|1975\cdot 12|}{(67.05)\cdot(1.00)} + \left(\left|\frac{353.5}{0.9} - 31.2\right| - 0.5\cdot 392.8\right)\cot(26.7)$$

1058 kips > 682 kips

In addition, at the inside edge of the bearing area of simple end supports to the section of critical shear, the longitudinal reinforcement on the flexural tension side of the member shall satisfy:

[5.8.3.5-2]

$$A_s f_y + A_{ps} f_{ps} \geq \left(\frac{V_u}{\varphi_v} - 0.5V_s - V_p \right) \cot\theta$$

Assuming a 12 inch long bearing pad, the development length is 9 + 6 = 15 inches. The transfer length is 30 inches so the effective prestress stress accounting for development is:

f_{ps} = (15 / 30)(151.48) = 75.74 ksi

$$(5.508)\cdot(75.74) \geq \left(\frac{353.5}{0.90} - 0.5\cdot(392.8) - 31.2 \right) \cot(26.7)$$

417 kips > 328 kips

$\therefore$ The bottom prestressing strands are adequate for longitudinal reinforcement without the addition of mild reinforcing.

Critical Shear At Pier

The shear calculations will also be shown in Span 1 near the pier where both a large shear and a large negative moment are present. The critical location is located a distance d_v from the face of the support. The equation, $d_v = 0.72\ h = 0.72(72 + 7.50) / 12 = 4.77$ feet will be used to determine the critical shear location near the pier.

Determine Shear

The shears based on a continuous beam analysis were determined for Span 1 from a computer program as follows:

	0.9	1.0	Unit
DC (Comp DL)	-13.4	-16.0	k
DW (FWS)	-10.5	-12.5	k
LL+IM Vehicle	-123.1	-136.6	k
SH + CR	-5.4	-5.4	k

The shear at a distance of 4.77 feet from the support is determined as follows:

Girder $V_{Crit} = 0.980\ [(110.75) / 2 - 4.77] = 49.6$ kips
Diaphragm $V_{Crit} = 4.92(1 / 2) = 2.5$ kips
Non-Comp $V_{Crit} = 1.068\ [(110.75) / 2 - 4.77] = 54.0$ kips
Comp DL $V_{Crit} = 16.0 - 0.228(4.77) = 14.9$ kips
DC $V_{Crit} = 49.6 + 2.5 + 54.0 + 14.9 = 121.0$ kips

DW $V_{Crit} = 12.5 - 0.178(4.77) = 11.7$ kips
Vehicle $V_{Crit} = 136.6 - (136.6 - 123.1)(4.77) / 11.20 = 130.9$ kips

Live Load Shear

Interior Girder
LL+IM $V_{Crit} = (130.9)(0.884)(1.076) = 124.5$ kips

Strength I Limit State [3.4.1]

$$V_u = 1.25(121.0) + 1.50(11.7) + 1.75(124.5) + 0.5(5.4) = 389.4 \text{ kips}$$

Shear Depth, d_v

[5.8.2.9]

For the shear design, the harped strands will be ignored in the determination of the shear depth. The shear depth is the maximum of the following criteria:

1) $d_v = 0.9\ d_e$ where $d_e = \dfrac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = d_s$ when $A_{ps} = 0$

$$d_s = 80.00 - 2.50 clr - 0.625 - 1.00/2 = 76.38\ in$$

$$d_v = 0.9d_s = 0.9 \cdot (76.38) = 68.74 \text{ in}$$

2) 0.72h = 0.72(72.00 + 7.50) = 57.24 in

[C5.8.2.9-1]

3) $$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{pu}}$$

From the negative moment continuity connection design, the flexural resistance M_n equals 5155 ft-k for #8 @ 6 inches. For mild reinforcing, this reinforcing must be properly developed on both sides of each location under investigation.

$$d_v = \frac{(5155) \cdot (12)}{(14.22) \cdot (60) + 0} = 72.50 \text{ in} \Leftarrow \text{Critical}$$

Based on the above, the shear depth, d_v, is controlled by criteria 3 and equals 72.50 inches.

Calculate, V_p

Due to symmetry the upward shear force near the pier equals that near the abutment.

V_p = (12)(0.153) [(0.75)(270) – 51.02] (0.11208) = 31.2 kips

Check Shear Width, b_v

The LRFD Specification requires that web width be checked for minimum width to protect against crushing.

[5.8.3.3-2]

$V_u <= \varphi V_n = \varphi(0.25 f'_c b_v d_v + V_p)$

$$\text{Required } b_v = \frac{\frac{389.4}{0.9} - 31.2}{(0.25) \cdot (5.0) \cdot (72.50)} = 4.43 \text{ inches}$$

Available b_v = 6.00 inches, ok

Evaluate Shear Stress

$$v_u = \frac{V_u - \varphi V_p}{\varphi b_v d_v} = \frac{389.4 - 0.90 \cdot (31.2)}{0.90 \cdot (6.00) \cdot (72.50)} = 0.923 \text{ ksi}$$

$$\frac{v_u}{f'_c} = \frac{0.923}{5.0} = 0.185$$

General Procedure [5.8.3.4.2]

Calculate strain, ε_x

A_c = area of concrete on the flexural tension side of the member.
A_c = (0.949)(108)(7.50) + 941.00 – 498.50 = 1211.19 in^2

A_{ps} = area of prestressing steel on the flexural tension side of the member. A_{ps} = 0 in^2. (There are 12 harped strands in the flexural tension side at this location but it is conservative to ignore them.)

A_s = area of nonprestressed steel on the flexural tension side of the member. A_s = 14.22 in^2

f_{po} = 0.70(270) = 189 ksi since the critical section is located more than the transfer length of 2.50 feet from the girder end.

N_u = factored axial force taken as positive if tensile = 0 kips

V_u = factored shear force = 389.4 kips

M_u = factored moment, not to be taken less than $V_u d_v$.

The moment at the critical shear location is required. The moments from the continuous beam analysis at the critical location near the pier follow:

	0.9	1.0	Units
DC (Comp DL)	-193	-358	ft-k
DW (FWS)	-151	-279	ft-k
One Vehicle	-1493	-2648	ft-k
SH + CR	-545	-605	ft-k

Girder M_{Crit} = 0.980(4.77)(110.75 – 4.77) / 2 = 248 ft-k
Diaphragm M_{Crit} =4.92(1 / 2)(4.77) = 12 ft-k
Non-Comp M_{Crit} =1.068(4.77)(110.75 – 4.77) / 2 = 270 ft-k
Comp DL M_{Crit} = -358 + 16.0(4.77) - 0.228$(4.77)^2$ / 2 =-284 ft-k

DW M_{Crit} = -279 + 12.5(4.77) – 0.178$(4.77)^2$ / 2 = -221 ft-k

LL + IM M_{Crit} = [-2648 + (2648 - 1493)(4.77) / 11.20](0.748)(0.956)
= -1542 ft-k

SH + CR M_{Crit} = [-605 + (605 - 545)(4.77) / 11.20 = -579 ft-k

M_{Crit} = 0.90(248 + 12 + 270) + 1.25(-284) + 1.50(-221) + 1.75(-1542)
+ 0.50(-579) = -3198 ft-k

M_u = 3198 ft-k but not less than $V_u d_v$ = (389.4)(72.50) / 12 = 2353 ft-k

$$\varepsilon_x = \left[\frac{\frac{|3198 \cdot 12|}{72.50} + 0 + 1.0 \cdot |389.4 - 31.2| - (0) \cdot (189)}{2(29000 \cdot 14.22 + 28500 \cdot 0)}\right]$$

ε_x = 0.00108 = 1.08 x 1000

Since the value is positive the first formula must be used.

Now go into [T5.8.3.4.2-1] to read the values for θ and β. From the previously calculated value of v_u / f'$_c$ = 0.185, enter the ≤ 0.200 row and from the calculated value of ε_x = 1.02 x 1000, enter the ≤ 1.00 column. For an initial cycle using the ≤ 1.00 column is fine. However, the strain limit for the final cycle must be within the stated limits. The new estimate for values is shown below:

θ = 36.1 degrees
β = 1.79

With the new value of θ, the strain must be recalculated.

$$\varepsilon_x = \left[\frac{\frac{|3198 \cdot 12|}{72.50} + 0 + 0.5 \cdot |389.4 - 31.2| \cot(36.1) - 0 \cdot (189)}{824{,}760}\right]$$

ε_x = 0.00094 = 0.94 x 1000

With this new estimate for strain, reenter the table and determine new values for θ and β. Since these new values are the same as assumed, our iterative portion of the design is complete.

Calculate Concrete Shear Strength, V_c

The nominal shear resistance from concrete, V_c, is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v$$

$$V_c = 0.0316\cdot(1.79)\cdot\sqrt{5.0}\cdot(6.00)\cdot(72.50) = 55.0 \text{ kips}$$

Determine Required Vertical Reinforcement, V_s

$$s = \frac{A_v f_y d_v \cot\theta}{\frac{V_u}{\phi} - V_c - V_p} = \frac{(0.62)\cdot(60)\cdot(72.50)\cdot\cot(36.1)}{\frac{389.4}{0.90} - 55.0 - 31.2} = 10.7 \text{ in}$$

Minimum Reinforcing

Based on minimum transverse reinforcing requirements previously calculated, s_{max} = 87.7 inches.

Maximum Spacing [5.8.2.7-2]

Based on the maximum spacing requirements:

v_u = 0.923 > 0.125(5.0) = 0.625
$\therefore$ $s_{max} = 0.4d_v$ = 0.4(72.50) = 29.0 inches but not greater than 12 inches.

Use #5 stirrups at 10 inch spacing

[5.8.3.3-4]

$$V_s = \frac{(0.62)\cdot(60)\cdot(72.50)\cot(36.1)}{10} = 369.9 \text{ kips}$$

The shear strength is the lesser of:

[5.8.3.3-1] $V_n = V_c + V_s + V_p$ = [55.0 + 369.9 + 31.2] = 456.1 kips

[5.8.3.3-2] $V_n = 0.25f'_c b_v d_v + V_p$ = [0.25(5.0)(6.00)(72.50) + 31.2] = 575.0 kips

φV_n = (0.90)(456.1) = 410.5 k > V_u = 389.4 k

$\therefore$ The transverse shear reinforcing is adequate.

Longitudinal Reinforcement

In addition to transverse reinforcement, shear requires a minimum amount of longitudinal reinforcement. The requirement for longitudinal reinforcement follows:

[5.8.3.5-1]

$$A_s f_y + A_{ps} f_{ps} \geq \frac{|M_u|}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

Where V_s is limited to V_u / φ = 389.4 / 0.9 = 432.7 kips

Considering the mild reinforcing steel produces the following:

$$(14.22)\cdot(60) \geq \frac{|3198\cdot 12|}{(72.50)\cdot(0.90)} + \left(\left|\frac{389.4}{0.90} - 31.2\right| - 0.5\cdot(369.9)\right)\cot(36.1)$$

853 kips < 885 kips

Since the demand is greater than the resistance, the longitudinal mild reinforcing steel does not appear to be adequate. However, the area of longitudinal reinforcement on the flexural tension side of the member need not exceed the area required to resist the maximum moment acting alone. Since the longitudinal reinforcing in the top deck is designed to resist the composite dead loads and live load the criteria is satisfied.

Interface Shear Transfer [5.8.4]

For precast girders, the cast-in-place deck is cast separately. Thus the shear transfer across this surface must be investigated. For this example, the shear transfer will be investigated at the critical shear location near the abutment. Only the composite dead loads and live load plus dynamic load allowance are considered.

Strength I Limit State

$V_u = 1.25(11.5) + 1.50(9.0) + 1.75(110.3) = 220.9$ k

d_e = the distance between the centroid of the steel in the tension side of the beam to the center of the compression block in the deck. For simplicity d_e may be taken as the distance between the centroid of the tension steel and the midthickness of the deck. Since this location is within the development length of the strand, use the approximate method.

$d_e = 79.50 - 5.00 - 7.50 / 2 = 70.75$ in

[C5.8.4.1-1]

$$V_{uh} = \frac{V_u}{d_e} = \frac{220.9}{70.75} = 3.12 \text{ k/in}$$

The nominal shear resistance of the interface plane is:

[5.8.4.1-1]

$V_n = cA_{cv} + \mu[A_{vf}f_y + P_c]$

A_{cv} = the area of concrete engaged in shear transfer.
= 40.00 in^2/in

A_{vf} = the area of shear reinforcement crossing the shear plane.
= (0.31)(2) / (12 in spacing) = 0.0517 in^2/in

[5.8.4.2]

For concrete placed against clean, hardened concrete with surface intentionally roughened to an amplitude of 0.25 in

$\mu = 1.0\ \lambda = 1.0(1.0) = 1.0$
$c = 0.100$ ksi

$V_n = 0.100(40.00) + 1.0[(0.0517)(60) + 0] = 7.10$ k/in

[5.8.4.2]

The nominal shear resistance used in the design shall not be greater than the lesser of:

[5.8.4.1-2]

$$V_n \le 0.2 f'_c A_{cv} = 0.2 \cdot (4.5) \cdot (40.00) = 36.00 \text{ k/in}$$

[5.8.4.1-3]

$$V_n \le 0.8 A_{cv} = 0.8 \cdot (40.00) = 32.00 \text{ k/in}$$

Since the nominal capacity of 7.10 k/in is less than the maximum allowed of 32.00 k/in, the nominal horizontal shear resistance is 7.10 k/in. The horizontal shear strength is:

$\varphi V_n = (0.90)(7.10) = 6.39$ k/in > $V_{uh} = 3.12$ k/in ok

The minimum required area of reinforcing crossing the interface is:

[5.8.4.1-4]

$$A_{vf} \geq \frac{0.050 b_v}{f_y} = \frac{(0.050)\cdot(40.00)}{60} = 0.0333\, in^2 / in$$

For #5 @ 12 inches $A_{vf} = (0.31)(2) / 12 = 0.0517$ in^2/in

$\therefore$ The minimum criteria is satisfied at the critical location near the abutment.

