## SUBSTRUCTURE EXAMPLE

Full Height
Abutment on Spread Footing

This example illustrates the design of a full height abutment on spread footings for a single span cast-in-place post-tensioned concrete box girder bridge. The bridge has a 160 feet span with a 15 degree skew. Standard ADOT 32-inch f-shape barriers will be used resulting in a typical deck section consisting of $1^{\prime}-5$ " barrier, $12^{\prime}-0$ " outside shoulder, two $12^{\prime}-0$ " lanes, a $6^{\prime}-0$ " inside shoulder and $1^{\prime}-5 "$ " barrier. The overall out-to-out width of the bridge is $44^{\prime}-10^{\prime \prime}$. A plan view and typical section of the bridge are shown in Figures 1 and 2.

The following legend is used for the references shown in the left-hand column:
[2.2.2] LRFD Specification Article Number
[2.2.2-1] LRFD Specification Table or Equation Number
[C2.2.2] LRFD Specification Commentary
[A2.2.2] LRFD Specification Appendix
[BDG] ADOT Bridge Design Guideline

## Superstructure

Design Example 1 demonstrates design of the superstructure and bearings for a single span cast-in-place post-tensioned concrete box girder bridge using
LRFD. Critical dimensions and loads are repeated here for ease of reference.

## Bridge Geometry

Bridge span length $\quad 160.00 \mathrm{ft}$
Bridge width $\quad 44.83 \mathrm{ft}$
Roadway width $\quad 42.00 \mathrm{ft}$

## Loads

DC Superstructure $\quad 1229.30 \mathrm{kips}$
DW Superstructure 85.63 kips

## Substructure

This example demonstrates basic design features for design of a full height abutment supported on a spread footing. The substructure has been analyzed in accordance with the AASHTO LRFD Bridge Design Specifications, $4^{\text {th }}$ Edition, 2007 and the 2008 Interim Revisions.

## Geotechnical

The soil profile used in this example is the one used for the Geotechnical Policy Memo Number 1: "Development of Factored Bearing Resistance Chart by a Geotechnical Engineer for Use by a Bridge Engineer to Size Spread Footings on Soils based on Service and Strength Limit States".


Figure 1


Figure 2

## Material Properties

[5.4.3.2]
[3.5.1-1]
[C3.5.1]
[C5.4.2.4]
[5.7.1]
[5.7.2.2]

Modulus of
Rupture
[5.4.2.6]

## Reinforcing Steel

Yield Strength $\quad f_{y}=60 \mathrm{ksi}$
Modulus of Elasticity $\mathrm{E}_{\mathrm{s}}=29,000 \mathrm{ksi}$

## Concrete

$$
\mathrm{f}_{\mathrm{c}}{ }_{\mathrm{c}}=3.5 \mathrm{ksi}
$$

Unit weight for normal weight concrete is listed below. The unit weight for reinforced concrete is increased 0.005 kcf greater than that for plain concrete.

Unit weight for computing $\mathrm{E}_{\mathrm{c}}=0.145 \mathrm{kcf}$
Unit weight for DL calculation $=0.150 \mathrm{kcf}$
The modulus of elasticity for normal weight concrete where the unit weight is 0.145 kcf may be taken as shown below:

$$
E_{c}=1820 \sqrt{f^{\prime}{ }_{c}}=1820 \sqrt{3.5}=3405 \mathrm{ksi}
$$

The modular ratio of reinforcing to concrete should be rounded to the nearest whole number.

$$
n=\frac{29000}{3405}=8.52 \text { Use } \mathrm{n}=9
$$

$\beta_{1}=$ the ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone stress block. For concrete strengths not exceeding $4.0 \mathrm{ksi}, \beta_{1}=0.85$.

The modulus of rupture for normal weight concrete has several values. When used to calculate service level cracking, as specified in Article 5.7.3.4 for side reinforcing or in Article 5.7.3.6.2 for determination of deflections, the following equation should be used:

$$
f_{r}=0.24 \sqrt{f_{c}^{\prime}}=0.24 \sqrt{3.5}=0.449 \mathrm{ksi}
$$

When the modulus of rupture is used to calculate the cracking moment of a member for determination of the minimum reinforcing requirement as specified in Article 5.7.3.3.2, the following equation should be used:

$$
f_{r}=0.37 \sqrt{f_{c}^{\prime}}=0.37 \sqrt{3.5}=0.692 \mathrm{ksi}
$$

## Existing Soil

The existing soil has the following properties:

| Depth <br> ft | Soil Type | Total unit weight, $\gamma_{s}$ <br> pcf | $\varphi ’$ <br> degrees |
| :---: | :--- | :---: | :---: |
| $0-25$ | Fine to coarse sands | 120 | 30 |
| $25-75$ | Gravelly sands | 125 | 36 |
| $75-90$ | Fine to coarse sands | 120 | 30 |
| $90-130$ | Gravels | 125 | 38 |

The following assumptions have been made. No groundwater is present. The soils will not experience any long-term (consolidation or creep) settlement.

The Factored Net Bearing Resistance Chart plots the factored net bearing resistance versus effective footing width for a range of immediate settlements as shown in Figure 3.


Figure 3

## Backfill Soil

The soil used for backfill has the following properties:

$$
\begin{aligned}
& \gamma_{\mathrm{s}}=0.120 \mathrm{kcf} \\
& \mathrm{k}_{\mathrm{a}}=0.292
\end{aligned}
$$

## Limit States

[1.3.2]
[1.3.2.1-1]
[1.3.2.1-2]
[1.3.2.1-3]
[1.3.3]
[1.3.4]
[1.3.5]
[BDG]

In the LRFD Specification, the general equation for design is shown below:

$$
\sum \eta_{i} \gamma_{i} Q_{i} \leq \varphi R_{n}=R_{r}
$$

For loads for which a maximum value of $\gamma_{i}$ is appropriate:

$$
\eta_{i}=\eta_{D} \eta_{R} \eta_{I} \geq 0.95
$$

For loads for which a minimum value of $\gamma_{i}$ is appropriate:

$$
\eta_{i}=\frac{1}{\eta_{D} \eta_{R} \eta_{I}} \leq 1.0
$$

## Ductility

For strength limit state for conventional design and details complying with the LRFD Specifications and for all other limit states:

$$
\eta_{\mathrm{D}}=1.0
$$

## Redundancy

For the strength limit state for conventional levels of redundancy and for all other limit states:

$$
\eta_{\mathrm{R}}=1.0
$$

## Operational Importance

For the strength limit state for typical bridges and for all other limit states:

$$
\eta_{I}=1.0
$$

For an ordinary structure with conventional design and details and conventional levels of ductility, redundancy, and operational importance, it can be seen that $\eta_{i}=1.0$ for all cases. Since multiplying by 1.0 will not change any answers, the load modifier $\eta_{\mathrm{i}}$ has not been included in this example.

For actual designs, the importance factor may be a value other than one. The importance factor should be selected in accordance with the ADOT Bridge Design Guidelines.

## SUBSTRUCTURE

Loads
Section 3
[10.5.2]
[10.5.3]

## Loads

There are several major changes and some minor changes concerning the determination of loads. The DC loads must be kept separate from the DW loads since different load factors apply. The live load is different as seen in the superstructure design. The dynamic load allowance is a constant rather than a function of the span and only applies to members above the ground. The Longitudinal Force in the Standard Specifications has been modified and replaced by the Braking Force. A vehicle collision force relating to protection of piers or abutments has been added. The wind and wind on live load is similar but has a modification factor for elevations above 30 feet. The vertical wind pressure is the same but the specification clarifies how to apply the force to the proper load group. The lateral earth pressure includes better clarification of the following items: when to use the Rankine or Coulomb Method, when to use active or at rest pressure, and when to use the equivalent fluid pressure method. The discussion of dead and live load surcharges is enhanced.


Figure 4
Abutment 1 is pinned while Abutment 2 is expansion. The pinned abutment will resist externally applied longitudinal forces. The expansion abutment will resist the friction and internal forces from the deformation of the bearing pads. Since determining which abutment is critical is not obvious, the forces at each abutment will be determined.

## Limit States

For substructure design, foundation design at the service limit state includes settlement, lateral displacement and overall stability.

Foundation design at the strength limit state includes bearing resistance, limiting eccentricity (excessive loss of contact), sliding at the base of the footing, and structural resistance. Three strength limit states require investigation. Strength I is the basic load combination without wind. Strength III is the load combination including wind exceeding 55 mph . Strength V is the load combination combining normal vehicular use with a wind of 55 mph .

For substructure design, Extreme Event I load combination includes seismic events while Extreme Event II load combination includes collision of substructure units by vehicles. These limit states are not considered in this example.

A diagram showing the general dimensions (feet) for the abutment follows:


Figure 5

## [3.5]

[3.5.1]

## PERMANENT LOADS

## DC - Dead Load Structural Components

DC superstructure dead load includes self-weight including intermediate and abutment diaphragms and barriers.

DC Superstructure $=1229.30 \mathrm{k}$
$\mathrm{e}_{\text {long }}=(16.00 / 2-5.50-1.25)=1.25 \mathrm{ft}$
$\mathrm{M}_{\text {long }}=(1229.30)(1.25)=1537 \mathrm{ft}-\mathrm{k}$
DC substructure dead load includes the weight of the abutment including end blocks, wingwalls and footing.

| Item | N |  | H | W | L | Weight | $\mathrm{X}_{\mathrm{A}}$ | $\mathrm{M}_{\mathrm{A}}$ |
| :--- | :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Backwall | 1 | 1 | 7.67 | 1.00 | 46.41 | 53.39 | 9.00 | 481 |
| Seat | 1 | 1 | 1.00 | 1.00 | 46.41 | 6.96 | 10.00 | 70 |
|  | 1 | $1 / 2$ | 1.00 | 1.00 | 46.41 | 3.48 | 9.83 | 34 |
| Stem | 1 | 1 | 13.33 | 4.00 | 46.41 | 371.19 | 7.50 | 2784 |
| Footing | 1 | 1 | 3.50 | 16.00 | 49.52 | 415.97 | 8.00 | 3328 |
| End Blk | 2 | 1 | 6.64 | 3.45 | 3.00 | 20.62 | 7.00 | 144 |
|  | 2 | $1 / 2$ | 6.46 | 2.67 | 3.00 | 7.76 | 7.00 | 54 |
| Wing | 2 | 1 | 21.00 | 1.00 | 10.87 | 68.48 | 10.75 | 736 |
|  |  |  |  |  |  |  |  |  |
| Total |  |  |  |  |  | 947.85 |  | 7631 |

DC Substructure $=947.85 \mathrm{k}$
c.g. $=7631 / 947.85=8.051 \mathrm{ft}$
$\mathrm{e}_{\text {long }}=16.00 / 2-8.051=-0.051 \mathrm{ft}$
$\mathrm{M}_{\text {long }}=(947.85)(-0.051)=-48 \mathrm{ft}-\mathrm{k}$

## DW - Dead Load Wearing Surface and Utilities

The DW superstructure load includes the future wearing surface and utility loads. This bridge has no utilities.

$$
\begin{aligned}
& \mathrm{DW}=85.63 \mathrm{k} \\
& \mathrm{M}_{\text {long }}=(85.63)(1.25)=107 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

## EV - Vertical Earth Pressure

The LRFD Specification does not provide data on unit weights of well compacted soils. For this example use a vertical earth pressure based on a unit weight of 0.120 kcf . In actual design use the values specified in the Geotechnical Report.

Design for the full height of the wall even though the soil only extends to the top of the seat. To avoid the complexities of how to deal with the weight of the approach slab it is simpler to design for the taller height of soil.

| Item | N |  | H | W | L | Weight | $\mathrm{X}_{\mathrm{A}}$ | $\mathrm{M}_{\mathrm{A}}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Toe | 1 | 1 | 3.00 | 5.50 | 49.52 | 98.05 | 2.75 | 270 |
| Heel | 1 | 1 | 21.00 | 6.50 | 46.41 | 760.20 | 12.75 | 9693 |
| Seat | 1 | 1 | 1.00 | 1.00 | 46.41 | -5.57 | 10.00 | -56 |
|  | 1 | $1 / 2$ | 1.00 | 1.00 | 46.41 | -2.78 | 9.83 | -27 |
| Total |  |  |  |  |  | 849.89 |  | 9879 |

Note: Some numbers may not add up due to rounding.

$$
\begin{aligned}
& \mathrm{EV}=850 \mathrm{kips} \\
& \text { c.g. }=9879 / 849.89=11.624 \mathrm{ft} \\
& \mathrm{e}_{\text {long }}=16.00 / 2-11.624=-3.624 \mathrm{ft} \\
& \mathrm{M}_{\text {long }}=(849.89)(-3.624)=-3080 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

## [3.11.5]

[C3.11.5.3]

## EH - Horizontal Earth Pressure

Two decisions must be made before analysis begins: (1) whether to use at-rest or active lateral earth pressure and (2) whether to use the Rankine or Coulomb theory.

Typical abutments supported on cohensionless soils with elastomeric bearings supporting the superstructure with structural grade backfill will deflect adequately to mobilize active soil pressure. Therefore, active pressure will be used in the design.

The LRFD Specification states that the Coulomb method is necessary for design of retaining walls where the back face of the wall interferes with the development of the full sliding surfaces in the backfill soil assumed in the Rankine theory. Abutment concrete cantilever walls with short heels will require the use of the Coulomb method. Abutment concrete cantilever walls with long heels may be designed with either the Rankine or Coulomb method.

The LRFD Specification indicates that the Rankine method of determining lateral earth pressure is not appropriate when the heel is determined to be a short heel. However, the use of the Coulomb Method is a major departure from ADOT past practice. In addition, a value of friction must be considered in the Coulomb Method yet the recommended value varies widely.

## [BDG]

[3.11.5.1-1]
"Foundation Analysis and Design" by Bowles partially agrees with the LRFD Specification but comes to the conclusion that neither method in its pure form can be used. However, either method can be used if the following modification is made: the soil loads are applied to a vertical line extending from the end of the heel and the soil on top of the heel is treated as a static load.

The Rankine formula provides more conservative designs, is allowed per ADOT Bridge Design Guidelines and will be used in this example.

The soil data will be provided in the Geotechnical Report. For this problem assume that the soil extends to the full height of the abutment with the following properties:

$$
\begin{aligned}
& \gamma_{\mathrm{s}}=0.120 \mathrm{kcf} \\
& \mathrm{k}_{\mathrm{a}}=0.295
\end{aligned}
$$

The lateral earth pressure is assumed to be linearly proportional to the depth of earth and taken as:

$$
\begin{aligned}
& \mathrm{p}=\mathrm{k}_{\mathrm{a}} \gamma_{\mathrm{s}} \mathrm{z}=(0.295)(0.120)(24.50)=0.867 \mathrm{ksf} / \mathrm{ft} \\
& \mathrm{EH}=0.5(0.867)(24.50)(46.41)=493.08 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

The resultant acts at a height of $\mathrm{H} / 3$ above the base of the wall.

$$
\mathrm{M}_{\text {long }}=(493.08)(24.50) / 3=4027 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

## [3.6.1.1]

[3.6.1.1.1]
[3.6.1.1.2]

## TRANSIENT LOADS

## LL - Vehicular Live Load

The number of design lanes is the integer part of the ratio $\mathrm{w} / 12=42.00 / 12=3$ where $w$ is the clear roadway width. The critical live load reaction is the combination of the design lane ( 52.19 kips ) and design truck ( 67.80 kips ). Refer to the Superstructure Example 1 for calculation of the live load reactions. Apply the multiple presence factor, $m$, for the reaction. Critical values are underlined.

| One Vehicle | $\mathrm{P}=(52.19+67.80)(1.20)(1)=143.99 \mathrm{k}$ |
| :--- | :--- |
|  | $\mathrm{e}_{\mathrm{L}}=16.00 / \cos (15)=16.56 \mathrm{ft}$ |
|  | $\mathrm{M}_{\text {trans }}=(143.99)(16.56)=2384 \mathrm{ft} \mathrm{k}$ |
|  | $\mathrm{M}_{\text {long }}=(143.99)(1.25)=180 \mathrm{ft} \mathrm{k}$ |
| Two Vehicles | $\mathrm{P}=(52.19+67.80)(1.00)(2)=239.98 \mathrm{k}$ |
|  | $\mathrm{e}_{\mathrm{L}}=10.00 / \cos (15)=10.35 \mathrm{ft}$ |
|  | $\mathrm{M}_{\text {trans }}=(239.98)(10.35)=\underline{2484 \mathrm{ft} \mathrm{k}}$ |
|  | $\mathrm{M}_{\text {long }}=(239.98)(1.25)=300 \mathrm{ft} \mathrm{k}$ |

Three Vehicles $\quad \mathrm{P}=(52.19+67.80)(0.85)(3)=\underline{305.97 \mathrm{k}}$
$\mathrm{e}_{\mathrm{L}}=4.00 / \cos (15)=4.14 \mathrm{ft}$
$\mathrm{M}_{\text {trans }}=(305.97)(4.14)=1267 \mathrm{ft}-\mathrm{k}$
$\mathrm{M}_{\text {long }}=(305.97)(1.25)=\underline{382 \mathrm{ft}-\mathrm{k}}$


Figure 6

To simplify the problem, the maximum reaction and moments will be used even though they do not occur simultaneously. This will reduce the number of load cases without substantially simplifying the design.

## [3.6.2]

IM - Dynamic Load Allowance
Dynamic load allowance need not apply for foundation components that are entirely below ground such as footings. For the portion of the abutment above the ground, the dynamic load allowance is only a design load for the stem.
[3.6.4]
BR - Vehicular Braking Force
The braking force shall be taken as the greater of:
25 percent of the axle weights of the design truck or design tandem

$$
\mathrm{V}=(0.25)(32+32+8)=18.00 \mathrm{k}<=\text { Critical }
$$

$$
\mathrm{V}=(0.25)(25+25)=12.50 \mathrm{k}
$$

5 percent of the design truck plus lane load or 5 percent of the design tandem plus lane load

$$
\begin{aligned}
& V=(0.05)[32+32+8+(160.00)(0.640)]=8.72 \mathrm{k} \\
& \mathrm{~V}=(0.05)[25+25+(160.00)(0.640)]=7.62 \mathrm{k}
\end{aligned}
$$

It should be noted that the truck load will always control and the tandem force need not be calculated.

The braking force shall be placed in all design lanes that are considered to be loaded which carry traffic in the same direction. For this bridge the number of lanes equals the clear roadway width of 42 feet divided by 12 foot lanes $=3.5$. Since only full lanes are used, use 3 lanes. The bridge is a one directional structure with all lanes headed in the same direction. Therefore, all design lanes shall be simultaneously loaded and the multiple presence factor shall apply.

$$
\mathrm{BR}=(18.00)(3)(0.85)=45.90 \mathrm{k}
$$

This load is applied 6 feet above the deck surface. However, due to the pinned restraint the longitudinal force will be applied at the seat level.

$$
\begin{aligned}
& \mathrm{V}_{\text {long }}=45.90 \cos (15)=44.34 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=45.90 \sin (15)=11.88 \mathrm{k} \\
& \mathrm{M}_{\text {long }}=(44.34)(16.83)=746 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=(11.88)(16.83)=200 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

## [3.11.6.4]

[3.11.6.4-1]
[3.11.6.4-1]
[C3.4.1]

## [3.11.6.5]

## LS - Live Load Surcharge

A live load surcharge shall be applied where a vehicular load is expected to act on the surface of the backfill within a distance equal to one-half the wall height behind the back face of the wall.

The increase in horizontal pressure due to live load surcharge may be estimated as:
$\Delta_{\mathrm{p}}=\mathrm{k} \gamma_{\mathrm{s}} \mathrm{h}_{\mathrm{eq}}$
where:
$\gamma_{\mathrm{s}}=$ total unit weight of soil $=0.120 \mathrm{kcf}$
$\mathrm{k}=$ coefficient of lateral earth pressure, $\mathrm{k}_{\mathrm{a}}$, for walls that deflect
$h_{\text {eq }}=$ equivalent height of soil for vehicular load from Table 1
$=2.0 \mathrm{ft}$ for abutment height $>20.0$ feet
$\Delta_{\mathrm{p}}=(0.295)(0.120)(2.0)=0.0708 \mathrm{ksf}$
$\mathrm{P}=(0.120)(2.0)(6.5)(46.41)=72.40 \mathrm{k}$
$\mathrm{V}_{\text {long }}=(0.0708)(24.50)(46.41)=80.50 \mathrm{k}$
$\mathrm{M}_{\text {long }}=(80.50)(24.50) / 2=986 \mathrm{ft}-\mathrm{k}$

The vertical weight of the soil surcharge is to be included for foundation designs where the load increases the load effect but ignored where the load increases the resistance. For bearing resistance the vertical soil weight on the heel will increase the total load and therefore the load effect and should be included. For sliding resistance and overturning the vertical soil weight will increase the resistance and therefore should be ignored.