Since the shear reinforcing spacing will typically vary across the span the interface shear must also be checked at numerous locations along the span.

Pre-Tensioned Anchor Zone [5.10.10]

Bursting Resistance [5.10.10.1]

The design of anchor zone for precast members is relatively simple. The amount of bursting reinforcing equals thearea required to resist 4 percent of the total prestress force acting at a unit stress of 20 ksi. The first reinforcing bar should be placed as close to the face as possible with all the required reinforcing placed within a distance equal to the member depth divided by 4.

$$A_s = (0.04)\cdot\frac{(0.75)\cdot(270)\cdot(48)\cdot(0.153)}{20.0} = 2.97\ \text{in}^2$$

Use five 2-legged #5 stirrups. $A_s = (5)(2)(0.31) = 3.10\ \text{in}^2$

The first stirrup should be placed 2 inches from the girder end. The remaining stirrups should be placed within a distance = 72.00 / 4 = 18.00 inches from the end. Use 4 inch spacing between the five stirrups.

Confinement Reinforcing [5.10.10.2]

The LRFD Specification has changed the criteria for confinement reinforcing. This reinforcing must now be spread over a length of 1.5 d from the end of the girder rather than over a length 'h' as specified in the Standard Specifications. The maximum spacing has also been reduced to 6 inches with the reinforcing shaped to enclose the strands. The confinement requirements do not have to be applied to the harped strands.

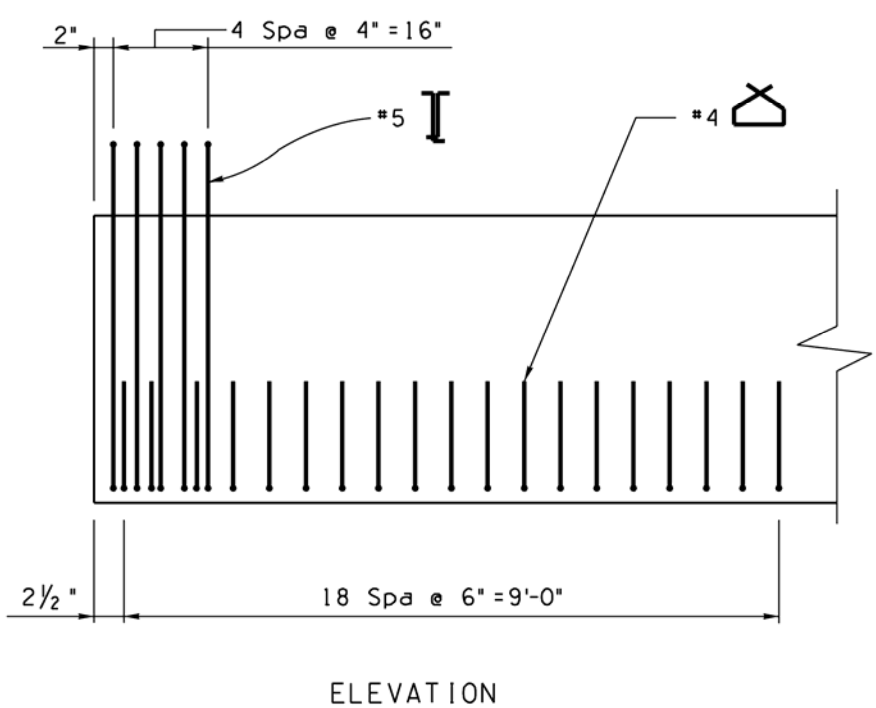


Figure 18

Deflections [5.7.3.6]

Deflections must be calculated so a camber can be put in the superstructure to provide for a smooth riding surface and the build-up can be estimated to determine the appropriate seat elevations and quantities.

Determination of the girder deflections is a tedious task complicated by the fact that the deck not only makes the girder composite but also continuous. If the continuity is not considered properly a rough ride can result.

There are four stages the girder experiences. The first stage is the release deflection when the prestress force is transferred to the girder. The release deflection is the summation of the deflections caused by self-weight of the girder and the prestress force under relaxation before transfer and elastic shortening losses.

The second stage is the initial deflection where some time dependent losses have occurred in the prestress force but where the concrete has also experienced creep and gained strength.

The third and fourth stages comprise the final deflection used to determine screeds. The third stage consists of the addition of non-composite loads with the girder simply supported. The fourth stage includes the addition of all other loads including the effects from time-dependent prestress losses and additional concrete creep under a composite continuous girder.

For calculations for deflection, the prismatic gross section properties and prestress losses at midspan will be used. While this may not be technically correct, it will provide sufficiently accurate deflections considering all the unknowns.

Release Deflection

Release Deflection:

The deflection at midspan from self-weight is due to the uniform load from the typical section. At this time the girder concrete is at its release strength.

$$\Delta_{girder} = \frac{5wl^4}{384E_{ci}I} = \frac{5\cdot(0.980)\cdot(110.75)^4\cdot(1728)}{384\cdot(3946)\cdot(671{,}108)} = 1.253 \text{ in}$$

The deflection from prestressing is more complicated but can be determined using moment area theory. The deflection at midspan from prestressing is:

$$\Delta_{p/s} = \frac{P}{24E_{ci}I}\left[e_m\left(2L^2+4aL-4a^2\right)+e_e\left(L^2-4aL+4a^2\right)\right]$$

$P = [(0.75)(270) - 2.23 - 18.35](0.153)(48) = 1336.02$ kips

$e_m = 36.439 - 5.500 = 30.939$ in

At the centerline of bearing of the abutment (See Figure 12):

$Y_{cl} = 2.00 + 58.00(42.375) / 43.125 + 10.00 = 68.991$ in

$$c.g. = \frac{(12)\cdot(68.991-5.0)+(8)\cdot(8.0)+(10)\cdot(6.0)+(10)\cdot(4.0)+(8)\cdot(2.0)}{48}$$

c.g. = 19.748 in

$e_e = 36.439 - 19.748 = 16.691$ in

$2L^2 + 4aL - 4a^2 = 2(110.75)^2 + 4(13.0)(110.75) - 4(13.0)^2 = 29{,}614$

$L^2 - 4aL + 4a^2 = (110.75)^2 - 4(13.0)(110.75) + 4(13.0)^2 = 7183$

$$\Delta_{p/s} = \frac{(1336.02)\cdot(144)}{24\cdot(3946)\cdot(671{,}108)}\cdot[(30.939)\cdot(29614)+(16.691)\cdot(7183)]$$

$\Delta_{p/s} = 3.136$ in

The deflection of the beam at release equals the algebraic sum of the above deflections:

$\Delta_R = 1.253 - 3.136 = -1.883$ in upward

Initial Deflection

Initial Deflection:

The initial deflection accounts for a loss of prestress over time and creep in the concrete. The deflection from the girder based on its final concrete strength is:

$$\Delta_{girder} = \frac{5wl^4}{384E_cI} = \frac{5\cdot(0.980)\cdot(110.75)^4\cdot(1728)}{384\cdot(4070)\cdot(671{,}108)} = 1.214 \text{ in}$$

The deflection from prestressing is complicated by the fact that some prestress loss has occurred. For this problem the time from transfer of prestressing force till deck pour is assumed to be 60 days.

The prestress loss equations do not include a time factor. The Bridge Practice Guidelines specify that lacking any better information that 40% of the creep and 50% of the time-dependent losses occur at the time of the deck pour.

P/S Loss 60 days = 2.23 + 18.35 + 0.50(34.00) = 37.58 ksi

The prestress force is:

P = [(0.75)(270) – 37.58](48)(0.153) = 1211.17 k

$$\Delta_{p/s} = \frac{(1211.17)\cdot(144)}{24\cdot(4070)\cdot(671{,}108)}\cdot\left[(30.939)\cdot(29614)+(16.691)\cdot(7183)\right]$$

$\Delta_{p/s}$ = 2.757 in

The deflection of the girder at 60 days equals the algebraic sum of the above deflections times a creep factor. The overall creep factor for deflections is 2.00. This means the total deflection is the sum of the initial deflection plus the creep factor at 60 days times the deflection. At 60 days the girder has a creep factor of 0.40(2.00) = 0.80 producing a total creep factor of 1.00 + 0.80 = 1.80.

Δ_i = 1.80(1.214 – 2.757) = -2.777 in upward

Final Deflections

<u>Final Deflection:</u>

The final deflection accounts for the remainder of the prestress loss and concrete creep and shrinkage. The final deflection is added to the profile grade to determine the screed elevations. The final deflections must be separated between those that occur while the girder is acting as a simple span and those that occur on the composite continuous girder.

The simple span deflections will occur instantaneously and will consist of the weight of the deck slab, build-up and the SIP forms with a uniform load equal to 0.900 + 0.083 + 0.085 = 1.068 k/ft and the intermediate diaphragm.

Simple Span Deflections

$$\Delta_{DL} = \frac{5wl^4}{384E_cI} = \frac{5 \cdot (1.068) \cdot (110.75)^4 \cdot (1728)}{384 \cdot (4070) \cdot (671{,}108)} = 1.324 \text{ in}$$

$$\Delta_{diaph} = \frac{PL^3}{48E_cI} = \frac{(4.92) \cdot (110.75)^3 \cdot (1728)}{48 \cdot (4070) \cdot (671{,}108)} = 0.088 \text{ in}$$

Continuous Deflections

The parapet, sidewalk and fence will be placed with the now composite girder acting as a continuous member. A standard continuous beam program is used to determine the deflection from the uniform load. The deflection at midspan is 0.060 inches. The above deflections are immediate so a creep factor of 1.00 is used. The resulting deflection equals 1.324 + 0.088 + 0.060 = 1.472 inches.

The remaining deflections are creep related and occur over time. These deflections are more complicated to determine due to continuity and a computer program is normally required. Input values are shown in Figure 19 but detailed calculations are not.

Long-term deflections include creep of the girder, deck slab, diaphragm, parapet, rail, fence, sidewalk, final prestress force, loss of prestress and differential shrinkage. Different creep factors are applied to the different loads. The girder dead load and prestress have already seen much of their creep with a creep value of 2.00 – 0.80 = 1.20 remaining for the long-term effects. The slab, diaphragm and parapet have not seen any creep so a value of 2.0 remains. A creep factor of 1.00 is applied for the loss of prestress and differential shrinkage. The differential shrinkage moment was calculated under the positive moment continuity connection section.

A summary of output from the continuous beam analysis is shown below:

	Deflection	Creep	Total
Girder	0.257	1.20	0.308
Diaphragm	0.020	2.00	0.040
Parapet & SW	0.060	2.00	0.120
Final P/S	-0.602	1.20	-0.722
Loss of P/S	0.162	1.00	0.162
Slab	0.277	2.00	0.554
Diff Shr	0.185	1.00	0.185
Total			0.647

The final downward long-term deflection at midspan is 1.472 + 0.647 = 2.119 inches.

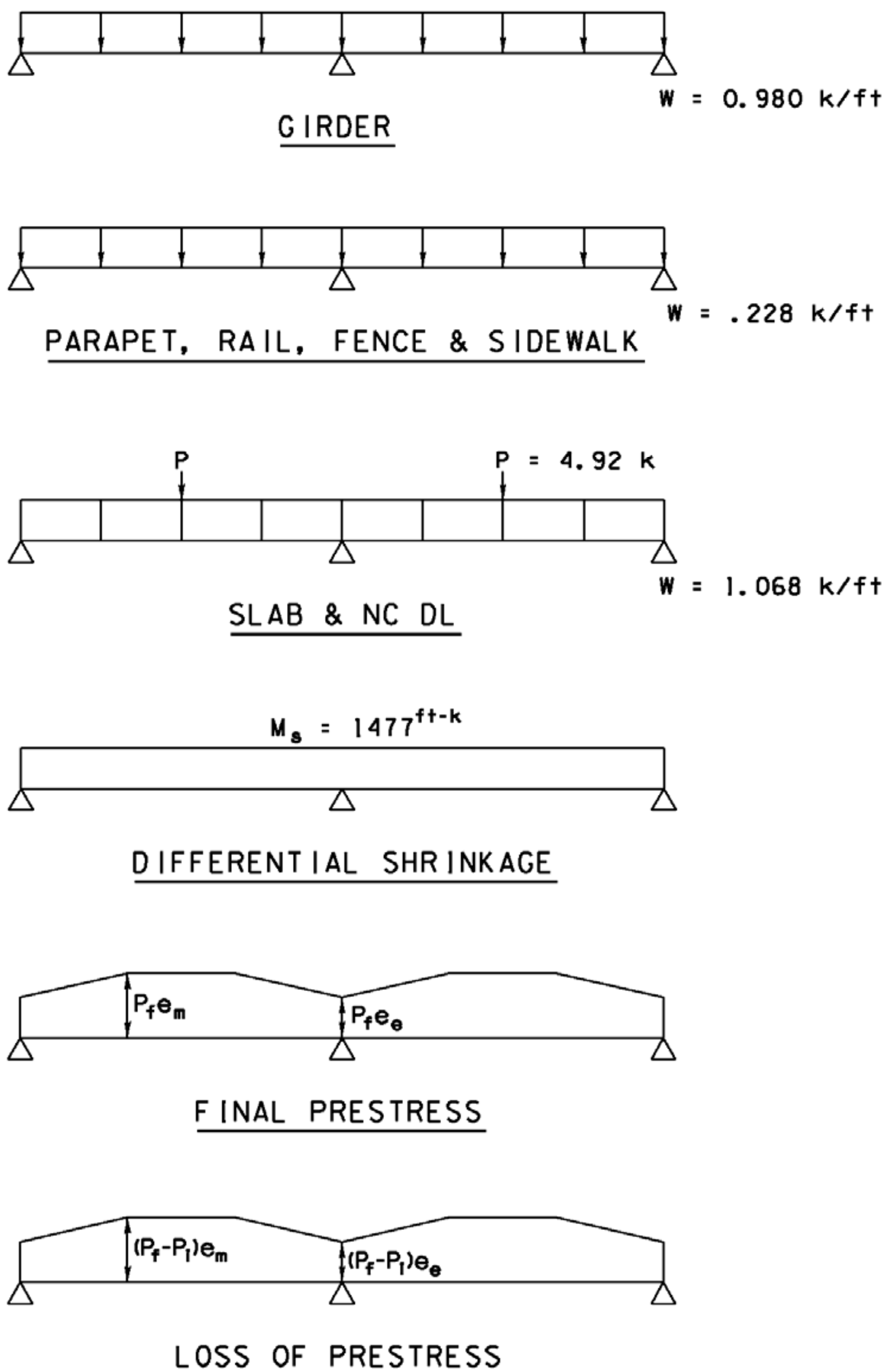

W = 0.980 k/ft
GIRDER
W = .228 k/ft
PARAPET, RAIL, FENCE & SIDEWALK
P
P = 4.92 k
W = 1.068 k/ft
SLAB & NC DL
M_s = 1477 ft-k
DIFFERENTIAL SHRINKAGE
P_f e_m
P_f e_e
FINAL PRESTRESS
(P_f-P_l)e_m
(P_f-P_l)e_e
LOSS OF PRESTRESS

Figure 19

APPENDIX A

PRECISE OVERHANG ANALYSIS

A simplified method of determining the adequacy of an overhang subjected to both tension and flexure is included in the example. This appendix shows the more complex and precise method along with the assumptions made to derive the approximate simplified equation.

Tension and Flexure [5.7.6.2]

The solution of the deck design problem involves determining the resistance of the deck overhang to a combination of tension and flexure. Members subjected to eccentric tension loading, which induces both tensile and compressive stresses in the cross section, shall be proportioned in accordance with the provisions of Article 5.7.2.

[5.7.2]

Assumptions for a valid analysis for an extreme event limit state are contained in Article 5.7.2. Factored resistance of concrete components shall be based on the conditions of equilibrium and strain compatibility and the following:

Strain is directly proportional to the distance from the neutral axis.

For unconfined concrete, the maximum usable strain at the extreme concrete compressive fiber is not greater than 0.003.

The stress in the reinforcement is based on a stress-strain curve of the steel or on an approved mathematical representation.

Tensile strength of the concrete is ignored.

The concrete compressive stress-strain distribution is assumed to be a rectangular stress block in accordance with Article 5.7.2.2.

The development of the reinforcing is considered.

[1.3.2.1]

While the article specifies the use of the reduction factors in Article 5.5.4.2, that requirement only applies to a strength limit state analysis. For an extreme event limit state, the resistance factor shall be taken as 1.0.

The above assumptions as shown in Figures A-1, A-2 and A-3 were used in the development of the equations for resistance from tension and flexure that occur with a vehicular collision with a traffic railing.

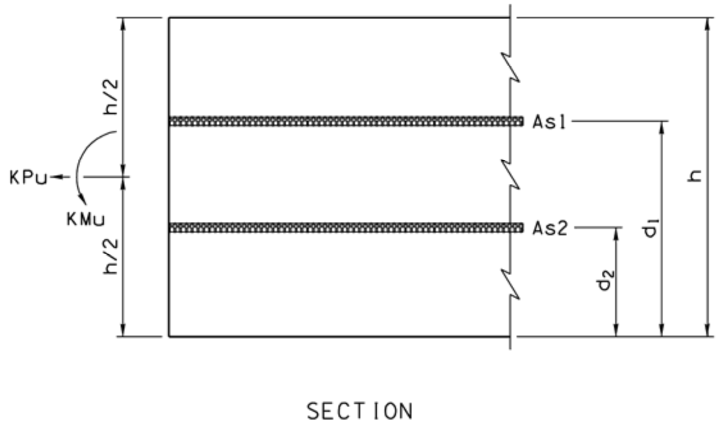


Figure A-1

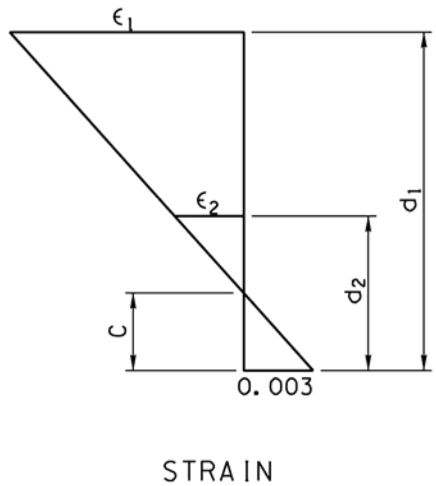


Figure A-2

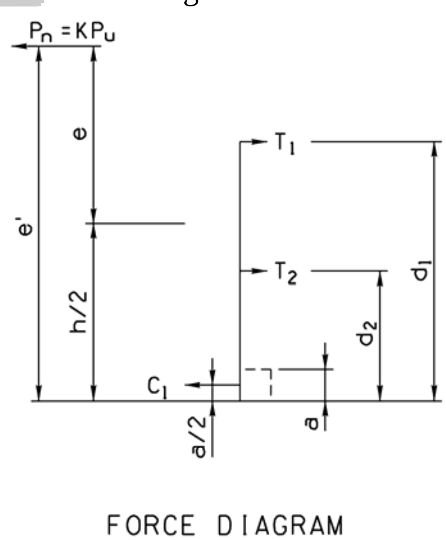


Figure A-3

The design of the deck overhang is complicated because both a bending moment and a tension force are applied. The problem can be solved using equilibrium and strain compatibility. The following trial and error approach may be used:

1. Assume a stress in the reinforcing
2. Determine force in reinforcing
3. Solve for k, the safety factor
4. Determine values for 'a' and 'c'
5. Determine corresponding strain
6. Determine stress in the reinforcing
7. Compare to assumed value and repeat if necessary

Face of Barrier
Location 1
Figure 4

The design horizontal force in the barrier is distributed over the length L_b equal to L_c plus twice the height of the barrier. See Figures 5 and 6.

$L_b = 9.39 + 2(2.76) = 14.91$ ft
$P_u = 98.17 / 14.91 = 6.584$ k/ft $\Rightarrow$ Use $P_u = 3.759$ k/ft per connection.

Dimensions

$h = 9.00 + (4.00)(1.00) / (1.333) = 12.00$ in
$d_1 = 12.00 - 2.50 \text{ clr} - 0.625 / 2 = 9.19$ in
$d_2 = 12.00 - 8.00 + 1.00 \text{ clr} + 0.625 / 2 = 5.31$ in

Moment at Face of Barrier

Deck $= 0.150(9.00 / 12)(1.00)^2 \div 2 = 0.06$ ft-k
$0.150(3.00 / 12)(1.00)^2 \div 6 = \underline{0.01 \text{ ft-k}}$
$= 0.07$ ft-k

Fence, Rail & Parapet: $w = 0.075 + 0.15(10/12)(2.76) = 0.420$ k/ft
FR & P $= 0.420(0.417) = 0.18$ ft-k

Collision $= 3.759[3.206 + (12.00/12) / 2] = 13.93$ ft-k

[A13.4.1]
Extreme Event II
[3.4.1]

The load factor for dead load shall be taken as 1.0.
$M_u = 1.00(0.07 + 0.18) + 1.00(13.93) = 14.18$ ft-k

$e = M_u / P_u = (14.18)(12) / (3.759) = 45.27$ in

1. Assume Stress

Assume both layers of reinforcing yield and $f_s = 60$ ksi

2. Determine Forces

Determine resulting forces in the reinforcing:

$T_1 = (0.744)(60) = 44.64$ k
$T_2 = (0.572)(60) = 34.32$ k

Strength Equation

Solving the equations of equilibrium by summing the forces on the section and summing the moments about the soffit and setting them equal to zero yields the following two equations. See Figure A-3.

Sum forces in horizontal direction

Eqn 1: $-kP_u + T_1 + T_2 - C_1 = 0$ where $C_1 = 0.85f'_c ab$

Sum of moments

Eqn 2: $kP_u(e') - T_1(d_1) - T_2(d_2) + C_1(a/2) = 0$

Solving the above equations for k, the ratio of strength to applied force and moment, results in a quadratic equation with the following coefficients:

$$A = \frac{P_u^{\,2}}{1.70\,f'_c\,b}$$

$$B = P_u\left(e + \frac{h}{2} - \frac{T_1 + T_2}{0.85\,f'_c\,b}\right)$$

$$C = -T_1 d_1 - T_2 d_2 + \frac{(T_1 + T_2)^2}{1.70\,f'_c\,b}$$

Solution

Substituting in specific values yields:

$$A = \frac{(3.759)^2}{1.70 \cdot (4.5) \cdot (12)} = 0.153922$$

$$B = (3.759) \cdot \left(45.27 + \frac{12.00}{2} - \frac{(44.64 + 34.32)}{0.85 \cdot (4.5) \cdot (12)}\right) = 186.2575$$

$$C = -(44.64) \cdot (9.19) - (34.32) \cdot (5.31) + \frac{(44.64 + 34.32)^2}{1.70 \cdot (4.5) \cdot (12)} = -524.5649$$

Solution of the quadratic equation yields the value k, the safety factor.

$$k = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

3. Determine k Safety Factor

$$k = \frac{-186.2575 + \sqrt{(186.2575)^2 - 4 \cdot (0.153922) \cdot (-524.5649)}}{2 \cdot (0.153922)} = 2.810$$

Since the value of k is greater than one the deck is adequately reinforced at this location.

4. Determine ‘a’ and ‘c’

Calculate the depth of the compression block from Eqn 1. See Figure A-3.

$$a = \frac{(T_1 + T_2 - kP_u)}{0.85 f'_c\, b}$$

$$a = \frac{(44.64 + 34.32 - (2.810)\cdot(3.759))}{0.85\cdot(4.5)\cdot(12)} = 1.49 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{1.49}{0.825} = 1.81 \text{ in}$$

5. Strains
6. Stresses

Determine the resulting strain in the two layers of reinforcing. See Figure A-2.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

$\varepsilon_1 = 0.003(d_1 / c-1) = 0.003(9.19 / 1.81 - 1) = 0.0122$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

$\varepsilon_2 = 0.003(d_2 / c - 1) = 0.003(5.31 / 1.81 - 1) = 0.00580$
Since $\varepsilon_2 > \varepsilon_y$ the bottom layer yields and $f_{s2} = 60$ ksi

7. Verify Assumption

Since both layers of reinforcing yield the assumptions made in the analysis are valid.

Maximum Strain

The LRFD Specification does not have an upper limit on the amount of strain in a reinforcing bar. ASTM does require that smaller diameter rebar have a minimum elongation at tensile strength of 8 percent. This appears to be a reasonable upper limit for an extreme event state where φ = 1.00. For this example the strain of 1.2 percent is well below this limit.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section. This step is not necessary for design but is included for educational purposes.

$\varphi P_n = \varphi k P_u = (1.00)(2.810)(3.759) = 10.56$ k

Solve for equilibrium from Figure 9 by substituting M_n for kP_ue and taking moments about the center of the compression block:

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) + T_2\left(d_2 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (44.64)\cdot\left(9.19 - \frac{1.49}{2}\right) + (34.32)\cdot\left(5.31 - \frac{1.49}{2}\right)$$

$$-(2.810)\cdot(3.759)\cdot\left(\frac{12.00}{2} - \frac{1.49}{2}\right) = 478.15 \text{ in-k}$$

φM_n = (1.00)(478.15) / 12 = 39.85 ft-k

The factor of safety for flexure is 39.85 / 14.18 = 2.810 the same as for axial strength. Thus this method provides both a tensile and flexural strength with the same safety factor.

Simplified Method

A simplified method of analysis is also available. If only the top layer of reinforcing is considered in determining strength, the assumption can be made that the reinforcing will yield. By assuming the safety factor for axial tension is 1.0 the strength equation can be solved directly. This method will determine whether the section has adequate strength. However, the method does not consider the bottom layer of reinforcing, does not maintain the required constant eccentricity and does not determine the maximum strain.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

$$\text{where } a = \frac{T_1 - P_u}{0.85 f'_c b} = \frac{44.64 - 3.759}{(0.85)\cdot(4.5)\cdot(12)} = 0.89 \text{ in}$$

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(9.19 - \frac{0.89}{2}\right) - (3.759)\cdot\left(\frac{12.00}{2} - \frac{0.89}{2}\right)\right] \div 12$$

φM_n = 30.79 ft-k

Since $\varphi M_n > M_u$ the overhang has adequate strength. Note that the resulting eccentricity equals (30.79)(12) ÷ 3.759 = 98.29 inches compared to the actual eccentricity of 45.27 inches that is fixed by the constant deck thickness, barrier height and dead load moment.

Independent analysis using the more complex method but considering only the top layer of reinforcing results in a flexural strength equal to 29.10 ft-k. Thus it would appear that the simplified analysis method produces a greater non-conservative result. However, the simplified method uses a safety factor of 1.0 for axial load leaving more resistance for flexure. As the applied load approaches the ultimate strength the two methods will converge to the same result.