If the vehicular loading is transmitted through a structural slab, which is also supported by means other than earth, a corresponding reduction in the surcharge loads may be permitted. The standard ADOT approach slab satisfies this criterion. However, the abutment is tall compared to the slab length and no method is provided to determine the amount of the reduction, so the full live load surcharge will be used. In addition, construction vehicles could produce a live load surcharge before the approach slab is constructed.

## [3.8]

## [3.8.1.1]

## Wind on

Superstructure
[3.8.1.2.1]

## WS - Wind Load on Structure

Wind pressures are based on a base design wind velocity of 100 mph . For structures with heights over 30 feet above the groundline, a formula is available to adjust the wind velocity. The wind is assumed to act uniformly on the area exposed to the wind. The exposed area is the sum of the areas of all components as seen in elevation taken perpendicular to the assumed wind direction.

$$
\begin{aligned}
& \text { Height }=7.50+2.67+0.02(44.83)=11.07 \mathrm{ft} \\
& \text { Area }=(11.07)(160.00)=1770 \mathrm{ft}^{2}
\end{aligned}
$$

The base pressure for girder bridges corresponding to the 100 mph wind is 0.050 psf . The minimum wind loading shall not be less than 0.30 klf. Since the girder bridge has spans greater than 125 feet, the wind must be evaluated for various angles of attack. The center of gravity of the loads is located 16.83 $+(11.07) / 2=22.36$ feet above the bottom of the footing. Wind force in the direction of the span will be applied at the top of the seat due to the pinned condition. Wind pressures for various angles of attack are taken from Table 3.8.1.2.2-1. Refer to Figure 4 for proper inclusion of the skew affect for the load combinations. Critical values are underlined.

## Pinned Abutment

0 Degree Skew Angle

$$
\begin{aligned}
\mathrm{V}_{\text {long }} & =[(1770)(0.050) \sin (15)] / 2+(1770)(0.000) \cos (15)=11.45 \mathrm{k} \\
\mathrm{~V}_{\text {trans }} & =[(1770)(0.050) \cos (15)] / 2+(1770)(0.000) \sin (15)=\underline{42.74 \mathrm{k}} \\
\mathrm{M}_{\text {long }} & =[(1770)(0.050)(22.36) \sin (15)] / 2+(1770)(0.000)(16.83) \cos (15) \\
& =256 \mathrm{ft}-\mathrm{k} \\
\mathrm{M}_{\text {trans }} & =[(1770)(0.050)(22.36) \cos (15)] / 2+(1770)(0.000)(16.83) \sin (15) \\
& =\underline{956 \mathrm{ft}-\mathrm{k}}
\end{aligned}
$$

15 Degree Skew Angle
$\mathrm{V}_{\text {long }}=[(1770)(0.044) \sin (15)] / 2+(1770)(0.006) \cos (15)=20.34 \mathrm{k}$
$\mathrm{V}_{\text {trans }}=[(1770)(0.044) \cos (15)] / 2+(1770)(0.006) \sin (15)=40.36 \mathrm{k}$
$\mathrm{M}_{\text {long }}=[(1770)(0.044)(22.36) \sin (15)] / 2+(1770)(0.006)(16.83) \cos (15)$
$=398 \mathrm{ft}-\mathrm{k}$
$\mathrm{M}_{\text {trans }}=[(1770)(0.044)(22.36) \cos (15)] / 2+(1770)(0.006)(16.83) \sin (15)$

$$
=887 \mathrm{ft}-\mathrm{k}
$$

$$
\begin{aligned}
& 30 \text { Degree Skew Angle } \\
& \mathrm{V}_{\text {long }}=[(1770)(0.041) \sin (15)] / 2+(1770)(0.012) \cos (15)=29.91 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=[(1770)(0.041) \cos (15)] / 2+(1770)(0.012) \sin (15)=40.55 \mathrm{k} \\
& \\
& \mathrm{M}_{\text {long }}=[(1770)(0.041)(22.36) \sin (15)] / 2+(1770)(0.012)(16.83) \cos (15) \\
&=555 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=[(1770)(0.041)(22.36) \cos (15)] / 2+(1770)(0.012)(16.83) \sin (15) \\
&=876 \mathrm{ft}-\mathrm{k} \\
& \\
& 45 \text { Degree Skew Angle } \\
& \mathrm{V}_{\text {long }}=[(1770)(0.033) \sin (15)] / 2+(1770)(0.016) \cos (15)=34.91 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=[(1770)(0.033) \cos (15)] / 2+(1770)(0.016) \sin (15)=35.54 \mathrm{k} \\
& \mathrm{M}_{\text {long }}=[(1770)(0.033)(22.36) \sin (15)] / 2+(1770)(0.016)(16.83) \cos (15) \\
&=629 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=[(1770)(0.033)(22.36) \cos (15)] / 2+(1770)(0.016)(16.83) \sin (15) \\
&=754 \mathrm{ft}-\mathrm{k} \\
& 60 \text { Degree } \text { Skew Angle } \\
& \mathrm{V}_{\text {long }}=[(1770)(0.017) \sin (15)] / 2+(1770)(0.019) \cos (15)=36.38 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=[(1770)(0.017) \cos (15)] / 2+(1770)(0.019) \sin (15)=23.24 \mathrm{k} \\
& \mathrm{M}_{\text {long }}=[(1770)(0.017)(22.36) \sin (15)] / 2+(1770)(0.019)(16.83) \cos (15) \\
&=634 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=[(1770)(0.017)(22.36) \cos (15)] / 2+(1770)(0.019)(16.83) \sin (15) \\
&=471 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

## Expansion Abutment

0 Degree Skew Angle

$$
\begin{aligned}
& \mathrm{V}_{\text {long }}=[(1770)(0.050) \sin (15)] / 2=\underline{11.45 \mathrm{k}} \\
& \mathrm{~V}_{\text {trans }}=[(1770)(0.050) \cos (15)] / 2=\underline{42.74 \mathrm{k}} \\
& \mathrm{M}_{\text {long }}=[(1770)(0.050)(22.36) \sin (15)] / 2=\underline{256 \mathrm{ft}-\mathrm{k}} \\
& \mathrm{M}_{\text {trans }}=[(1770)(0.050)(22.36) \cos (15)] / 2=\underline{956 \mathrm{ft}-\mathrm{k}}
\end{aligned}
$$

15 Degree Skew Angle
$\mathrm{V}_{\text {long }}=[(1770)(0.044) \sin (15)] / 2=10.08 \mathrm{k}$
$\mathrm{V}_{\text {trans }}=[(1770)(0.044) \cos (15)] / 2=37.61 \mathrm{k}$
$\mathrm{M}_{\text {long }}=[(1770)(0.044)(22.36) \sin (15)] / 2=225 \mathrm{ft}-\mathrm{k}$
$\mathrm{M}_{\text {trans }}=[(1770)(0.044)(22.36) \cos (15)] / 2=841 \mathrm{ft}-\mathrm{k}$

$$
\begin{aligned}
& 30 \text { Degree Skew Angle } \\
& \mathrm{V}_{\text {long }}=[(1770)(0.041) \sin (15)] / 2=9.39 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=[(1770)(0.041) \cos (15)] / 2=35.05 \mathrm{k} \\
& \mathrm{M}_{\text {long }}=[(1770)(0.041)(22.36) \sin (15)] / 2=210 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=[(1770)(0.041)(22.36) \cos (15)] / 2=784 \mathrm{ft}-\mathrm{k} \\
& 45 \text { Degree Skew Angle } \\
& \mathrm{V}_{\text {long }}=[(1770)(0.033) \sin (15)] / 2=7.56 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=[(1770)(0.033) \cos (15)] / 2=28.21 \mathrm{k} \\
& \mathrm{M}_{\text {long }}=[(1770)(0.033)(22.36) \sin (15)] / 2=169 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=[(1770)(0.033)(22.36) \cos (15)] / 2=631 \mathrm{ft}-\mathrm{k} \\
& 60 \text { Degree } \\
& \mathrm{V}_{\text {long }}=[(1770)(0.017) \sin (15)] / 2=3.89 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=[(1770)(0.017) \cos (15)] / 2=14.53 \mathrm{k} \\
& \mathrm{M}_{\text {long }}=[(1770)(0.017)(22.36) \sin (15)] / 2=87 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=[(1770)(0.017)(22.36) \cos (15)] / 2=325 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

A conservative answer can be achieved by simplifying the problem by using the maximum values in each direction acting simultaneously. If wind controls the design, the complexities of combining 5 wind combinations should be performed. A summary of wind forces used in the design follows:

| Pinned | Expansion |
| :---: | :---: |
| $\mathrm{V}_{\text {long }}=36.38 \mathrm{k}$ | 11.45 k |
| $\mathrm{V}_{\text {trans }}=42.74 \mathrm{k}$ | 42.74 k |
| $\mathrm{M}_{\text {long }}=634 \mathrm{ft}-\mathrm{k}$ | $256 \mathrm{ft}-\mathrm{k}$ |
| $\mathrm{M}_{\text {trans }}=956 \mathrm{ft}-\mathrm{k}$ | $956 \mathrm{ft}-\mathrm{k}$ |

The transverse and longitudinal forces to be applied directly to the substructure are calculated from an assumed base wind pressure of 0.040 ksf . Because the longitudinal wind blows opposite the earth pressure, the critical wind on substructure load in the longitudinal direction will be zero.

$$
\begin{aligned}
& \mathrm{V}_{\text {long }}=0 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=0.040[(10.50)(18.00)]=7.56 \mathrm{k} \\
& \mathrm{M}_{\text {long }}=0 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=0.040(10.50)(18.00)(15.50)=117 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

\section*{| [3.8.1.3] | WL - Wind Pressure on Vehicles |
| :--- | :--- |}

Wind pressure on vehicles is represented by a moving force of 0.10 klf acting normal to and 6.0 feet above the roadway. Loads normal to the span should be applied at a height of $24.50+6.00=30.50 \mathrm{ft}$