Exterior Support Location 2
Figure 4

The deck slab must also be evaluated at the exterior overhang support. At this location the design horizontal force is distributed over a length L_{s1} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the exterior support. See Figures 4, 5 and 6. Assume composite action between deck and girder for this extreme event. Using a distribution of 30 degrees from the face of barrier to the exterior support results in the following:

L_{S1} = 9.39 + 2(2.76) + (2)[tan(30)](1.04) = 16.11 ft
P_u = 98.17 / 16.11 = 6.094 k/ft ⇒ Use P_u = 3.759 k/ft per connection

Dimensions

h = 8.00 + 5.00 + 3.00(8.5/17) = 14.50 in
d_1 = 14.50 – 2.50 clr – 0.625 / 2 = 11.69 in
d_2 = 5.00 + 3.00(8.5/17) + 1.00 clr + 0.625 / 2 = 7.81 in

Moment at Exterior Support

.

DC Loads

Deck	= 0.150(9.00 / 12)$(2.04)^2$ / 2		= 0.23 ft-k
	= 0.150(5.50 / 12)$(2.04)^2$ /6		= 0.05 ft-k
Parapet	= 0.420(0.417 + 1.042)		= 0.61 ft-k
Sidewalk	= 0.15(9.16 / 12)$(1.04)^2$ / 2		= 0.06 ft-k
		DC	= 0.95 ft-k

DW Loads

FWS	=	= 0.00 ft-k

Collision = 3.759[3.206 + (14.50 / 12) / 2] = 14.32 ft-k

[A13.4.1]

The load factor for dead load shall be taken as 1.0.

Extreme Event II [3.4.1]

M_u = 1.00(0.95) + 1.00(0.00) + 1.00(14.32) = 15.27 ft-k

e = M_u / P_u = (15.27)(12) / (3.759) = 48.75 in

1. Assume Stress

Assume both layers of reinforcing yield and f_s = 60 ksi

2. Determine Forces

Determine resulting forces in the reinforcing:

T_1 = (0.744)(60) = 44.64 k
T_2 = (0.572)(60) = 34.32 k

$T_1 + T_2$ = 44.64 + 34.32 = 78.96 k

Solution

Using the previously derived equations for safety factor yields the following:

$$A = \frac{(3.759)^2}{1.70 \cdot (4.5) \cdot (12)} = 0.153922$$

$$B = (3.759) \cdot \left(48.75 + \frac{14.50}{2} - \frac{(78.96)}{0.85 \cdot (4.5) \cdot (12)} \right) = 204.0375$$

$$C = -(44.64) \cdot (11.69) - (34.32) \cdot (7.81) + \frac{(78.96)^2}{1.70 \cdot (4.5) \cdot (12)} = -721.9649$$

Solution of the quadratic equation yields the value k, the safety factor.

3. Determine k Safety Factor

$$k = \frac{-204.0375 + \sqrt{(204.0375)^2 - 4 \cdot (0.153922) \cdot (-721.9649)}}{2 \cdot (0.153922)} = 3.529$$

Since the value of k is greater than one, the deck is adequately reinforced at this location.

4. Determine 'a' and 'c'

Calculate the depth of the compression block.

$$a = \frac{(T_1 + T_2 - kP_u)}{0.85 f'_c\, b}$$

$$a = \frac{(78.96 - (3.529) \cdot (3.759))}{0.85 \cdot (4.5) \cdot (12)} = 1.43 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{1.43}{0.825} = 1.73 \text{ in}$$

5. Strains
6. Stress

Determine the resulting strain in the two layers of reinforcing. See Figure A-2.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

$\varepsilon_1 = 0.003(d_1 / c - 1) = 0.003(11.69 / 1.73 - 1) = 0.01727$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

$\varepsilon_2 = 0.003(d_2 / c - 1) = 0.003(7.81 / 1.73 - 1) = 0.01054$
Since $\varepsilon_2 > \varepsilon_y$ the bottom layer yields and $f_{s2} = 60$ ksi

7. Verify Assumption

Since both layers of reinforcing yield the assumptions made in the analysis are valid.

Maximum Strain

The maximum strain of 1.7 percent is less than the ADOT limit of 8 percent and is therefore satisfactory.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section.

$\varphi P_n = \varphi k P_u = (1.00)(3.529)(3.759) = 13.27$ k

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) + T_2\left(d_2 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (44.64)\cdot\left(11.69 - \frac{1.43}{2}\right) + (34.32)\cdot\left(7.81 - \frac{1.43}{2}\right)$$

$$-(3.529)\cdot(3.759)\cdot\left(\frac{14.50}{2} - \frac{1.43}{2}\right) = 646.73 \text{ in-k}$$

$\varphi M_n = (1.00)(646.73) / 12 = 53.89$ ft-k

The factor of safety for flexure is 53.89 / 15.27 = 3.529 the same as for axial strength.

Simplified Method

A simplified method of analysis is available based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{44.64 - 3.759}{(0.85)\cdot(4.5)\cdot(12)} = 0.89$ in

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(11.69 - \frac{0.89}{2}\right) - (3.759)\cdot\left(\frac{14.50}{2} - \frac{0.89}{2}\right)\right] \div 12$$

$\varphi M_n = 39.70$ ft-k

Interior Support Location 3 Figure 4

The deck slab must also be evaluated at the interior point of support. The critical location will be at the edge of the girder flange where the deck will be the thinnest. Only the top layer of reinforcing will be considered. At this location the design horizontal force is distributed over a length L_{s2} equal to the length L_c plus twice the height of the barrier plus a distribution length from the face of the barrier to the interior support. See Figures 4, 5 and 6. Using a distribution of 30 degree from the face of the barrier to the interior support results in the following:

L_{S2} = 9.39 + 2(2.76) + (2)[tan(30)](3.67) = 19.15 ft
P_u = 98.17 / 19.15 = 5.126 k/ft ⇒ Use P_u = 3.759 k/ft per connection

Dimensions

h = 8.00 in
d_1 = 8.00 – 2.50 clr – 0.625 / 2 = 5.19 in

Moment at Interior Support

For dead loads use the maximum negative moments for the interior cells used in the interior deck analysis
.
DC = 0.56 ft-k
DW = 0.13 ft-k
Collision = 3.759[3.206 + (8.00 / 12) / 2] = 13.30 ft-k

[A13.4.1] Extreme Event II [3.4.1]

The load factor for dead load shall be taken as 1.0.
M_u = 1.00(0.56) + 1.00(0.13) + 1.00(13.30) = 13.86 ft-k
e = M_u / P_u = (13.86)(12) / (3.759) = 44.25 in

1. Assume Stress

Assume the top layer of reinforcing yields and f_s = 60 ksi

2. Determine Force

Determine resulting force in the reinforcing:

T_1 = (0.744)(60) = 44.64 k

Using the previously derived equations for safety factor yields the following:

$$A = \frac{(3.759)^2}{1.70 \cdot (4.5) \cdot (12)} = 0.153922$$

$$B = (3.759) \cdot \left(44.25 + \frac{8.00}{2} - \frac{(44.64)}{0.85 \cdot (4.5) \cdot (12)} \right) = 177.7159$$

$$C = -(44.64) \cdot (5.19) + \frac{(44.64)^2}{1.70 \cdot (4.5) \cdot (12)} = -209.9743$$

3. Determine k Safety Factor

Solution of the quadratic equation yields the value k, the safety factor.

$$k = \frac{-177.7159 + \sqrt{(177.7159)^2 - 4\cdot(0.153922)\cdot(-209.9743)}}{2\cdot(0.153922)} = 1.180$$

Since the value of k is greater than one, the deck is adequately reinforced at this location.

4. Determine ‘a’ and ‘c’

Calculate the depth of the compression block.

$$a = \frac{(T_1 - kP_u)}{0.85 f'_c\, b}$$

$$a = \frac{(44.64 - (1.180)\cdot(3.759))}{0.85\cdot(4.5)\cdot(12)} = 0.88 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{0.88}{0.825} = 1.06 \text{ in}$$

5. Strain

Determine the resulting strain in the top layer of reinforcing. See Figure A-2.

$\varepsilon_y = f_y / E_s = 60 / 29000 = 0.00207$

6. Stress

$\varepsilon_1 = 0.003(d_1 / c - 1) = 0.003(5.19 / 1.06 - 1) = 0.0117$
Since $\varepsilon_1 > \varepsilon_y$ the top layer yields and $f_{s1} = 60$ ksi

7. Verify Assumption

Since the top layer of reinforcing yields the assumption made in the analysis is valid.

Maximum Strain

The maximum strain of 1.2 percent is less than the ADOT limit of 8 percent and is therefore satisfactory.

Verify Results

Verify the results by calculating the tensile strength and flexural resistance of the section.

$\varphi P_n = \varphi k P_u = (1.00)(1.180)(3.759) = 4.44$ k

$$M_n = T_1\left(d_1 - \frac{a}{2}\right) - kP_u\left(\frac{h}{2} - \frac{a}{2}\right)$$

$$M_n = (44.64)\cdot\left(5.19 - \frac{0.88}{2}\right) - (1.180)\cdot(3.759)\cdot\left(\frac{8.00}{2} - \frac{0.88}{2}\right)$$

M_n = 196.25 in-k

φM_n = (1.00)(196.25) / 12 = 16.35 ft-k

The factor of safety for flexure is 16.35 / 13.86 = 1.180 the same as for axial strength.

Simplified Method

A simplified method of analysis is available based on the limitations previously stated.

$$\varphi M_n = \varphi\left[T_1\left(d_1 - \frac{a}{2}\right) - P_u\left(\frac{h}{2} - \frac{a}{2}\right)\right]$$

where $a = \dfrac{T_1 - P_u}{0.85 f'_c\, b} = \dfrac{44.64 - 3.759}{(0.85)\cdot(4.5)\cdot(12)} = 0.26$ in

$$\varphi M_n = (1.00)\cdot\left[(44.64)\cdot\left(5.19 - \frac{0.89}{2}\right) - (3.759)\cdot\left(\frac{8.00}{2} - \frac{0.89}{2}\right)\right] \div 12$$

φM_n = 16.54 ft-k

It is interesting to note that for a safety factor close to one that the results of the precise and approximate methods are the similar.

APPENDIX B

PRECISE TRANSFORMED SECTION PROPERTIES

The approximate method of calculating transformed section properties assumes that all the strands act at the center of gravity of the strand pattern. The precise method assumes the strands in each row act as a single strand equal to the area of all the strands in that row.

Both methods produce the same values for the area and location of the neutral axis. At the midspan, where all the strands are closely spaced near the bottom, the moment of inertia of the two methods is very close. Near the ends of the girder where the strands are draped there are some differences in the moment of inertia. However, these differences are small and can be ignored.

Midspan Transformed Properties

The following transformed section properties are calculated at the midspan based on the strand pattern shown in Figures 11 and 12:

Net Section – I-Girder

No.	As	A	Y	Ay	Io	$A(y-yb)^2$
		941.00	36.439	34289	671108	56
10	0.153	-1.53	2.000	-3	0	-1841
12	0.153	-1.84	4.000	-7	0	-1966
12	0.153	-1.84	6.000	-11	0	-1732
10	0.153	-1.53	8.000	-12	0	-1259
2	0.153	-0.31	10.000	-3	0	-221
2	0.153	-0.31	12.000	-3	0	-189
		933.64		34250	671108	-7152

A = 933.64 in^2
y_b = 34250 / 933.64 = 36.684 in
y_t = 72.00 – 36.684 = 35.316 in
I = 671,108 - 7152 = 663,956 in^4
r^2 = I / A = 663,956 / 933.64 = 711.15 in^2

Transformed Section – I-Girder (n = 7.00)

n	No.	As	A	Y	Ay	Io	$A(y-yb)^2$
			933.64	36.684	34250	663956	2475
7	10	0.153	10.71	2.000	21	0	11703
7	12	0.153	12.85	4.000	51	0	12394
7	12	0.153	12.85	6.000	77	0	10849
7	10	0.153	10.71	8.000	86	0	7840
7	2	0.153	2.14	10.000	21	0	1343
7	2	0.153	2.14	12.000	26	0	1138
			985.04		34532	663956	47742

A = 985.04 in^2
y_b = 34532 / 985.04 = 35.056 in
y_t = 72.00 – 35.056 = 36.944 in
I = 663,956 + 47742 = 711,698 in^4

Composite Section – I-Girder & Deck

n = 3861 / 4070 = 0.949

Interior Girder

n	W	H	A	y	Ay	Io	$A(y\text{-}yb)^2$
			985.04	35.056	34532	711698	313399
0.949	108.00	7.50	768.69	75.750	58228	3603	401596
			1753.73		92760	715301	714995

$A = 1753.73\ in^2$
$y_b = 92760 / 1753.73 = 52.893$ in
$y_t = 72.00 – 52.893= 19.107$ in
$I = 715{,}301 + 714{,}995 = 1{,}430{,}296\ in^4$

Exterior Girder

n	W	H	A	y	Ay	Io	$A(y\text{-}yb)^2$
			985.04	35.056	34532	711698	270098
0.949	90.00	7.50	640.58	75.75	48524	3003	373137
0.949	16.00	1.00	15.18	71.50	1086	1	6002
0.949	½*16.0	4.00	30.37	69.67	2116	27	9900
			1671.17		86258	714729	659137

$A = 1671.17\ in^2$
$y_b = 86258 / 1671.17 = 51.615$ in
$y_t = 72.00 – 51.615 = 20.385$ in
$I = 714{,}729 + 659{,}137 = 1{,}373{,}866\ in^4$

Transfer Length

[6.11.4.1]

The section properties are also required near the ends of the beam at a distance equal to the transfer length from the end of the beam. Since transformed section properties are being used, the section properties will vary with the change in center of gravity of the strands. The transfer length of the bonded prestressing strands is 60 times the strand diameter. For 0.5 inch diameter strand the transfer length equals 30 inches. The centerline of bearing is 9 inches from the end. Therefore the critical location is 21 inches from the centerline of bearing.

See Figure 12 for a diagram of the harped strands. The rise in the top strand at the end of the transfer length is:

$Y = 12.00 + (58.00)(40.625) / (43.125) = 66.638$ in

$$cg = \frac{12\cdot(66.638-5.00)+8\cdot(2.0)+10\cdot(4.0)+10\cdot(6.0)+8\cdot(8.0)}{48} = 19.160$$

Transfer Length Transformed Properties

At transfer length from the girder end:

Net Section – I-Girder

No.	As	A	y	Ay	Io	A(y-yb)^2
		941.00	36.439	34289	671108	18
2	0.153	-0.31	66.638	-20	0	-280
2	0.153	-0.31	64.638	-20	0	-244
2	0.153	-0.31	62.638	-19	0	-211
2	0.153	-0.31	60.638	-19	0	-179
2	0.153	-0.31	58.638	-18	0	-151
2	0.153	-0.31	56.638	-17	0	-125
8	0.153	-1.22	8.00	-10	0	-996
10	0.153	-1.53	6.00	-9	0	-1430
10	0.153	-1.53	4.00	-6	0	-1624
8	0.153	-1.22	2.00	-2	0	-1459
		933.64		34149	671108	-6681

A = 933.64 in^2
y_b = 34149 / 933.64 = 36.576 in
y_t = 72.00 – 36.576 = 35.424 in
I = 671,108 - 6681 = 664,427 in^4
r^2 = 664,427 / 933.64 = 711.65 in^2

Transformed Section – I-Girder (n = 7)

n	No.	As	A	y	Ay	Io	A(y-yb)^2
			933.64	36.576	34149	664427	770
7	2	0.153	2.14	66.638	143	0	2053
7	2	0.153	2.14	64.638	138	0	1796
7	2	0.153	2.14	62.638	134	0	1557
7	2	0.153	2.14	60.638	130	0	1334
7	2	0.153	2.14	58.638	126	0	1129
7	2	0.153	2.14	56.638	121	0	941
7	8	0.153	8.57	8.00	69	0	6560
7	10	0.153	10.71	6.00	64	0	9427
7	10	0.153	10.71	4.00	43	0	10741
7	8	0.153	8.57	2.00	17	0	9714
			985.04		35134	664427	46022

A = 985.04 in^2
y_b = 35134 / 985.04 = 35.668 in
y_t = 72.00 – 35.668 = 36.332 in
I = 664,427 + 46,022 = 710,449 in^4

<u>Composite Section</u> – I-Girder & Deck

n = 3861 / 4070 = 0.949

Interior Girder

n	W	H	A	y	Ay	Io	A(y-yb)^2
			985.04	35.668	35134	710449	304017
0.949	108.00	7.50	768.69	75.75	58228	3603	389634
			1753.73		93362	714052	693651

A = 1753.73 in^2
y_b = 93362 / 1753.73 = 53.236 in
y_t = 72.00 – 53.236 = 18.764 in
I = 714,052 + 693,651 = 1,407,703 in^4

Exterior Girder

n	W	H	A	y	Ay	Io	A(y-yb)^2
			985.04	35.668	35134	710449	261972
0.949	90.00	7.50	640.58	75.75	48524	3003	362058
0.949	16.00	1.00	15.18	71.50	1086	1	5786
0.949	½*16	4.00	30.37	69.67	2116	27	9508
			1671.17		86860	713480	639324

A = 1671.17 in^2
y_b = 86860 / 1671.17 = 51.976 in
y_t = 72.00 – 51.976 = 20.024 in
I = 713,480 + 639,324 = 1,352,804 in^4

Comparison Properties

	Midspan		Transfer Length	
	Precise	Approx	Precise	Approx
At	985.04	985.07	985.04	985.07
yb	35.056	35.055	35.668	35.666
It	711,698	711,399	710,449	683,675
Ac	1753.73	1753.76	1753.73	1753.76
Yb	52.893	52.892	53.236	53.235
Ic	1,430,296	1,430,042	1,407,703	1,381,008

These results show that at the midspan where the strands are closely spaced, the results are nearly identical but where the strands are more widely spread at the ends differences appear. Use of the precise method could reduce the required concrete strength near the ends for some designs but the extra effort to calculate them is usually not warranted.

APPENDIX C

REFINED PRESTRESSED LOSSES

The refined prestress losses are based on the following timeline of events.

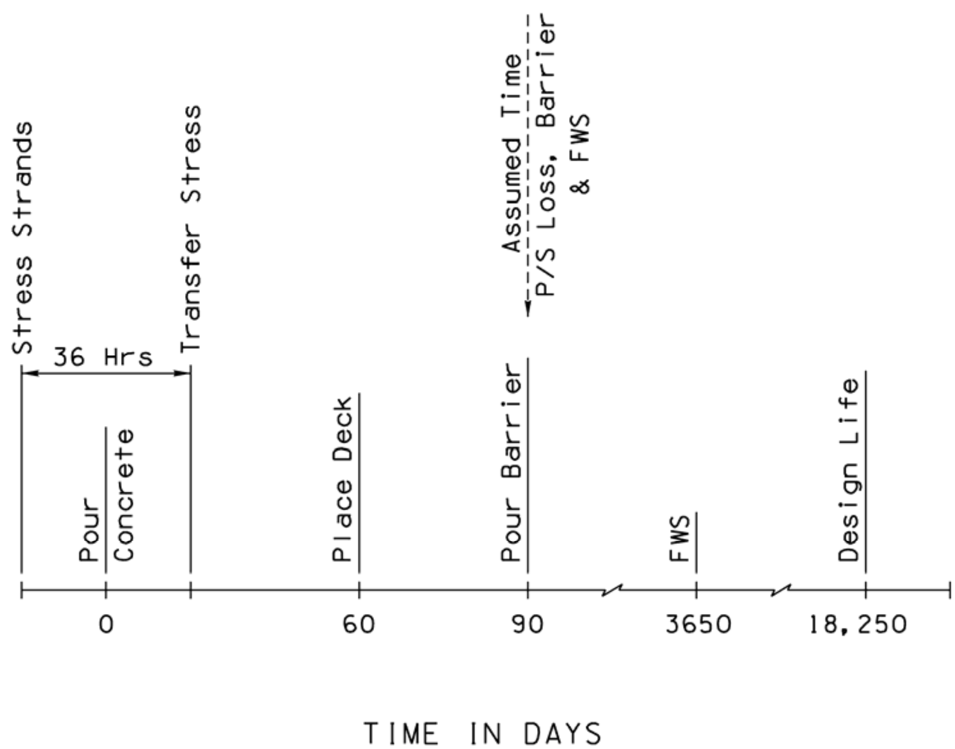


Figure C-1

Refined Time-Dependent Losses - 2006 [5.9.5.4]

For precast pretensioned members more accurate values of creep, shrinkage and relaxation related losses are determined as follows:

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df}$$

Δf_{pSR} = prestress loss due to shrinkage of girder concrete between transfer and deck placement.

Δf_{pCR} = prestress loss due to creep of girder concrete between transfer and deck placement.

Δf_{pR1} = prestress loss due to relaxation of prestressing strands between time of transfer and deck placement.

Δf_{pSD} = prestress loss due to shrinkage of girder concrete between time of deck placement and final time.

Δf_{pCD} = prestress loss due to creep of girder concrete between time of deck placement and final time.

Δf_{pR2} = prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time.

Δf_{pSS} = prestress loss due to shrinkage of deck composite section.

$(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id}$ = sum of time dependent prestress losses between transfer and deck placement.

$(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df}$ = sum of time-dependent prestress losses after deck placement.

The first and last equations show a plus sign before the term $\Delta f_{pSS.}$ The 2006 Interims show a negative sign in front of the variable. The term Δf_{pSS} is a gain in stress and should be subtracted from the losses. However, the term Δf_{pSS} is a negative number so the negative sign is not required here.

Shrinkage of Girder Concrete [5.9.5.4.2a] [5.9.5.4.2a-1]

The prestress loss due to shrinkage of girder concrete between time of transfer and deck placement shall be determined as follows:

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id}$$

Where:

ε_{bid} = concrete shrinkage strain of girder between the time of transfer and deck placement per Eq. 5.4.2.3.3-1.

K_{id} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between transfer and deck placement.

To determine the value of ε_{bid} the shrinkage strain of the girder must be determined at the time of deck placement. The basic equation for shrinkage is:

[5.4.2.3.3-1]

$$\varepsilon_{sh} = -k_{vs}\, k_{hs}\, k_f\, k_{td}\, 0.48 \times 10^{-3}$$

[5.4.2.3.2-2]

$$k_{vs} = 1.45 - 0.13(V/S) = 1.45 - 0.13(3.89) = 0.944$$

A humidity of 40 percent is used for this design.

[5.4.2.3.3-2]

$$k_{hs} = 2.00 - 0.014H = 2.00 - 0.014(40) = 1.440$$

[5.4.2.3.2-4]

$$k_f = \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 4.7} = 0.877$$

[5.4.2.3.2-5]

$$k_{td} = \frac{t}{61 - 4f'_{ci} + t}$$

At time of deck placement, assumed at 60 days:

$$k_{td} = \frac{60}{61 - (4)\cdot(4.7) + 60} = 0.587$$

$$\varepsilon_{bid} = -(0.944)(1.440)(0.877)(0.587)(0.48 \times 10^{-3}) = -0.336 \times 10^{-3}$$

To determine the value of K_{id} the girder creep must be determined at final age as follows:

$\psi_b(t_f, t_i)$ = girder creep coefficient at final time due to loading introduced at transfer per Eq. 5.4.2.3.2-1

t_f = final age = (50 years)(365 days / year) = 18,250 days

t_i = age at transfer = 1 day.

The time between pouring of the concrete and transfer of prestress may be taken as 18 hours for release strengths up to 4.5 ksi. Article [5.9.2.3.2] states that one day of accelerated curing by steam or radian heat may be taken as equal to 7 days of normal curing. This statement has caused confusion since it only applies to girders not ssteam cured and is being deleted.

[5.4.2.3.2-1]

$\psi_b(t_f, t_i) = 1.9\ k_{vs}\ k_{hc}\ k_f\ k_{td}\ t_i^{-0.118}$

[5.4.2.3.2-3]

$k_{hc} = 1.56 - 0.008H = 1.56 - (0.008)(40) = 1.240$

[5.4.2.3.2-5]

$$k_{td} = \frac{18{,}250}{61-(4)\cdot(4.7)+18{,}250} = 0.998 \text{ Use 1.0 for design}$$

$\psi_b(t_f, t_i) = (1.9)(0.944)(1.240)(0.877)(1.0)(1)^{-0.118} = 1.951$

Use gross section properties to determine K_{id}.

[5.9.5.4.2a-2]

$$K_{id} = \frac{1}{1+\frac{E_p}{E_{ci}}\frac{A_{ps}}{A_g}\left(1+\frac{A_g e^2{}_{pg}}{I_g}\right)\left[1+0.7\psi_b(t_f,t_i)\right]}$$

$$K_{id} = \frac{1}{1+\frac{28500}{3946}\frac{7.344}{941}\left(1+\frac{(941)\cdot(30.939)^2}{671{,}108}\right)\left[1+(0.7)\cdot(1.951)\right]}$$

$K_{id} = 0.762$

The prestress loss due to shrinkage between the time of transfer and the time of deck placement can be determined as follows:

$$\Delta f_{pSR} = (0.000336)\cdot(28500)\cdot(0.762) = 7.30 \text{ ksi}$$

Creep of Girder Concrete [5.9.5.4.2b]

The prestress loss due to creep of girder concrete between time of transfer and deck placement is determined as follows:

[5.9.5.4.2b-1]

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \psi_b(t_d, t_i) K_{id}$$

f_{cgp} = concrete stress at center of gravity of prestressing tendons.

$\psi_b(t_d, t_i)$ = girder creep coefficient at time of deck placement due to loading introduced at transfer per Eq. 5.4.2.3.2-1

$\psi_b(t_d, t_i) = 1.9\ k_{vs}\ k_{hc}\ k_f\ k_{td}\ t_i^{-0.118}$

$\psi_b(t_d, t_i) = (1.9)(0.944)(1.240)(0.877)(0.587)(1)^{-0.118} = 1.145$

The effective prestress force is the jacking stress minus the relaxation loss from time of stressing till time of transfer.

$P_t = [(0.75)(270) – 2.23](7.344) = 1470.78$ k

The concrete stress at the centroid of the prestress steel is calculated as follows using the transformed section properties at transfer.

$$f_{cgp} = (1470.78)\cdot\left[\frac{1}{986.68} + \frac{(29.507)^2}{712{,}809}\right] - \frac{(1503)\cdot(12)\cdot(29.507)}{712{,}809}\ \text{ksi}$$

$f_{cgp} = 2.540$ ksi

All the remaining variables have already been determined. The prestress loss due to creep is calculated as follows:

$$\Delta f_{pCR} = \frac{28500}{3946}\cdot(2.540)\cdot(1.145)\cdot(0.762) = 16.01\ \text{ksi}$$

Relaxation of Prestressing Strands [5.9.5.4.2c]

The prestress loss due to relaxation of prestressing strands between time of transfer and deck placement is determined as follows:

[5.9.5.4.2c-1]

$$\Delta f_{pR1} = \frac{f_{pt}}{K_L}\left(\frac{f_{pt}}{f_{py}} - 0.55\right)$$

f_{pt} = stress in prestressing strands immediately after transfer but shall not be less than $0.55f_{py}$ = 0.55(243) = 133.65 ksi. The stress after transfer includes the relaxation loss prior to transfer and the elastic shortening loss.