## Pinned Abutment

0 Degree Skew Angle

$$
\mathrm{V}_{\text {long }}=160.00[(0.100) \sin (15) / 2+(0.000) \cos (15)]=2.07 \mathrm{k}
$$

$$
\mathrm{V}_{\text {trans }}=160.00[(0.100) \cos (15) / 2+(0.000) \sin (15)]=\underline{7.73 \mathrm{k}}
$$

$$
\mathrm{M}_{\text {long }}=160.00[(0.100)(30.50) \sin (15) / 2+(0.000)(16.83) \cos (15)]
$$

$$
=63 \mathrm{ft}-\mathrm{k}
$$

$$
\mathrm{M}_{\text {trans }}=160.00[(0.100)(30.50) \cos (15) / 2+(0.000)(16.83) \sin (15)]
$$

$$
=\underline{236 \mathrm{ft}-\mathrm{k}}
$$

15 Degree Skew Angle
$\mathrm{V}_{\text {long }}=160.00[(0.088) \sin (15) / 2+(0.012) \cos (15)]=3.68 \mathrm{k}$
$\mathrm{V}_{\text {trans }}=160.00[(0.088) \cos (15) / 2+(0.012) \sin (15)]=7.30 \mathrm{k}$
$\mathrm{M}_{\text {long }}=160.00[(0.088)(30.50) \sin (15) / 2+(0.012)(16.83) \cos (15)]$
$=87 \mathrm{ft}-\mathrm{k}$
$\mathrm{M}_{\text {trans }}=160.00[(0.088)(30.50) \cos (15) / 2+(0.012)(16.83) \sin (15)]$
$=216 \mathrm{ft}-\mathrm{k}$
30 Degree Skew Angle
$\mathrm{V}_{\text {long }}=160.00[(0.082) \sin (15) / 2+(0.024) \cos (15)]=5.41 \mathrm{k}$
$\mathrm{V}_{\text {trans }}=160.00[(0.082) \cos (15) / 2+(0.024) \sin (15)]=7.33 \mathrm{k}$
$\mathrm{M}_{\text {long }}=160.00[(0.082)(30.50) \sin (15) / 2+(0.024)(16.83) \cos (15)]$
$=114 \mathrm{ft}-\mathrm{k}$
$\mathrm{M}_{\text {trans }}=160.00[(0.082)(30.50) \cos (15) / 2+(0.024)(16.83) \sin (15)]$
$=210 \mathrm{ft}-\mathrm{k}$

45 Degree Skew Angle
$\mathrm{V}_{\text {long }}=160.00[(0.066) \sin (15) / 2+(0.032) \cos (15)]=6.31 \mathrm{k}$
$\mathrm{V}_{\text {trans }}=160.00[(0.066) \cos (15) / 2+(0.032) \sin (15)]=6.43 \mathrm{k}$
$\mathrm{M}_{\text {long }}=160.00[(0.066)(30.50) \sin (15) / 2+(0.032)(16.83) \cos (15)]$
$=\underline{125 \mathrm{ft}-\mathrm{k}}$
$\mathrm{M}_{\text {trans }}=160.00[(0.066)(30.50) \cos (15) / 2+(0.032)(16.83) \sin (15)]$
$=178 \mathrm{ft}-\mathrm{k}$

```
60 Degree Skew Angle
    \(\mathrm{V}_{\text {long }}=160.00[(0.034) \sin (15) / 2+(0.038) \cos (15)]=\underline{6.58 \mathrm{k}}\)
    \(\mathrm{V}_{\text {trans }}=160.00[(0.034) \cos (15) / 2+(0.038) \sin (15)]=4.20 \mathrm{k}\)
    \(\mathrm{M}_{\text {long }}=160.00[(0.034)(30.50) \sin (15) / 2+(0.038)(16.83) \cos (15)]\)
    \(=120 \mathrm{ft}-\mathrm{k}\)
    \(\mathrm{M}_{\text {trans }}=160.00[(0.034)(30.50) \cos (15) / 2+(0.038)(16.83) \sin (15)]\)
        \(=107 \mathrm{ft}-\mathrm{k}\)
```

    Expansion Abutment
    0 Degree Skew Angle
$\mathrm{V}_{\text {long }}=160.00(0.100) \sin (15) / 2=\underline{2.07 \mathrm{k}}$
$\mathrm{V}_{\text {trans }}=160.00(0.100) \cos (15) / 2=\underline{7.73 \mathrm{k}}$
$\mathrm{M}_{\text {long }}=160.00(0.100)(30.50) \sin (15) / 2=63 \mathrm{ft}-\mathrm{k}$
$\mathrm{M}_{\text {trans }}=160.00(0.100)(30.50) \cos (15) / 2=\underline{236 \mathrm{ft}-\mathrm{k}}$
15 Degree Skew Angle
$\mathrm{V}_{\text {long }}=160.00(0.088) \sin (15) / 2=1.82 \mathrm{k}$
$\mathrm{V}_{\text {trans }}=160.00(0.088) \cos (15) / 2=6.80 \mathrm{k}$
$\mathrm{M}_{\text {long }}=160.00(0.088)(30.50) \sin (15) / 2=56 \mathrm{ft}-\mathrm{k}$
$\mathrm{M}_{\text {trans }}=160.00(0.088)(30.50) \cos (15) / 2=207 \mathrm{ft}-\mathrm{k}$

30 Degree Skew Angle
$\mathrm{V}_{\text {long }}=160.00(0.082) \sin (15) / 2=1.70 \mathrm{k}$
$\mathrm{V}_{\text {trans }}=160.00(0.082) \cos (15) / 2=6.34 \mathrm{k}$
$\mathrm{M}_{\text {long }}=160.00(0.082)(30.50) \sin (15) / 2=52 \mathrm{ft}-\mathrm{k}$
$\mathrm{M}_{\text {trans }}=160.00(0.082)(30.50) \cos (15) / 2=193 \mathrm{ft}-\mathrm{k}$
45 Degree Skew Angle
$\mathrm{V}_{\text {long }}=160.00(0.066) \sin (15) / 2=1.37 \mathrm{k}$
$\mathrm{V}_{\text {trans }}=160.00(0.066) \cos (15) / 2=5.10 \mathrm{k}$
$\mathrm{M}_{\text {long }}=160.00(0.066)(30.50) \sin (15) / 2=42 \mathrm{ft}-\mathrm{k}$
$\mathrm{M}_{\text {trans }}=160.00(0.066)(30.50) \cos (15) / 2=156 \mathrm{ft}-\mathrm{k}$
60 Degree Skew Angle
$\mathrm{V}_{\text {long }}=160.00(0.034) \sin (15) / 2=0.70 \mathrm{k}$
$\mathrm{V}_{\text {trans }}=160.00(0.034) \cos (15) / 2=2.63 \mathrm{k}$
$\mathrm{M}_{\text {long }}=160.00(0.034)(30.50) \sin (15) / 2=21 \mathrm{ft}-\mathrm{k}$
$M_{\text {trans }}=160.00(0.034)(30.50) \cos (15) / 2=80 \mathrm{ft}-\mathrm{k}$

A conservative answer for wind on live load can be achieved by using the maximum values in each direction acting simultaneously. If wind controls the design, the complexities of combining 5 wind directions should be performed.

$$
\begin{array}{lll}
\quad \begin{array}{l}
\text { Pinned } \\
\mathrm{V}_{\text {long }}
\end{array}=6.58 \mathrm{k} & & \underline{\text { Expansion }} \\
\mathrm{V}_{\text {trans }}=7.07 \mathrm{k} & & 7.73 \mathrm{k} \\
\mathrm{M}_{\text {long }}=125 \mathrm{ft}-\mathrm{k} & & 63 \mathrm{ft}-\mathrm{k} \\
\mathrm{M}_{\text {trans }}=236 \mathrm{ft}-\mathrm{k} & & 236 \mathrm{ft}-\mathrm{k}
\end{array}
$$

[3.8.2]

## [3.13]

[14.6.3.1]
[14.6.3.1-1]

## Vertical Wind Pressure

A vertical upward wind force of 0.020 ksf times the width of the deck shall be applied at the windward quarter point of the deck. This load is only applied for limit states which include wind but not wind on live load (Strength III Limit State) and only when the direction of wind is taken to be perpendicular to the longitudinal axis of the bridge. When applicable the wind loads are as shown:

$$
\begin{aligned}
& \mathrm{P}=(0.020)(44.83)(160) / 2=-71.73 \text { upward } \\
& \mathrm{M}_{\text {trans }}=[71.73(44.83) / 4] \cos (15)=777 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {long }}=[71.73(44.83) / 4] \sin (15)=208 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

## FR - Friction Forces

Friction forces from the greased bearings caused by superstructure movement will be transmitted to the substructure for Abutment 2, the expansion abutment. These forces will occur during the stressing operation and for a short period of time afterwards while the bridge undergoes long term prestress shortening. This force was calculated for the elastomeric bearing for the Superstructure Example 1 as repeated below:

$$
\begin{aligned}
& \mathrm{H}_{\mathrm{bu}}=\mu \mathrm{P}_{\mathrm{u}} \\
& \mathrm{P}_{\mathrm{u}}=1.25 \mathrm{DC}+1.50 \mathrm{DW} \\
& \mathrm{P}_{\mathrm{u}}=1.25(245.86)+1.50(17.13)=333.02 \mathrm{k} \\
& \mathrm{H}_{\mathrm{bu}}=(0.10)(333.02)=33.30 \mathrm{k} \text { per bearing pad }
\end{aligned}
$$

It is important to note that this force is already factored and only applies to strength and extreme event limit states. Table 3.4.1-1 does list FR as a load with a load factor of 1.0 for all limit states including service. However, this FR does not apply to service limit states.

$$
\begin{aligned}
& \mathrm{V}_{\text {trans }}=(33.30)(5 \text { bearings }) \sin (15)=43.09 \mathrm{k} \\
& \mathrm{~V}_{\text {long }}=(33.30)(5 \text { bearings }) \cos (15)=160.83 \mathrm{k} \\
& \mathrm{M}_{\text {trans }}=(43.09)(16.83)=725 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {long }}=(160.83)(16.83)=2707 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

[14.6.3.1-2]

## [BDG]

[BDG]

## Bearing Translation

The elastomeric bearing pad will also transmit forces to the substructure due to horizontal displacements caused by temperature, shrinkage, creep and prestress shortening. For the greased pad the shrinkage, creep and prestress shortening are resisted as a friction load so there is no direct load for SH and CR. The force due to deformation of an elastomeric bearing pad due to TU shall be taken as:

$$
H_{b u}=G A \frac{\Delta_{u}}{h_{r t}}
$$

$\Delta_{\mathrm{u}}=$ shear deformation from applicable strength and extreme event load combinations in Table 3.4.1-1.

$$
h_{r t}=(0.5000-0.0747)(3 \text { interior })+(0.2500-0.0374)(2 \text { exterior })=1.70^{\prime \prime}
$$

For a post-tensioned box girder with greased sliding pads the elastic shortening and creep are assumed to be taken by the greased pad in a sliding mode.
Afterwards the grease hardens and the pad resists temperature movement by deformation of the pad. The strength limit state load factor for TU deformations is 0.50 . The 0.65 factor reflects the fact that the pads are not always constructed at the mean temperature. The temperature range for elevations less than 3000 feet is 90 degrees.

$$
\begin{aligned}
\Delta_{\mathrm{u}} & =(0.50)(0.65)(0.000006)(90)(160)(12)=0.337 \mathrm{in} \\
H_{b u} & =(0.130) \cdot(28) \cdot(14) \cdot \frac{0.337}{1.70}=10.10 \mathrm{kips}
\end{aligned}
$$

The force from friction ( $33.30 \mathrm{k} / \mathrm{pad}$ ) is higher than the force resulting from the internal deformation of the elastomeric bearing ( $10.10 \mathrm{k} / \mathrm{pad}$ ). This bearing translation force only applies to the strength and extreme event load combinations and is also already factored. Since the FR forces are greater than the TU forces and only one force can occur at a time, only the FR forces will be considered further.