Determine the elastic shortening loss as follows:

$$\Delta f_{pES} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{3946} \cdot (2.540) = 18.35 \text{ ksi}$$

f_{pt} = (0.75)(270) – 2.23 - 18.35 = 181.92 ksi

K_L = 30 for low relaxation strands.

$$\Delta f_{pR1} = \frac{181.92}{30} \cdot \left(\frac{181.92}{243} - 0.55\right) = 1.20 \text{ ksi}$$

Time-Dependent Losses Prior to Deck Placement

The sum of these first three losses is the time-dependent loss prior to placement of deck.

$(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id}$ = 7.30 + 16.01 + 1.20 = 24.51 ksi

The losses from the time of deck placement to final time consist of four components: shrinkage of girder concrete, creep of girder concrete, relaxation of prestressing strand and shrinkage of deck concrete.

Shrinkage of Girder Concrete [5.9.5.4.3a]

The prestress loss due to shrinkage of the girder concrete between time of deck placement and final time is determined as follows:

[5.9.5.4.3a-1]

$$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df}$$

where:

ε_{bdf} = shrinkage strain of girder between time of deck placement and final time per Eq. 5.4.2.3.3-1.

K_{df} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between deck placement and final time.

e_{pc} = eccentricity of strands with respect to centroid of the net composite section.

A_c = area of section calculated using the net composite concrete section properties of the girder and the deck and the deck-to-girder modular ratio.

I_c = moment of inertia of section calculated using the net composite concrete section properties of the girder and the deck and the deck-to-girder modular ratio at service.

The 2006 Interims state that the variable K_{df} is determined using net composite section properties. However, the variable K_{df} should be determined using gross composite section propertiesto be consistent with the usage of pross section properties in determination of K_{id}.

For determination of K_{df} use gross composite section properties.

[5.9.5.4.3a-2]

$$K_{df} = \frac{1}{1+\frac{E_p}{E_{ci}}\frac{A_{ps}}{A_c}\left(1+\frac{A_c e^2{}_{pc}}{I_c}\right)\left[1+0.7\psi_b\left(t_f,t_i\right)\right]}$$

$$K_{df} = \frac{1}{1+\frac{28500}{3946}\frac{7.344}{1710}\cdot\left(1+\frac{(1710)\cdot(48.613)^2}{1{,}328{,}521}\right)\cdot\left[1+(0.7)\cdot(1.951)\right]}$$

$K_{df} = 0.771$

The value of shrinkage at the time of deck placement has already been calculated. The shrinkage at final time is calculated as follows:

$\varepsilon_{sh} = -k_{vs}\, k_{hs}\, k_f\, k_{td}\; 0.48 \times 10^{-3}$

$\varepsilon_{sh} = -(0.944)(1.440)(0.877)(1.0)(0.48 \times 10^{-3}) = -0.572 \times 10^{-3}$

The difference in strain between time of deck placement and final time is:

$\varepsilon_{bid} = 0.572 \times 10^{-3} - 0.336 \times 10^{-3} = 0.236 \times 10^{-3}$

The prestress loss due to shrinkage after deck placement is:

$\Delta f_{pSR} = \varepsilon_{bdf}\, E_p\, K_{df} = (0.000236)(28500)(0.771) = 5.19$ ksi

Creep of Girder Concrete [5.9.5.4.3b]

The prestress loss due to creep of girder concrete between time of deck placement and final time is determined as:

[5.9.5.4.3b-1]

$$\Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{cgp}\left[\psi_b\left(t_f,t_i\right)-\psi_b\left(t_d,t_i\right)\right]K_{df} + \frac{E_p}{E_c}\Delta f_{cd}\psi_b\left(t_f,t_d\right)K_{df} \geq 0.0$$

where:

$\psi_b(t_f\,,\,t_d)$ = girder creep coefficient at final time due to loading at deck placement per Eq. 5.4.2.3.2-1.

$\psi_b(t_f\,,\,t_d) = (1.9)(0.944)(1.240)(0.877)(1.0)(60)^{-0.118} = 1.203$

Δf_{cd} = change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement, combined with deck weight and superimposed loads.

The long-term time-dependent loss between transfer and deck placement is 24.51 ksi. This change in stress will result in the following tensile forces in the concrete at the centroid of the prestressing using transformed section properties at service (28 days).

$\Delta P = \Delta f_{pid}\ A_{ps} = (24.51)(7.344) = 180.00$ k

$$\Delta f_{cd1} = (180.00)\cdot\left(\frac{1}{985.07} + \frac{(29.555)^2}{711{,}399}\right) = -0.404\ \text{ksi}$$

The addition of the weight of the deck and superimposed loads will result in the following tensile forces in the concrete using transformed section properties.

$$\Delta f_{cd2} = \frac{(1773)\cdot(12)\cdot(29.555)}{711{,}399} + \frac{(350+273)\cdot(12)\cdot(47.392)}{1{,}430{,}042} = -1.132\ \text{ksi}$$

Δf_{cd} = -0.404 – 1.132 = -1.536 ksi

The loss of prestress force is applied at the time of deck placement, the barrier load is applied 30 days afterwards and the FWS is applied at an assumed time of 3650 days. Determine creep losses assuming the three loads are all applied at a time of 90 days as follows:

$$\Delta f_{pCD} = \frac{28500}{3946}\cdot(2.540)\cdot[1.951 - 1.145]\cdot(0.771)$$

$$+\frac{28500}{4070}\cdot(-1.536)\cdot[1.203]\cdot(0.771) = 11.40 - 9.98 = 1.42 \geq 0.0\ \text{ksi}$$

Δf_{pCD} = 1.42 ksi

Relaxation of Prestressing Strands [5.9.5.4.3c] [5.9.5.4.3c-1]

The prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and final time is determined as follows:

$$\Delta f_{pR2} = \Delta f_{pR1} = 1.20\ \text{ksi}$$

Shrinkage of Deck Concrete [5.9.5.4.3d]

The prestress gain due to shrinkage of deck composite section is determined as follows:

[5.9.5.4.3d-1]

$$\Delta f_{pSS} = \frac{E_p}{E_c}\Delta f_{cdf} K_{df}\left[1+0.7\psi_b\left(t_f, t_d\right)\right]$$

in which:

$$\Delta f_{cdf} = \frac{\varepsilon_{ddf} A_d E_{cd}}{\left[1+0.7\psi_d\left(t_f, t_d\right)\right]}\left(\frac{1}{A_c}+\frac{e_{pc}e_d}{I_c}\right)$$

Δf_{cdf} = change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete.

ε_{ddf} = shrinkage strain of deck concrete between placement and final time per Eq. 5.4.2.3.3-1.

The basic equation for shrinkage is:

[5.4.2.3.3-1]

$\varepsilon_{sh} = -k_{vs}\, k_{hs}\, k_f\, k_{td}\ 0.48 \times 10^{-3}$

The volume-to-surface ratio is determined as follows:

$V = (7.50)(108) = 810\ in^2$

S = 108 top +108 bottom – 40 top flange = 176 in

V / S = 810 / 176 = 4.60 in

k_{vs} = 1.45 – 0.13(V/S) = 1.45 – 0.13(4.60) = 0.852

k_{hs} = 2.00 – 0.014H = 2.00 – 0.014(40) = 1.440

Since there is no specified release strength for the deck, use 80% f'_c.

$$k_f = \frac{5}{1+f'_{ci}} = \frac{5}{1+(0.80)\cdot(4.5)} = 1.087$$

$$k_{td} = \frac{t}{61-4f'_{ci}+t} = \frac{18{,}250}{61-(4)\cdot(0.80)\cdot(4.5)+18{,}250} = 0.997 \text{ Use 1.0}$$

$\varepsilon_{ddf} = -(0.852)(1.440)(1.087)(1.0)\ 0.48 \times 10^{-3} = 0.640 \times 10^{-3}$

Other variables are defined as follows:

A_d = area of deck concrete = (7.50)(108) = 810 in^2

E_{cd} = modulus of elasticity of deck concrete = 3861 ksi

e_d = eccentricity of deck with respect to the transformed net composite section, taken as negative in common construction. Use the gross composite properties to be consistent with other calculations.

e_d = -17.887 – 7.50 / 2 = -21.637 in

$\psi_d(t_f, t_d)$ = creep coefficient of deck concrete at final time due to loading introduced shortly after deck placement per Eq. 5.4.2.3.2-1

Once the deck sets any shortening will be transmitted to the girder. Therefore assume t_i = 1 day.

[5.4.2.3.2-1]

$$\psi_d(t_f, t_i) = 1.9\, k_{vs}\, k_{hc}\, k_f\, k_{td}\, t_i^{-0.118}$$

$$\psi_d(t_f, t_d) = (1.9)(0.852)(1.240)(1.087)(1.0)(1)^{-0.118} = 2.182$$

[5.9.5.4.3d-2]

$$\Delta f_{cdf} = \frac{(0.000640)\cdot(810)\cdot(3861)}{[1+(0.7)\cdot(2.182)]}\cdot\left(\frac{1}{1710}+\frac{(48.613)\cdot(-21.637)}{1{,}328{,}521}\right) = -0.164$$

$$\Delta f_{pSS} = \frac{28500}{4070}\cdot(-0.164)\cdot(0.771)\cdot[1+0.7\cdot(1.203)] = -1.63\,\text{ksi}$$

Time-Dependent Losses After Deck Placement

The sum of the time-dependent prestress losses after deck placement is:

$$(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS})_{df} = 5.19 + 1.42 + 1.20 - 1.63 = 6.18 \text{ ksi}$$

Time-Dependent Losses

The final sum of time-dependent losses is:

$$\Delta f_{pLT} = 24.51 + 6.18 = 30.69 \text{ ksi}$$

This compares to the time-dependent loss of 34.00 ksi using the approximate method.

Total Losses

The sum of all losses including relaxation before transfer and elastic shortening is:

Total Loss = 2.23 + 18.35 + 30.69 = 51.27 ksi

APPENDIX D

APPROXIMATE PRESTRESS LOSSES

DEVELOPMENT OF EQUATION

The approximate method for determining time-dependent prestress losses is derived from the refined method shown in Appendix C. The time dependent loss is the sum of the losses before deck placement and those after deck placement. The following equation results:

$$\Delta f_{pLT} = (\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1})_{id} + (\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} + \Delta f_{pSS})_{df}$$

Substituting the appropriate equation for each loss yields:

$$\begin{aligned}\Delta f_{pLT} = {} & \varepsilon_{bid} E_p K_{id} + \frac{E_p}{E_{ci}} f_{cgp} \psi_b(t_d, t_i) K_{id} + \Delta f_{pR1} \\ & + \varepsilon_{bdf} E_p K_{df} + \frac{E_p}{E_{ci}} f_{cgp} \left[\psi_b(t_f, t_i) - \psi_b(t_d, t_i)\right] K_{df} \\ & + \frac{E_p}{E_c} \Delta f_{cd} \psi_b(t_f, t_d) K_{df} + \Delta f_{pR2} + \Delta f_{pSS}\end{aligned}$$

Shrinkage Loss

The shrinkage loss is the sum of the shrinkage loss before deck placement and that after deck placement.

$$\text{Shrinkage} = \varepsilon_{bid} E_p K_{id} + \varepsilon_{bdf} E_p K_{df}$$

Assumption 1

Based on investigation of many examples, assume that $K_{id} = K_{df} = 0.8$

The equation simplifies to:

$$\text{Shrinkage} = 0.8 E_p(\varepsilon_{bid} + \varepsilon_{bdf})$$

Where $\varepsilon_{bid} + \varepsilon_{bd}$ is the total shrinkage strain as shown below:

$$\varepsilon_{sh} = -k_{vs}\, k_{hs}\, k_f\, k_{td}\, 0.48 \times 10^{-3}$$

Assumption 2

Assume that the volume to surface ratio is 3.5, then

$$k_{vs} = 1.45 - 0.13(V/S) = 1.45 - 0.13(3.5) = 1.0$$

$k_{td} = 1.0$ for final time

$$\varepsilon_{sh} = -k_{vs}\, k_{hs}\, k_f\, k_{td}\, 0.48 \times 10^{-3} = (1.0)\, k_{hs}\, k_f\, (1.0)\, 0.48 \times 10^{-3}$$
$$\varepsilon_{sh} = k_{hs}\, k_f\, 0.48 \times 10^{-3}$$

The equation for shrinkage loss reduces to:

$$\text{Shrinkage} = 0.8 E_p(\varepsilon_{bid} + \varepsilon_{bdf}) = 0.8(28{,}500) k_{hs}\, k_f\, 0.48 \times 10^{-3}$$
$$\text{Shrinkage} = 10.94\, k_{hs}\, k_f \Rightarrow \underline{\text{Use} = 12.0\, k_{hs}\, k_f}$$

Creep Loss

The creep loss is the sum of the creep loss before deck placement plus the creep loss after deck placement.

$$\text{Creep} = \frac{E_p}{E_{ci}} f_{cgp}\psi_b(t_d, t_i)K_{id} + \frac{E_p}{E_{ci}} f_{cgp}\left[\psi_b(t_f, t_i) - \psi_b(t_d, t_i)\right]K_{df} + \frac{E_p}{E_c}\Delta f_{cd}\psi_b(t_f, t_d)K_{df}$$

Again assume that $K_{id} = K_{df} = 0.8$ and combine terms resulting in:

$$\text{Creep} = \frac{E_p}{E_{ci}} f_{cgp}\psi_b(t_f, t_i)\cdot(0.8) + \frac{E_p}{E_c}\Delta f_{cd}\psi_b(t_f, t_d)\cdot(0.8)$$

Where:

$\psi_b(t_f, t_i) = 1.9\ k_{vs}\ k_{hc}\ k_f\ k_{td}\ t_i^{-0.118}$

Assume the following:

Volume to surface ratio equals 3.5 $\Rightarrow k_{vs} = 1.0$

Assumption 3 The load is applied at one day $\Rightarrow t_i^{-0.118} = 1.0$

$\psi_b(t_f, t_i) = 1.9\ (1.0)\ k_{hc}\ k_f\ (1.0)\ (1)^{-0.118} = 1.9\ k_{hc}\ k_f$

Assumption 4 Assume that the creep factor for loads applied after the deck pour equals:

$\psi_b(t_f, t_d) = 0.4\ \psi_b(t_f, t_i)$

Assume the following:

Assumption 5 1) $\frac{E_p}{E_{ci}} = 7$ based on $f'_{ci} = 5.0$ ksi

Assumption 6 2) $\frac{E_p}{E_c} = 6$ based on $f'_c = 6.8$ ksi

The equation for creep loss then reduces to the following:

$\text{Creep} = (7)f_{cgp}(1.9\ k_{hc}\ k_f)(0.8) + (6)\Delta f_{cd}(0.4)(1.9\ k_{hc}\ k_f)(0.8)$

$\text{Creep} = 10.64\ k_{hc}\ k_f\ f_{cgp} + 3.65\ k_{hc}\ k_f\ \Delta f_{cd}$

This is similar in form to the current prestress creep equation. Additional assumptions are now made to eliminate the two terms f_{cgp} and Δf_{cd}.