## Bearing Rotation

Rotations in the elastomeric bearing pads will cause bending moments that will be transmitted to the substructure. For unconfined elastomeric bearings the moment shall be taken as:
$M_{u}=1.60\left(0.5 E_{c} I\right) \frac{\theta_{s}}{h_{r t}}$
where:
$\mathrm{I}=$ moment of inertia of plan shape of bearing
$\mathrm{I}=\mathrm{WL}^{3} / 12=(28)(14)^{3} / 12=6403 \mathrm{in}^{3}$
$\mathrm{E}_{\mathrm{c}}=$ effective modulus of elastomeric bearing in compression
$\mathrm{E}_{\mathrm{c}}=6 \mathrm{GS}^{2}=6(0.130)(11)^{2}=94.38 \mathrm{ksi}$
Refer to Example 1 Superstructure Bearing calculations. $\theta_{\mathrm{S}}=0.008384$ radians

Strength Limit States

$$
\begin{aligned}
& M_{u}=(1.60) \cdot(0.5) \cdot(94.38) \cdot(6403) \cdot \frac{0.008384}{1.70} \div 12=199 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {long }}=(199)(5 \text { bearings }) \cos (15)=961 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=(199)(5 \text { bearings }) \sin (15)=258 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

Again this moment is already factored. The 1.60 factor in the formula is the load factor that allows for use of service limit rotations. This load only applies to the strength and extreme limit states.

A summary of unfactored axial loads, shears and moments except as otherwise noted follows:

Abutment 1 (Pinned)

| Load | $\mathrm{P}_{\text {max }}$ | $\mathrm{P}_{\text {min }}$ | $\mathrm{V}_{\text {long }}$ | $\mathrm{V}_{\text {trans }}$ | $\mathrm{M}_{\text {long }}$ | $\mathrm{M}_{\text {trans }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | kip | kip | kip | kip | ft-k | ft-k |
| $\mathrm{DC}_{\text {super }}$ | 1229 | 1229 | 0 | 0 | 1537 | 0 |
| $\mathrm{DC}_{\text {sub }}$ | 948 | 948 | 0 | 0 | -48 | 0 |
| DC | 2177 | 2177 | 0 | 0 | 1489 | 0 |
| DW | 86 | 0 | 0 | 0 | 107 | 0 |
| EV | 850 | 850 | 0 | 0 | -3080 | 0 |
| EH | 0 | 0 | 493 | 0 | 4027 | 0 |
| LL | 306 | 0 | 0 | 0 | 382 | 2484 |
| BR | 0 | 0 | 44 | 12 | 746 | 200 |
| LS | 72 | 0 | 81 | 0 | 986 | 0 |
| $\mathrm{WS}_{\text {super }}$ | 0 | 0 | 36 | 43 | 634 | 956 |
| $\mathrm{WS}_{\text {sub }}$ | 0 | 0 | 0 | 8 | 0 | 117 |
| WS | 0 | 0 | 36 | 51 | 634 | 1073 |
| $\mathrm{WS}_{\text {vertical }}$ | 0 | -72 | 0 | 0 | 208 | 777 |
| WL | 0 | 0 | 7 | 8 | 125 | 236 |
| FR * | 0 | 0 | 0 | 0 | 0 | 0 |
| Bearing <br> Rotation* | 0 | 0 | 0 | 0 | 961 | 258 |


| Abutment 2 (Expansion) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Load | $\mathrm{P}_{\text {max }}$ | $\mathrm{P}_{\text {min }}$ | $\mathrm{V}_{\text {long }}$ | $\mathrm{V}_{\text {trans }}$ | $\mathrm{M}_{\text {long }}$ | $\mathrm{M}_{\text {trans }}$ |
|  | kip | kip | kip | kip | ft-k | ft-k |
| $\mathrm{DC}_{\text {super }}$ | 1229 | 1229 | 0 | 0 | 1537 | 0 |
| $\mathrm{DC}_{\text {sub }}$ | 948 | 948 | 0 | 0 | -48 | 0 |
| DC | 2177 | 2177 | 0 | 0 | 1489 | 0 |
| DW | 86 | 0 | 0 | 0 | 107 | 0 |
| EV | 850 | 850 | 0 | 0 | -3080 | 0 |
| EH | 0 | 0 | 493 | 0 | 4027 | 0 |
| LL | 306 | 0 | 0 | 0 | 382 | 2484 |
| BR | 0 | 0 | 0 | 0 | 0 | 0 |
| LS | 72 | 0 | 81 | 0 | 986 | 0 |
| $\mathrm{WS}_{\text {super }}$ | 0 | 0 | 11 | 43 | 256 | 956 |
| $\mathrm{WS}_{\text {sub }}$ | 0 | 0 | 0 | 8 | 0 | 117 |
| WS | 0 | 0 | 11 | 51 | 256 | 1073 |
| $\mathrm{WS}_{\text {vertical }}$ | 0 | -72 | 0 | 0 | 208 | 777 |
| WL | 0 | 0 | 2 | 8 | 63 | 236 |
| FR* | 0 | 0 | 161 | 43 | 2707 | 725 |
| Bearing <br> Rotation* | 0 | 0 | 0 | 0 | 961 | 258 |

## [3.4.1-1]

## LOAD COMBINATIONS STRENGTH I

$$
\begin{aligned}
\mathrm{Max}= & 1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV} \\
& +1.75(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+\mathrm{FR}+\text { Bearing } \\
\mathrm{Min}= & 0.90 \mathrm{DC}+0.65 \mathrm{DW}+0.90 \mathrm{EH}+1.00 \mathrm{EV} \\
& +1.75(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+\mathrm{FR}+\text { Bearing }
\end{aligned}
$$

## STRENGTH III

$$
\begin{aligned}
\mathrm{Max}= & 1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV} \\
& +1.40\left(\mathrm{WS}+\mathrm{WS}_{\mathrm{vert}}\right)+\mathrm{FR}+\text { Bearing }
\end{aligned}
$$

$$
\operatorname{Min}=0.90 \mathrm{DC}+0.65 \mathrm{DW}+0.90 \mathrm{EH}+1.00 \mathrm{EV}
$$

$$
+1.40\left(\mathrm{WS}+\mathrm{WS}_{\mathrm{vert}}\right)+\mathrm{FR}+\text { Bearing }
$$

## STRENGTH IV

$\mathrm{Max}=1.50 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV}+\mathrm{FR}+$ Bearing

## STRENGTH V

$$
\begin{aligned}
& \text { Max }= 1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV} \\
&+1.35(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+0.40 \mathrm{WS}+1.00 \mathrm{WL}+\mathrm{FR}+\text { Bearing } \\
& \mathrm{Min}= \\
& 0.90 \mathrm{DC}+0.65 \mathrm{DW}+0.90 \mathrm{EH}+1.00 \mathrm{EV} \\
&+1.35(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+0.40 \mathrm{WS}+1.00 \mathrm{WL}+\mathrm{FR}+\text { Bearing }
\end{aligned}
$$

## SERVICE I

$$
\begin{aligned}
\mathrm{Max}= & 1.00(\mathrm{DC}+\mathrm{DW}+\mathrm{EH}+\mathrm{EV})+1.00(\mathrm{LL}+\mathrm{BR}+\mathrm{LS}) \\
& +0.30 \mathrm{WS}+1.00 \mathrm{WL}
\end{aligned}
$$

The moment due to bearing rotation and friction forces from the bearing only apply to strength limit states and are already factored.

As previously discussed, CR and SH forces are not critical for this bridge and are not included in the load combinations above.

## General

The methods used to estimate loads for the design of foundations using LRFD are fundamentally the same as the procedures used in the past for ASD. What has changed is the way the loads are considered for evaluation of foundation stability (bearing and sliding resistance of spread footing foundations) and foundation deformation. The design of foundations supporting bridge abutments should consider all limit states loading conditions applicable to the structure being designed. The following Strength Limit States may control the design and should be investigated:

Strength I Limit State will control for high live to dead load ratios.
Strength III or V will control for structures subjected to high wind loads
Strength IV Limit State will control for high dead to live load ratios
A spread footing foundation will be evaluated for the following failure conditions:

1. Bearing Resistance - Strength Limit States
2. Settlement - Service I Limit State
3. Sliding Resistance - Strength Limit States
4. Load Eccentricity (Overturning) - Strength Limit States
5. Overall Stability - Service I Limit State
6. Structural Resistance - Service I and Strength Limit States
[11.5.3]
[C11.5.5-1]
[11.5.5]
[11.6.3.2-1]

## 1. Bearing Resistance

Bearing resistance check is a strength limit state. The appropriate strength limit states are I, III and V. The maximum bearing stress will be found by applying the maximum load factors to each applicable load. The Factored Net Bearing Resistance Chart will be provided in the Geotechnical Report. The load factors and loads are shown below:

## Strength I (max) Limit State:



Figure 7

$$
\begin{aligned}
& \text { Strength } \mathrm{I}= 1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV}+ \\
& 1.75(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+\mathrm{FR}+\text { Bearing } \\
& \mathrm{P}_{\max }=1.25(2177)+1.50(86)+1.35(850)+1.75(306+72)=4659 \mathrm{k}
\end{aligned}
$$

Pinned Abutment

$$
\begin{aligned}
\hline \mathrm{M}_{\text {long }}= & 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
& +1.75(382+746+986)+961=9643 \mathrm{ft}-\mathrm{k} \\
\mathrm{M}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.75(2484+200+0) \\
& +258=4955 \mathrm{ft}-\mathrm{k} \\
& \\
\mathrm{e}_{\text {long }}= & \mathrm{e}_{\mathrm{B}}=9643 / 4659=2.070 \mathrm{ft} \\
\mathrm{e}_{\text {trans }}= & \mathrm{e}_{\mathrm{L}}=4955 / 4659=1.064 \mathrm{ft}
\end{aligned}
$$

$$
\mathrm{B}^{\prime}=16.00-2(2.070)=11.86 \mathrm{ft}(\text { effective footing width })
$$

$$
\mathrm{L}^{\prime}=49.52-2(1.064)=47.39 \mathrm{ft}(\text { effective footing length })
$$

$$
\mathrm{q}_{\max }=4659 /[(11.86)(47.39)]=8.29 \mathrm{ksf}
$$

$$
\mathrm{q}_{\mathrm{nveu}}=8.29-1.35(0.120)(6.50)=7.24 \mathrm{ksf}
$$

## Expansion Abutment

$$
\begin{aligned}
& \mathrm{M}_{\text {long }}= 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
&+1.75(382+0+986)+2707+961=11044 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}= 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.75(2484+0+0) \\
&+725+258=5330 \mathrm{ft}-\mathrm{k} \\
& \\
& \mathrm{e}_{\text {long }}= \mathrm{e}_{\mathrm{B}}=11044 / 4659=2.370 \mathrm{ft} \\
& \mathrm{e}_{\text {trans }}= \mathrm{e}_{\mathrm{L}}=5330 / 4659=1.144 \mathrm{ft} \\
& \mathrm{~B}^{\prime}=16.00-2(2.370)=11.26 \mathrm{ft} \text { (effective footing width) } \\
& \mathrm{L}^{\prime}=49.52-2(1.144)=47.23 \mathrm{ft} \text { (effective footing length) } \\
& \mathrm{q}_{\text {max }}= 4659 /[(11.26)(47.23)]=8.76 \mathrm{ksf} \\
& \mathrm{q}_{\text {nveu }}= 8.76-1.35(0.120)(6.50)=\underline{7.71 \mathrm{ksf}}<=\text { Critical }
\end{aligned}
$$

## Strength III (max) Limit State:



BEARING RESISTANCE STRENGTH III

Figure 8

$$
\begin{aligned}
& \text { Strength } \mathrm{III}= 1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV}+1.40\left(\mathrm{WS}+\mathrm{WS}_{\text {vert }}\right) \\
& \quad+\mathrm{FR}+\text { Bearing } \\
& \mathrm{P}_{\max }=1.25(2177)+1.50(86)+1.35(850)=3998 \mathrm{k}
\end{aligned}
$$

## Pinned Abutment

$$
\begin{aligned}
& \mathrm{M}_{\text {long }}= 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
&+1.40(634+208)+961=7122 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}= 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.40(1073+777) \\
&+258=2848 \mathrm{ft}-\mathrm{k} \\
& \\
& \mathrm{e}_{\text {long }}= \mathrm{e}_{\mathrm{B}}=7122 / 3998=1.781 \mathrm{ft} \\
& \mathrm{e}_{\text {trans }}= \mathrm{e}_{\mathrm{L}}=2848 / 3998=0.712 \mathrm{ft} \\
& \mathrm{~B}^{\prime}=16.00-2(1.781)=12.44 \mathrm{ft} \text { (effective footing width) } \\
& \mathrm{L}^{\prime}=49.52-2(0.712)=48.10 \mathrm{ft} \text { (effective footing length) } \\
& \\
& \mathrm{q}_{\text {max }}=3998 /[(12.44)(48.10)]=6.68 \mathrm{ksf} \\
& \mathrm{q}_{\text {nveu }}=6.68-1.35(0.120)(6.50)=5.63 \mathrm{ksf}
\end{aligned}
$$

Expansion Abutment

$$
\begin{aligned}
\mathrm{M}_{\text {long }}= & 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
& +1.40(256+208)+2707+961=9300 \mathrm{ft}-\mathrm{k} \\
\mathrm{M}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.40(1073+777) \\
& +725+258=3573 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

$$
\mathrm{e}_{\text {long }}=\mathrm{e}_{\mathrm{B}}=9300 / 3998=2.326 \mathrm{ft}
$$

$$
e_{\text {trans }}=e_{L}=3573 / 3998=0.894 \mathrm{ft}
$$

$$
\mathrm{B}^{\prime}=16.00-2(2.326)=11.35 \mathrm{ft}(\text { effective footing width })
$$

$$
\left.\mathrm{L}^{\prime}=49.52-2(0.894)=47.73 \mathrm{ft} \text { (effective footing length }\right)
$$

$$
\mathrm{q}_{\max }=3998 /[(11.35)(47.73)]=7.38 \mathrm{ksf}
$$

$$
\mathrm{q}_{\text {nveu }}=7.38-1.35(0.120)(6.50)=6.33 \mathrm{ksf}
$$

## Strength V (max) Limit State:



Figure 9

$$
\left.\begin{array}{rl}
\text { Strength } \mathrm{V}= & 1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV} \\
& +1.35(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+0.40 \mathrm{WS}+1.00 \mathrm{WL}+\mathrm{FR}+\text { Bearing }
\end{array}\right] \begin{array}{r}
\mathrm{P}_{\max }=1.25(2177)+1.50(86)+1.35(850)+1.35(306+72)=4508 \mathrm{k}
\end{array}
$$

Pinned Abutment

$$
\begin{aligned}
\mathrm{M}_{\text {long }}= & 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
& +1.35(382+746+986)+0.40(634)+1.00(125)+961=9176 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

$$
\mathrm{M}_{\text {trans }}=1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.35(2484+200+0)
$$

$$
+0.40(1073)+1.00(236)+258=4547 \mathrm{ft}-\mathrm{k}
$$

$\mathrm{e}_{\text {long }}=\mathrm{e}_{\mathrm{B}}=9176 / 4508=2.035 \mathrm{ft}$
$e_{\text {trans }}=e_{L}=4547 / 4508=1.009 \mathrm{ft}$
$\mathrm{B}^{\prime}=16.00-2(2.035)=11.93 \mathrm{ft}$
$\mathrm{L}^{\prime}=49.52-2(1.009)=47.50 \mathrm{ft}$

$$
\mathrm{q}_{\max }=4508 /[(11.93)(47.50)]=7.96 \mathrm{ksf}
$$

$$
\mathrm{q}_{\mathrm{nveu}}=7.96-1.35(0.120)(6.50)=6.91 \mathrm{ksf}
$$

## Expansion Abutment

$$
\begin{aligned}
\mathrm{M}_{\text {long }}= & 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
& +1.35(382+0+986)+0.40(256)+1.00(63)+2707+961 \\
= & 10662 \mathrm{ft}-\mathrm{k} \\
\mathrm{M}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.35(2484+0+0) \\
& +0.40(1073)+1.00(236)+725+258=5002 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

$\mathrm{e}_{\text {long }}=\mathrm{e}_{\mathrm{B}}=10662 / 4508=2.365 \mathrm{ft}$

$$
\mathrm{e}_{\text {trans }}=\mathrm{e}_{\mathrm{L}}=5002 / 4508=1.110 \mathrm{ft}
$$

$\mathrm{B}^{\prime}=16.00-2(2.365)=11.27 \mathrm{ft}($ effective footing width $)$
$L^{\prime}=49.52-2(1.110)=47.30 \mathrm{ft}($ effective footing length $)$

$$
\begin{aligned}
& \mathrm{q}_{\max }=4508 /[(11.27)(47.30)]=8.46 \mathrm{ksf} \\
& \mathrm{q}_{\mathrm{nveu}}=8.46-1.35(0.120)(6.50)=7.41 \mathrm{ksf}
\end{aligned}
$$

The maximum factored net bearing stress is 7.71 ksf for Strength I (max) Limit State at the expansion abutment. The factored net bearing resistance from the Factored Net Bearing Resistance Chart from the geotechnical report as shown in Figure 10 below is 9.20 ksf. Therefore, the bearing resistance criterion is satisfied.


Figure 10

## [11.5.2]

[BDG]

## 2. Settlement

Settlement is a service limit state. For a single span bridge settlement and differential settlement will not cause structural distress to the superstructure but must be considered for the bearing and joint design. There are also limits to settlement to ensure a smooth ride. The Geotechnical Foundation Report will provide a chart that plots the factored net bearing stress versus the effective footing width for various settlement curves. This chart is specific for a given effective footing length and embedment depth. The geotechnical engineer will include the bearing resistance factor in the chart since the factor is a function of variables only the geotechnical engineer can determine.

The bridge engineer will determine the amount of settlement that the bridge can tolerate, determine the actual bearing stress and compare the corresponding settlement determined from the chart to the tolerable settlement. As an alternative the bridge engineer may determine the amount of settlement that the bridge can tolerate, determine the maximum bearing stress from the chart for a given settlement and compare the actual bearing stress to the maximum. The bridge engineer will also evaluate whether the structure can handle the estimated horizontal movement.

Bridge Group guidance on this topic is under development. Refer to the Bridge Design Guidelines for the most current guideline. The proposed criterion is to limit the maximum settlement to 3 inches per 100 feet for simple span bridges corresponding to a tolerable angle of (3)/[(100)(12)] $=0.0025$ radians. For a span of 160 feet the corresponding tolerable settlement equals (3)[(160) / (100)] $=4.80$ inches. The example was originally developed for a rotation limit of 0.0004 radians. The 0.004 radian value is used in the elastomeric bearing design and subsequent load combinations and was not recalculated using the 0.0025 radian criterion. For the service limit state, forces from the bearing rotation and bearing friction are not included.