Assume the following:

Assumption 7

1) Final stress at the cg of the strands = 0

Assumption 8

2) Moment from the girder, deck placement and live load are equal, resulting in $\sum M = 3M_g$.

Assumption 9

3) $\left(1+\frac{Ae^2}{I}\right) = 2$

Assumption 10

4) The effective prestress equals 80% of the initial prestress.

Sum the stresses at the c.g. of the strands as follows:

$$f_{cg} = \frac{3M_g e_p}{I_g} - 0.8P_i\left(\frac{1}{A_g}+\frac{e^2}{I_g}\right) = 0$$

$$\frac{3M_g e_p}{I_g} = \frac{0.8P_i}{A_g}\left(1+\frac{Ae_p^{\ 2}}{I_g}\right) = \frac{0.8P_i}{A_g}(2.0)$$

Girder Moment

$$M_g = \frac{1.6P_i I_g}{3A_g e_p}$$

$$f_{cgp} = \frac{P_i}{A_g}\left(1+\frac{A_g e_p^{\ 2}}{I_g}\right) - \frac{M_g e_p}{I_g}$$

$$f_{cgp} = \frac{0.8P_i}{A_g}(2) - \frac{M_g e_p}{I_g} = \frac{1.6P_i}{A_g} - \frac{1.6P_i I_g}{3A_g e_p}\cdot\frac{e_p}{I_g} = \frac{3.2P_i}{3A_g}$$

Deck Stress

$$\Delta f_{cd} = \frac{M_g e_p}{I_g} = \frac{1.6P_i I_g}{3A_g e_p}\cdot\frac{e_p}{I_g} = \frac{-1.6P_i}{3A_g}$$

Creep = 10.64 k_{hc} k_f (3.2P_i / 3A_g) + 3.65 k_{hc} k_f (-1.6P_i / 3A_g)

Creep = 9.40 (P_i / A_g) k_{hc} k_f

Rounding and Substituting f_{pi} A_{ps} = P_i Yields $\underline{10.0(f_{pi}\ A_{ps} / A_g)\ k_{hc}\ k_f}$

Assumption 11

Ignore the gain in prestress from the deck shrinkage.

Relaxation Loss
Assumption 12

Assume the relaxation from the prestressing strands equals 2.5 ksi for low relaxation strands.

The resulting equation is then:

$$\Delta f_{pLT} = 10.0 \frac{f_{pi} A_{ps}}{A_g} \gamma_h \gamma_{st} + 12.0 \gamma_h \gamma_{st} + \Delta f_{pR}$$

in which:

$\gamma_h = 1.7 - 0.01H$ is an average humidity factor for shrinkage and creep.

$$\gamma_{st} = \frac{5}{1 + f'_{ci}}$$

A summary of the assumptions made in the development of the approximate formula are listed below with the corresponding value from this example.

	Assumption	Actual
Assumption 1	$K_{id} = 0.8$ $K_{df} = 0.8$	$K_{id} = 0.762$ $K_{df} = 0.771$
Assumption 2	$V / S = 3.5$ $k_{VS} = 1.0$	$V / S = 3.89$ $k_{VS} = 0.944$
Assumption 3	$t_i = 1.0$	$t_i = 1.0$
Assumption 4	$\psi_b(t_f, t_d) = 0.4\ \psi_b(t_f, t_i)$	$\psi_b(t_f, t_d) = 0.617\ \psi_b(t_f, t_i)$
Assumption 5	$E_p / E_{ci} = 7$	$E_p / E_{ci} = 7.22$
Assumption 6	$E_p / E_c = 6$	$E_p / E_c = 7.00$
Assumption 7	$f_{cgp} = 0$	$f_{cgp} = 0.481$ ksi
Assumption 8	$M_g = M_d = M_{ll}$	$M_g = 1503$ ft-k $M_d = 2396$ ft-k $M_{ll} = 2332$ ft-k

Assumption 9	$1 + A\ e^2 / I = 2$	$1 + A\ e^2 / I = 2.34$ gross $1 + A\ e^2 / I = 4.04$ gross composite
Assumption 10	$P_{eff} = 0.80\ P_i$	$P_{eff} = 0.76\ P_i$
Assumption 11	$\Delta f_{pSS} = 0$ ksi	$\Delta f_{pSS} = 1.63$ ksi
Assumption 12	$\Delta f_r = 2.50$ ksi	$\Delta f_r = 2.40$ ksi

Summary

Assumption 2 is a function of the type of girder/beam. This assumption is close for the I-girder.

Assumption 4 appears to be questionable.

Assumptions 5 and 6 do not reflect current ADOT practice on concrete strengths.

Assumption 8 is not bad for this example but could differ considerably for some girder spacings.

APPENDIX E

ANALYSIS COMPARISON

1) Transformed Section

2) Net Section with Elastic Gain

3) Gross Section with Elastic Gain

4) Gross Section

1) Transformed Section

The concrete and prestress strand stresses will be calculated using transformed section properties.

Transfer Stresses P/S and Beam

The effective prestress force prior to transfer is:

P_i = [(0.75)(270) – 2.23](48)(0.153) = 1470.78 kips

The stress in the concrete using transformed section properties at transfer is:

$$f_t = (1470.78)\cdot\left(\frac{1}{986.68} - \frac{(29.507)\cdot(36.993)}{712{,}809}\right) + \frac{(1503)\cdot(12)\cdot(36.993)}{712{,}809}$$

f_t = -0.762 + 0.936 = 0.174 ksi

$$f_{cgp} = (1470.78)\cdot\left(\frac{1}{986.68} + \frac{(29.507)\cdot(29.507)}{712{,}809}\right) - \frac{(1503)\cdot(12)\cdot(29.507)}{712{,}809}$$

f_{cgp} = 3.287 – 0.747 = 2.540 ksi

$$f_b = (1470.78)\cdot\left(\frac{1}{986.68} + \frac{(29.507)\cdot(35.007)}{712{,}809}\right) - \frac{(1503)\cdot(12)\cdot(35.007)}{712{,}809}$$

f_b = 3.622 – 0.886 = 2.736 ksi

The loss of stress in the prestress strand = (2.540)(28,500) / (3946) = 18.35 ksi

Deck Placement

The stress in the concrete from the deck pour using transformed section properties at service is:

$$f_t = \frac{(1773)\cdot(12)\cdot(36.945)}{711{,}399} = 1.105$$

$$f_{cgp} = -\frac{(1773)\cdot(12)\cdot(29.555)}{711{,}399} = -0.884$$

$$f_b = -\frac{(1773)\cdot(12)\cdot(35.055)}{711{,}399} = -1.048$$

The gain in stress in the prestress strand = (-0.884)(28,500) / (4070) = -6.19 ksi

Composite DL

The stress in the concrete from the composite dead load is:

$$f_t = \frac{(623)\cdot(12)\cdot(19.108)}{1{,}430{,}042} = 0.100$$

$$f_{cgp} = -\frac{(623)\cdot(12)\cdot(47.392)}{1{,}430{,}042} = -0.248$$

$$f_b = -\frac{(623)\cdot(12)\cdot(52.892)}{1{,}430{,}042} = -0.277$$

The gain in stress in the prestress strand = (-0.248)(28,500) / (4070) = -1.74 ksi

Live Load +IM

The stress in the concrete from the live load plus dynamic load allowance is:

$$f_t = \frac{(2332)\cdot(12)\cdot(19.108)}{1{,}430{,}042} = 0.374$$

$$f_{cgp} = -\frac{(2332)\cdot(12)\cdot(47.392)}{1{,}430{,}042} = -0.927$$

$$f_b = -\frac{(2332)\cdot(12)\cdot(52.892)}{1{,}430{,}042} = -1.035$$

The gain in stress in the prestress strand = (-0.927)(28,500) / (4070) = -6.49 ksi

Time-Dependent Loss

The 34.00 ksi prestress loss results in a loss in prestress force of:

P_i = (34.00)(48)(0.153) = 249.70 kips

The stress in the concrete using net section properties is:

$$f_t = (-249.70)\cdot\left(\frac{1}{933.66} - \frac{(31.182)\cdot(35.318)}{664{,}023}\right) = 0.147$$

$$f_b = (-249.70)\cdot\left(\frac{1}{933.66} + \frac{(31.182)\cdot(36.682)}{664{,}023}\right) = -0.698$$

Stress Summary

A summary of stresses in the concrete follows:

Service I

$$f_t = 1.0(0.174 + 0.147 + 1.105 + 0.100) + 1.0(0.374) = 1.900 \text{ ksi}$$

Service III

$$f_b = 1.0(2.736 - 0.698 - 1.048 – 0.277) + 0.8(– 1.035) = -0.115 \text{ ksi}$$

A summary of stress in the strand is:

$$f_{ps} = (0.75)(270) – 2.23 – 18.35 + 6.19 + 1.74 + 6.49 – 34.00 = 162.34 \text{ ksi}$$

2) Net Section With Elastic Gain

The concrete and prestress strand stresses will be calculated using net section properties and including the elastic gain in prestress.

Transfer Stresses P/S and Beam

The effective prestress force after transfer including elastic shortening loss is:

P_i = [(0.75)(270) – 2.23 – 18.35](48)(0.153) = 1336.02 kips

The stress in the concrete is:

$$f_t = (1336.02)\cdot\left(\frac{1}{933.66} - \frac{(31.182)\cdot(35.318)}{664{,}023}\right) + \frac{(1503)\cdot(12)\cdot(35.318)}{664{,}023}$$

f_t = -0.785 + 0.959 = 0.174 ksi

$$f_{cgp} = (1336.02)\cdot\left(\frac{1}{933.66} + \frac{(31.182)\cdot(31.182)}{664{,}023}\right) - \frac{(1503)\cdot(12)\cdot(31.182)}{664{,}023}$$

f_{cgp} = 3.387 – 0.847 = 2.540 ksi

$$f_b = (1336.02)\cdot\left(\frac{1}{933.66} + \frac{(31.182)\cdot(36.682)}{664{,}023}\right) - \frac{(1503)\cdot(12)\cdot(36.682)}{664{,}023}$$

f_b = 3.732 - 0.996 = 2.736 ksi

The loss of stress in the prestress strand = (2.540)(28,500) / (3946) = 18.35 ksi

Deck Placement

Since the strands are bonded to the concrete, the addition of external loads will add tension to the prestress strands. Solution of the problem of determining the concrete stress at the c.g. of the strands is similar to that for elastic shortening as shown in the following formula.