Service I Limit State

$$
\begin{aligned}
\mathrm{P}_{\max }= & 1.0(\mathrm{DC}+\mathrm{DW}+\mathrm{EH}+\mathrm{EV})+1.0(\mathrm{LL}+\mathrm{BR}+\mathrm{LS}) \\
& +0.3 \mathrm{WS}+1.0 \mathrm{WL} \\
\mathrm{P}_{\max }= & 1.0(2177+86+0+850)+1.0(306+0+72)=3491 \mathrm{kips}
\end{aligned}
$$

Pinned Abutment

$$
\begin{aligned}
\mathrm{M}_{\text {long }}= & 1.0(1489+107+4027-3080)+1.0(382+746+986) \\
& +0.3(634)+1.0(125)=4972 \mathrm{ft}-\mathrm{k} \\
\mathrm{M}_{\text {trans }}= & 1.0(0)+1.0(2484+200)+0.3(1073)+1.0(236) \\
= & 3242 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{e}_{\text {long }}=\mathrm{e}_{\mathrm{B}}=4972 / 3491=1.424 \mathrm{ft} \\
& \mathrm{e}_{\text {trans }}=\mathrm{e}_{\mathrm{L}}=3242 / 3491=0.929 \mathrm{ft} \\
& \mathrm{~B}^{\prime}=16.00-2(1.424)=13.15 \mathrm{ft} \text { (footing effective width) } \\
& \mathrm{L}^{\prime}=49.52-2(0.929)=47.66 \mathrm{ft} \text { (footing effective length) } \\
& \\
& \mathrm{q}_{\text {tveu }}=3491 /[(13.15)(47.66)]=5.57 \mathrm{ksf}
\end{aligned}
$$

The total factored equivalent uniform vertical bearing stress, $\mathrm{q}_{\mathrm{tve}}$, at the base of the footing is 5.57 ksf . To determine the corresponding settlement for this bearing stress, the equivalent net uniform bearing stress is required. To determine the equivalent net uniform bearing stress, the factored overburden stress is subtracted.

$$
\mathrm{q}_{\mathrm{nveu}}=5.57-1.0(0.120)(6.50)=4.79 \mathrm{ksf}
$$

The Factored Net Bearing Resistance Chart is only valid for a specific effective footing length, $L^{\prime}$, and embedment depth of 6.50 feet. The bridge engineer will enter the chart for an effective footing width, $\mathrm{B}^{\prime}=13.15$ feet, for a net equivalent uniform bearing stress of 4.79 ksf . From the chart the corresponding settlement is approximately 0.95 inch. Refer to Figure 11.


Figure 11

## Expansion Abutment

$$
\begin{aligned}
& \mathrm{M}_{\text {long }}= 1.0(1489+107+4027-3080)+1.0(382+986) \\
&+0.3(256)+1.0(63)=4051 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}= 1.0(0)+1.0(2484)+0.3(1073)+1.0(236) \\
&= 3042 \mathrm{ft}-\mathrm{k} \\
& \\
& \mathrm{e}_{\text {long }}= \mathrm{e}_{\mathrm{B}}=4051 / 3491=1.160 \mathrm{ft} \\
& \mathrm{e}_{\text {trans }}= \mathrm{e}_{\mathrm{L}}=3042 / 3491=0.871 \mathrm{ft} \\
&\left.\mathrm{~B}^{\prime}=16.00-2(1.160)=13.68 \mathrm{ft} \text { (effective footing width }\right) \\
& \mathrm{L}^{\prime}=49.52-2(0.871)=47.78 \mathrm{ft}(\text { effective footing length }) \\
& \\
& \mathrm{q}_{\text {tveu }}= 3491 /[(13.68)(47.78)]=5.34 \mathrm{ksf}
\end{aligned}
$$

The gross equivalent uniform bearing stress at the base of the footing is 5.34 ksf. To determine the corresponding settlement for this bearing stress, the equivalent net uniform bearing stress is required. To determine the equivalent net uniform bearing stress, the factored overburden stress is subtracted.

$$
\mathrm{q}_{\mathrm{nveu}}=5.34-1.0(0.120)(6.50)=4.56 \mathrm{ksf}
$$

The bridge engineer will enter the Factored Net Bearing Resistance Chart for an effective footing width B' of 13.68 feet and a net equivalent uniform bearing stress of 4.56 ksf to determine the corresponding settlement of 0.90 inch as seen in Figure 12.


Figure 12

The estimated settlement is less than the target settlement of 4.80 inches and the settlement design limit state is satisfied. While not required, a more precise method of analysis will be used to demonstrate how to determine the settlement considering the phased application of the loads.

Phase 1 consists of construction of the abutment and earth fill. At this point any differential settlement can be corrected with construction of the superstructure since the expansion joint closure pour for the backwall has not been poured.

$$
\begin{aligned}
& \mathrm{P}_{\text {max }}=1.0(948+850)=1798 \mathrm{kips} \\
& \mathrm{M}_{\text {long }}=1.0(-48-3080+4027)=899 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}=0 \mathrm{ft}-\mathrm{k} \\
& \mathrm{e}_{\text {long }}=\mathrm{e}_{\mathrm{B}}=899 / 1798=0.500 \mathrm{ft} \\
& \mathrm{e}_{\text {trans }}=\mathrm{e}_{\mathrm{L}}=0 \mathrm{ft} \\
& \mathrm{~B}^{\prime}=16.00-2(0.500)=15.00 \mathrm{ft} \text { (effective footing width) } \\
& \mathrm{L}^{\prime}=49.52-2(0.000)=49.52 \mathrm{ft} \text { (effective footing length) } \\
& \mathrm{q}_{\text {tveu }}=1798 /[(15.00)(49.52)]=2.42 \mathrm{ksf} \\
& \mathrm{q}_{\text {nveu }}=2.42-1.0(0.120)(6.50)=1.64 \mathrm{ksf}
\end{aligned}
$$



Figure 13

From the design chart the settlement is approximately 0.25 inches.
Phase 2 consists of the completed structure with all design loads applied. The settlement from this case has been calculated to be 0.90 inch. The difference between the settlement for Phase 1 and Phase 2 is both the design differential settlement. The differential settlement equals $0.90-0.25=0.65$ inches. Since this value is less than the tolerable settlement of 4.80 inches the settlement design limit state is satisfied.

## [11.6.3.6]

[10.6.3.4]
[11.5.3]
[C11.5.5-2]

## 3. Sliding

Spread footings must be designed to resist lateral loads without sliding failure of the foundation. The sliding resistance of a footing on cohesionless soil is based on the normal stress and the interface friction between the foundation and the soil. The Geotechnical Foundation Report should provide the coefficient of sliding resistance, $\mu$, for use in design. For this example, assuming a cast-in-place footing, $\mu=\tan \varphi^{\prime}=\tan (30)=0.577$.

The Strength Limit States are used for this check. Since the resistance is based on the reaction, minimum factors are used for all vertical loads and the vertical weight of the live load surcharge is ignored on the footing heel. The maximum factors are used with the horizontal forces.

## Strength I (min) Limit State:



SLIDING \& ECCENTRICITY STRENGTH I
Figure 14

Max Strength $\mathrm{I}=1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV}$ $+1.75(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+\mathrm{FR}$

Min Strength $\mathrm{I}=0.90 \mathrm{DC}+0.65 \mathrm{DW}+0.90 \mathrm{EH}+1.00 \mathrm{EV}$
$+1.75(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+\mathrm{FR}$
Since DW is the future wearing surface, the surface may not be present for some time if at all and the weight should be zero.

$$
\mathrm{P}_{\min }=0.90(2177)+0.65(0)+1.50(0)+1.00(850)=2809 \mathrm{k}
$$

Pinned Abutment

$$
\begin{aligned}
& \mathrm{V}_{\text {long }}=1.25(0)+1.50(0)+1.50(493)+1.35(0)+1.75(0+44+81)=958 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.75(0+12+0)=21 \mathrm{k} \\
& V_{u}=\sqrt{(958)^{2}+(21)^{2}}=958 \mathrm{k} \\
& \varphi \mathrm{~V}_{\mathrm{n}}=0.80(0.577)(2809)=1297 \mathrm{k}
\end{aligned}
$$

Expansion Abutment

$$
\begin{aligned}
\hline \mathrm{V}_{\text {long }}= & 1.25(0)+1.50(0)+1.50(493)+1.35(0)+1.75(0+0+81) \\
& +161=1042 \mathrm{k} \\
\mathrm{~V}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.75(0+0+0)+43 \\
= & 43 \mathrm{k} \\
V_{u}= & \sqrt{(1042)^{2}+(43)^{2}}=1043 \mathrm{k} \\
\varphi \mathrm{~V}_{\mathrm{n}}= & 0.80(0.577)(2809)=1297 \mathrm{k}
\end{aligned}
$$

## Strength III (min) Limit State:



SLIDING \& ECCENTRICITY STRENGTH III
Figure 15

$$
\begin{aligned}
\text { Max Strength III }= & 1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV} \\
& +1.40\left(\mathrm{WS}+\mathrm{WS}_{\text {vert }}\right)+\mathrm{FR} \\
& \\
\text { Min Strength III }= & 0.90 \mathrm{DC}+0.65 \mathrm{DW}+0.90 \mathrm{EH}+1.00 \mathrm{EV} \\
& +1.40\left(\mathrm{WS}+\mathrm{WS}_{\text {vert }}\right)+\mathrm{FR} \\
\mathrm{P}_{\min }= & 0.90(2177)+0.65(0)+1.50(0)+1.00(850)+1.40(-72) \\
= & 2709 \mathrm{k}
\end{aligned}
$$

Pinned Abutment

$$
\begin{aligned}
& \mathrm{V}_{\text {long }}=1.25(0)+1.50(0)+1.50(493)+1.35(0)+1.40(36)=790 \mathrm{k} \\
& \mathrm{~V}_{\text {trans }}=1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.40(51)=71 \mathrm{k} \\
& V_{u}=\sqrt{(790)^{2}+(71)^{2}}=793 \mathrm{k} \\
& \varphi \mathrm{~V}_{\mathrm{n}}=0.80(0.577)(2709)=1250 \mathrm{k}
\end{aligned}
$$

Expansion Abutment

$$
\begin{aligned}
\begin{aligned}
\mathrm{V}_{\text {long }} & =1.25(0)+1.50(0)+1.50(493)+1.35(0)+1.40(11)+161 \\
& =916 \mathrm{k} \\
\mathrm{~V}_{\text {trans }} & =1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.40(51)+43 \\
& =114 \mathrm{k} \\
V_{u}= & \sqrt{(916)^{2}+(114)^{2}}=923 k \\
\varphi V_{\mathrm{n}} & =0.80(0.577)(2709)=1250 \mathrm{k}
\end{aligned}
\end{aligned}
$$