The elastic gain from the applied moment, M is shown below:

$$f_{cgp} = \frac{-Me}{I + \frac{E_p}{E_{ct}} A_{ps}\left(r^2 + e^2\right)}$$

$$f_{cgp} = \frac{-(1773)\cdot(12)\cdot(31.182)}{664{,}023 + \frac{28500}{4070}\cdot(7.344)\cdot\left(711.20 + (31.182)^2\right)} = -0.884 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.884) = -6.19\,\text{ksi}$$

P_{gain} = (6.19)(7.344) = 45.45 k

The stress in the concrete from the deck pour is:

$$f_t = \frac{(1773)\cdot(12)\cdot(35.318)}{664{,}023} + (45.45)\cdot\left(\frac{1}{933.66} - \frac{(31.182)\cdot(35.318)}{664{,}023}\right)$$

f_t = 1.132 - 0.027 = 1.105 ksi

$$f_b = -\frac{(1773)\cdot(12)\cdot(36.682)}{664{,}023} + (45.45)\cdot\left(\frac{1}{933.66} + \frac{(31.182)\cdot(36.682)}{664{,}023}\right)$$

f_b = -1.175 + 0.127 = -1.048 ksi

Time-Dependent Loss

The time-dependent loss of 34.00 ksi will not cause any elastic gain so net section properties should be used. The loss of stress in the concrete will be:

$$f_t = -(34.00)\cdot(7.344)\cdot\left(\frac{1}{933.66} - \frac{(31.182)\cdot(35.318)}{664{,}023}\right)$$

f_t = 0.147 ksi

$$f_b = -(34.00)\cdot(7.344)\cdot\left(\frac{1}{933.66} + \frac{(31.182)\cdot(36.682)}{664{,}023}\right)$$

f_b = -0.698 ksi

Net Composite Section Properties

The net composite section properties must be calculated as follows:

A_c = 933.66 + (0.949)(108.00)(7.50) = 1702.35 in^2

y_b = [(933.66)(36.682) + 0.949(7.50)(108)(75.75)] ÷ 1702.35 = 54.323 in

e = 54.323 – 5.50 = 48.823 in

y_t = 72.00 – 54.323 = 17.677 in

I_t = 664,023 + 933.66(36.682 – 54.323)^2 + 0.949(108)(7.50)^3 ÷12
+ 0.949(108)(7.50)(75.75 – 54.323)^2 = 1,311,104 in^4

r^2 = 1,311,104 / 1702.35 = 770.17 in^2

Composite Dead Load

The elastic gain from the applied moment, M is shown below:

Elastic Gain: $$f_{cgp} = \frac{-Me}{I + \frac{E_p}{E_{ct}} A_{ps}\left(r^2 + e^2\right)}$$

$$f_{cgp} = \frac{-(623)\cdot(12)\cdot(48.823)}{1{,}311{,}104 + \frac{28500}{4070}\cdot(7.344)\cdot\left(770.17 + (48.823)^2\right)} = -0.248 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.248) = 1.73\,\text{ksi}$$

P_{gain} = (1.73)(7.344) = 12.74 k

The stress in the concrete from the composite dead load is:

$$f_t = \frac{(623)\cdot(12)\cdot(17.677)}{1{,}311{,}104} + (12.74)\cdot\left(\frac{1}{1702.35} - \frac{(48.823)\cdot(17.677)}{1{,}311{,}104}\right)$$

f_t = 0.101 - 0.001 = 0.100 ksi

$$f_b = -\frac{(623)\cdot(12)\cdot(54.323)}{1{,}311{,}104} + (12.74)\cdot\left(\frac{1}{1702.35} + \frac{(48.823)\cdot(54.323)}{1{,}311{,}104}\right)$$

f_b = -0.310 + 0.033 = -0.277 ksi

Live Load + IM

The elastic gain from the applied moment, M is shown below:

$$f_{cgp} = \frac{-(2332)\cdot(12)\cdot(48.823)}{1{,}311{,}104 + \frac{28500}{4070}\cdot(7.344)\cdot\left(770.17 + (48.823)^2\right)} = -0.927 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.927) = -6.49\,\text{ksi}$$

P_{gain} = (6.49)(7.344) = 47.69 k

The stress in the concrete from the live load is:

$$f_t = \frac{(2332)\cdot(12)\cdot(17.677)}{1{,}311{,}104} + (47.69)\cdot\left(\frac{1}{1702.35} - \frac{(48.823)\cdot(17.677)}{1{,}311{,}104}\right)$$

f_t = 0.377 – 0.003 = 0.374 ksi

$$f_b = -\frac{(2332)\cdot(12)\cdot(54.323)}{1{,}311{,}104} + (47.69)\cdot\left(\frac{1}{1702.35} + \frac{(48.823)\cdot(54.323)}{1{,}311{,}104}\right)$$

f_b = -1.159 + 0.124 = -1.035 ksi

A summary of stresses in the concrete follows:

Service I

f_t = 1.0(0.174 + 0.147 + 1.105 + 0.100) + 1.0(0.374) = 1.900 ksi

Service III

f_b = 1.0(2.736 – 0.698 – 1.048 – 0.277) + 0.8(– 1.035) = -0.115 ksi

A summary of stress in the strand is:

f_{ps} = (0.75)(270) – 2.23 – 18.35 + 6.19 + 1.73 + 6.49 – 34.00
= 162.33 ksi

3) Gross Section With Elastic Gain

The concrete and prestress strand stresses will be calculated using gross section properties and including the elastic gain in prestress.

Elastic Shortening

The elastic shortening loss is determined using gross section properties as follows:

$$\Delta f_{pES} = \frac{f_{pbt} A_{ps}\left(r^2 + e_m{}^2\right) - e_m M_g}{A_{ps}\left(r^2 + e_m{}^2\right) + \dfrac{I \cdot E_{ci}}{E_p}}$$

$$\Delta f_{pES} = \frac{(200.27)\cdot(7.344)\cdot\left(713.19 + (30.939)^2\right) - (30.939)\cdot(1503)\cdot(12)}{(7.344)\cdot\left(713.19 + (30.939)^2\right) + \dfrac{(671{,}108)\cdot(3946)}{28{,}500}}$$

Δf_{pES} = 18.05 ksi

Transfer Stresses P/S and Beam

The effective prestress force after transfer including elastic shortening loss is:

P_i = [(0.75)(270) – 2.23 – 18.05](48)(0.153) = 1338.22 kips

The stress in the concrete is:

$$f_t = (1338.22)\cdot\left(\frac{1}{941} - \frac{(30.939)\cdot(35.561)}{671{,}108}\right) + \frac{(1503)\cdot(12)\cdot(35.561)}{671{,}108}$$

f_t = -0.772 + 0.956 = 0.184 ksi

$$f_{cgp} = (1338.22)\cdot\left(\frac{1}{941} + \frac{(30.939)\cdot(30.939)}{671{,}108}\right) - \frac{(1503)\cdot(12)\cdot(30.939)}{671{,}108}$$

f_{cgp} = 3.331 – 0.831 = 2.500 ksi

$$f_b = (1338.22)\cdot\left(\frac{1}{941} + \frac{(30.939)\cdot(36.439)}{671{,}108}\right) - \frac{(1503)\cdot(12)\cdot(36.439)}{671{,}108}$$

f_b = 3.670 – 0.979 = 2.691 ksi

The loss in stress in the prestress strand = (2.500)(28,500) / (3946) = 18.06 ksi

Deck Placement

Since the strands are bonded to the concrete, the addition of external loads will add tensile stress to the strands. Solution of the problem of determining the concrete stress at the c.g. of the strands is similar to that of elastic shortening but in this case the prestress component is a function of the applied load.

The elastic gain from the applied moment, M is shown below:

$$f_{cgp} = \frac{-Me}{I + \frac{E_p}{E_{ct}} A_{ps}\left(r^2 + e^2\right)}$$

$$f_{cgp} = \frac{-(1773)\cdot(12)\cdot(30.939)}{671{,}108 + \frac{28500}{4070}\cdot(7.344)\cdot\left(713.19 + (30.939)^2\right)} = -0.870 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.870) = -6.09 \text{ ksi}$$

P_{gain} = (6.09)(7.344) = 44.72 k

The stress in the concrete from the deck pour is:

$$f_t = \frac{(1773)\cdot(12)\cdot(35.561)}{671{,}108} + (44.72)\cdot\left(\frac{1}{941} - \frac{(30.939)\cdot(35.561)}{671{,}108}\right)$$

f_t = 1.127 - 0.026 = 1.101 ksi

$$f_b = -\frac{(1773)\cdot(12)\cdot(36.439)}{671{,}108} + (44.72)\cdot\left(\frac{1}{941} + \frac{(30.939)\cdot(36.439)}{671{,}108}\right)$$

f_b = -1.155 + 0.123 = -1.032 ksi

Time-Dependent Loss

The time-dependent loss of 34.00 ksi will not cause any elastic gain. The stress in the concrete from the time-dependent loss is:

$$f_t = -(34.00)\cdot(7.344)\cdot\left(\frac{1}{941} - \frac{(30.939)\cdot(35.561)}{671{,}108}\right)$$

f_t = 0.144 ksi

$$f_b = -(34.00)\cdot(7.344)\cdot\left(\frac{1}{941} + \frac{(30.939)\cdot(36.439)}{671{,}108}\right)$$

f_b = -0.685 ksi

Composite Dead Load

The elastic gain from the applied moment, M is shown below where gross composite section properties are used.

$$f_{cgp} = \frac{-Me}{I + \frac{E_p}{E_{ct}} A_{ps}\left(r^2 + e^2\right)}$$

$$f_{cgp} = \frac{-(623)\cdot(12)\cdot(48.613)}{1{,}328{,}521 + \frac{28500}{4070}\cdot(7.344)\cdot\left(777.05 + (48.613)^2\right)} = -0.244 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.244) = -1.71 \text{ ksi}$$

P_{gain} = (1.71)(7.344) = 12.54 k

The stress in the concrete from the composite dead load is:

$$f_t = \frac{(623)\cdot(12)\cdot(17.887)}{1{,}328{,}521} + (12.54)\cdot\left(\frac{1}{1709.69} - \frac{(48.613)\cdot(17.887)}{1{,}328{,}521}\right)$$

f_t = 0.101 - 0.001 = 0.100 ksi

$$f_b = -\frac{(623)\cdot(12)\cdot(54.113)}{1{,}328{,}521} + (12.54)\cdot\left(\frac{1}{1709.69} + \frac{(48.613)\cdot(54.113)}{1{,}328{,}521}\right)$$

f_b = -0.305 + 0.032 = -0.273 ksi

Live Load + IM

The elastic gain from the applied moment, M is shown below:

$$f_{cgp} = \frac{-(2332)\cdot(12)\cdot(48.613)}{1{,}328{,}521 + \frac{28500}{4070}\cdot(7.344)\cdot\left(777.05 + (48.613)^2\right)} = -0.913 \text{ ksi}$$

$$\Delta f_{gain} = \frac{E_p}{E_{ct}} f_{cgp} = \frac{28500}{4070}\cdot(-0.913) = -6.39 \text{ ksi}$$

P_{gain} = (6.39)(7.344) = 46.95 k

The stress in the concrete from the live load is:

$$f_t = \frac{(2332)\cdot(12)\cdot(17.887)}{1{,}328{,}521} + (46.95)\cdot\left(\frac{1}{1709.69} - \frac{(48.613)\cdot(17.887)}{1{,}328{,}521}\right)$$

f_t = 0.377 - 0.003 = 0.374 ksi

$$f_b = -\frac{(2332)\cdot(12)\cdot(54.113)}{1{,}328{,}521} + (46.95)\cdot\left(\frac{1}{1709.69} + \frac{(48.613)\cdot(54.113)}{1{,}328{,}521}\right)$$

f_b = -1.140 + 0.120 = -1.020 ksi

A summary of stresses in the concrete follows:

Service I

f_t = 1.0(0.184 + 0.144 + 1.101 + 0.100) + 1.0(0.374) = 1.903 ksi

Service III

f_b = 1.0(2.691 – 0.685 – 1.032 – 0.273) + 0.8(-1.020) = -0.115 ksi

A summary of stress in the strand is:

f_{ps} = (0.75)(270) – 2.23 – 18.05 + 6.09 + 1.71 + 6.39 – 34.00
= 162.41 ksi

Summary
Top Fiber Stress

	Method 1	Method 2	Method 3	Method 4
Girder	0.936	0.959	0.956	0.956
P/S	-0.762	-0.785	-0.772	-0.772
Subtotal	0.174	0.174	0.184	0.184
Deck	1.105	1.132	1.127	1.127
Elastic Gain		-0.027	-0.026	
Subtotal	1.105	1.105	1.101	1.127
Comp DL	0.100	0.101	0.101	0.101
Elastic Gain		-0.001	-0.001	
Subtotal	0.100	0.100	0.100	0.101
LL + IM	0.374	0.377	0.377	0.377
Elastic Gain		-0.003	-0.003	
Subtotal	0.374	0.374	0.374	0.377
P/S Losses	0.147	0.147	0.144	0.144
Total	**1.900**	**1.900**	**1.903**	**1.933**

Bottom Fiber Stress

	Method 1	Method 2	Method 3	Method 4
Girder	-0.886	-0.996	-0.979	-0.979
P/S	3.622	3.732	3.670	3.670
Subtotal	2.736	2.736	2.691	2.691
Deck	-1.048	-1.175	-1.155	-1.155
Elastic Gain		0.127	0.123	
Subtotal	-1.048	-1.048	-1.032	-1.155
Comp DL	-0.277	-0.310	-0.305	-0.305
Elastic Gain		0.033	0.032	
Subtotal	-0.277	-0.277	-0.273	-0.305
LL + IM	-1.035	-1.159	-1.140	-1.140
Elastic Gain		0.124	0.120	
Subtotal	-1.035	-1.035	-1.020	-1.140
0.80	-0.828	-0.828	-0.816	-0.912
P/S Losses	-0.698	-0.698	-0.685	-0.685
Total	**-0.115**	**-0.115**	**-0.115**	**-0.366**

Summary

As can be seen in the above table, use of transformed section properties (Method 1) and use of net section properties considering elastic gain (Method 2) produce nearly identical results for each load except for minor differences due to rounding. Use of gross section properties considering elastic gain (Method 3) produces stresses close to the first two methods with the final sum almost identical. Any of these three methods should be acceptable.

Use of gross section properties without considering elastic gain (Method 4) produces higher stresses than the other three methods. While this method is the simpliest to use it is overly conservative and should not be used.

Method 1 requires maximum effort to determine the section properties but once determined the calculation of concrete stresses is simple. Method 2 requires some effort to determine the section properties and also requires consideration of elastic gain. Use of gross section properties is simpliest for calculation of section properties but does require use of the elastic gain to produce reliable results.