## Strength V (min) Limit State:



SLIDING \& ECCENTRICITY STRENGTH V
Figure 16

$$
\begin{aligned}
& \text { Max Strength } \mathrm{V}= 1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV} \\
&+1.35(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+0.40 \mathrm{WS}+1.00 \mathrm{WL}+\mathrm{FR} \\
& \text { Min Strength } \mathrm{V}= 0.90 \mathrm{DC}+0.65 \mathrm{DW}+0.90 \mathrm{EH}+1.00 \mathrm{EV} \\
&+1.35(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+0.40 \mathrm{WS}+1.00 \mathrm{WL}+\mathrm{FR} \\
& \mathrm{P}_{\min }=0.90(2177)+0.65(0)+1.50(0)+1.00(850)=2809 \mathrm{k}
\end{aligned}
$$

Pinned Abutment

$$
\begin{aligned}
\mathrm{V}_{\text {long }}= & 1.25(0)+1.50(0)+1.50(493)+1.35(0)+1.35(0+44+81) \\
& +0.40(36)+1.00(7)=930 \mathrm{k} \\
& \\
\mathrm{~V}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.35(0+12+0) \\
& +0.40(51)+1.00(8)=45 \mathrm{k} \\
V_{u}= & \sqrt{(930)^{2}+(45)^{2}}=931 k \\
\varphi V_{\mathrm{n}}= & 0.80(0.577)(2809)=1297 \mathrm{k}
\end{aligned}
$$

Expansion Abutment

$$
\begin{aligned}
\mathrm{V}_{\text {long }}= & 1.25(0)+1.50(0)+1.50(493)+1.35(0)+1.35(0+0+81) \\
& +0.40(11)+1.00(2)+161=1016 \mathrm{k} \\
\mathrm{~V}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.35(0+0+0)+0.40(51) \\
& +1.00(8)+43=71 \mathrm{k} \\
V_{u}= & \sqrt{(1016)^{2}+(71)^{2}}=1018 \mathrm{k} \\
\varphi \mathrm{~V}_{\mathrm{n}}= & 0.80(0.577)(2809)=1297 \mathrm{k}
\end{aligned}
$$

Since the resistance to sliding, $\varphi V_{n}$, is greater than the factored load, $V_{u}$, for all strength limit states, the sliding criteria is satisfied.
[10.6.3.3]

## 4. Limiting Eccentricity (Overturning or Excessive Loss of Contact)

Spread footing foundations must be designed to resist overturning which results from lateral and eccentric vertical loads. For LRFD, the criteria were revised to reflect the factoring of loads. As a result, the eccentricity of footings for factored loads must be less than $\mathrm{B} / 4$ and $\mathrm{L} / 4$ for footings on soil. These new limits were developed by direct calibration with ASD. The effect of factoring the loads is to increase the eccentricity of the load resultant such that the permissible eccentricity is increased.

The appropriate strength limit states are I, III and V. The maximum eccentricity will be found by applying the maximum load factors to each lateral or eccentrically applied load but to apply the minimum load factors to the resisting loads. The load combinations are the same as for sliding except moments are grouped instead of lateral loads.

## Strength I (min) Limit State

$$
\begin{aligned}
& \text { Max Strength } \mathrm{I}= 1.25 \mathrm{DC}+1.50 \mathrm{DW}+1.50 \mathrm{EH}+1.35 \mathrm{EV} \\
&+1.75(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+\mathrm{FR}+\text { Bearing } \\
& \text { Min Strength } \mathrm{I}= 0.90 \mathrm{DC}+0.65 \mathrm{DW}+0.90 \mathrm{EH}+1.00 \mathrm{EV} \\
&+1.75(\mathrm{LL}+\mathrm{BR}+\mathrm{LS})+\mathrm{FR}+\text { Bearing } \\
& \mathrm{P}_{\min }=0.90(2177)+0.65(0)+0.90(0)+1.00(850)=2809 \mathrm{k}
\end{aligned}
$$

Pinned Abutment

$$
\begin{aligned}
\mathrm{M}_{\text {long }}= & 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
& +1.75(382+746+986)+961=9643 \mathrm{ft}-\mathrm{k} \\
\mathrm{M}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.75(2484+200+0) \\
& +258=4955 \mathrm{ft}-\mathrm{k} \\
& \\
\mathrm{e}_{\text {long }}= & 9643 / 2809=3.43 \mathrm{ft}<\mathrm{B} / 4=16.00 / 4=4.00 \mathrm{ft} \\
\mathrm{e}_{\text {trans }}= & 4955 / 2809=1.76 \mathrm{ft}<\mathrm{L} / 4=49.52 / 4=12.38 \mathrm{ft}
\end{aligned}
$$

Expansion Abutment

$$
\begin{aligned}
& \mathrm{M}_{\text {long }}= 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
&+1.75(382+0+986)+2707+961=11044 \mathrm{k} \\
& \mathrm{M}_{\text {trans }}= 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.75(2484+0+0) \\
&+725+258=5330 \mathrm{ft}-\mathrm{k} \\
& \\
& \mathrm{e}_{\text {long }}= 11044 / 2809=3.93 \mathrm{ft}<4.00 \mathrm{ft} \\
& \mathrm{e}_{\text {trans }}= 5330 / 2809=1.90 \mathrm{ft}<12.38 \mathrm{ft}
\end{aligned}
$$

## Strength III (min) Limit State:

$$
\mathrm{P}_{\min }=0.90(2177)+0.65(0)+0.90(0)+1.00(850)+1.4(-72)=2709 \mathrm{k}
$$

Pinned Abutment

$$
\begin{aligned}
\mathrm{M}_{\text {long }}= & 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
& +1.40(634+208)+961=7122 \mathrm{ft}-\mathrm{k} \\
\mathrm{M}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.40(1073+777) \\
& +258=2848 \mathrm{ft}-\mathrm{k} \\
& \\
\mathrm{e}_{\text {long }}= & 7122 / 2709=2.63 \mathrm{ft}<4.00 \mathrm{ft} \\
\mathrm{e}_{\text {trans }}= & 2848 / 2709=1.05 \mathrm{ft}<12.38 \mathrm{ft}
\end{aligned}
$$

## Expansion Abutment

$$
\begin{aligned}
\mathrm{M}_{\text {long }}= & 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
& +1.40(256+208)+2707+961=9300 \mathrm{ft}-\mathrm{k} \\
\mathrm{M}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.40(1073+777) \\
& +725+258=3573 \mathrm{ft}-\mathrm{k} \\
& \\
\mathrm{e}_{\text {long }}= & 9300 / 2709=3.43 \mathrm{ft}<4.00 \mathrm{ft} \\
\mathrm{e}_{\text {trans }}= & 3573 / 2709=1.32 \mathrm{ft}<12.38 \mathrm{ft}
\end{aligned}
$$

## Strength V (min) Limit State:

$$
\mathrm{P}_{\min }=0.90(2177)+0.65(0)+0.90(0)+1.00(850)=2809 \mathrm{k}
$$

Pinned Abutment

$$
\begin{aligned}
& \mathrm{M}_{\text {long }}= 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
&+1.35(382+746+986)+0.40(634)+1.00(125)+961 \\
&= 9176 \mathrm{ft}-\mathrm{k} \\
& \mathrm{M}_{\text {trans }}= 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.35(2484+200+0) \\
&+0.40(1073)+1.00(236)+258=4547 \mathrm{ft}-\mathrm{k} \\
& \\
& \mathrm{e}_{\text {long }}= 9176 / 2809=3.27 \mathrm{ft}<4.00 \mathrm{ft} \\
& \mathrm{e}_{\text {trans }}= 4547 / 2809=1.62 \mathrm{ft}<12.38 \mathrm{ft}
\end{aligned}
$$

$$
\left.\begin{array}{rl}
\text { Expansion Abutment } \\
\mathrm{M}_{\text {long }}= & 1.25(1489)+1.50(107)+1.50(4027)+1.00(-3080) \\
& +1.35(382+0+986)+0.40(256)+1.00(63)+2707+961 \\
= & 10662 \mathrm{k}
\end{array}\right] \begin{aligned}
\mathrm{M}_{\text {trans }}= & 1.25(0)+1.50(0)+1.50(0)+1.35(0)+1.35(2484+0+0) \\
& +0.40(1073)+1.00(236)+725+258=5002 \mathrm{ft}-\mathrm{k} \\
\mathrm{e}_{\text {long }}= & 10662 / 2809=3.80 \mathrm{ft}<4.00 \mathrm{ft} \\
\mathrm{e}_{\text {trans }}= & 5002 / 2809=1.78 \mathrm{ft}<12.38 \mathrm{ft}
\end{aligned}
$$

## [11.5.2]

[11.5.3]

## BACKWALL DESIGN

## 5. Overall Stability

Overall stability is a service limit state. This will depend upon the properties of the supporting soil as well as the geometry of the land including any slopes. This design check is the responsibility of the geotechnical engineer. The results of this analysis should be included in the Final Foundation Report.

## 6. Structural Resistance

All components of the abutment must satisfy the appropriate strength and serviceability requirements. The three major parts of the abutment consist of the backwall, stem and footing. The design will be based on a foot wide strip.

## Backwall Design



BACKWALL
Figure 17
[3.5.1]
[3.11.5.3]
[3.11.6.4]
[3.6.1]
[3.6.2]

DC Loads: The backwall must support the eccentric load from the seat including the self-weight of half the approach slab. A conservative assumption is to ignore the approach slab where this load reduces the critical moment.

> Seat
> $\mathrm{M}=0.15[(1.00)(1.00)(1.00)+0.50(1.00)(1.00)(0.83)]=-0.21 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$

Approach Slab
$\mathrm{M}=0.15(1.00) \cos (15)(7.50)(1.00)=-1.09 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
DW Load: The approach slab could have a wearing surface added in the future.

Wearing surface

$$
\mathrm{M}=0.025(1.00) \cos (15)(7.50)(1.00)=-0.18 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

EH Load: The horizontal soil pressure will exert an inward force on the backwall. To simplify the problem, the soil is conservatively assumed to extend to the top of the backwall.

$$
\begin{aligned}
& \mathrm{V}=(0.295)(0.120)(7.67)^{2} / 2=1.041 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}=(1.041)(7.67) / 3=2.66 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

LS Load: The live load surcharge will not be seen by the backwall after the approach slab is constructed. Live load vehicles acting within a distance equal to one-half the wall height behind the back face of the wall will be carried by the approach slab. For this short wall height the live load surcharge can be ignored.

LL Load: The live load vehicle will react through the approach slab seat producing a moment in the backwall. Assuming a 45 degree angle of distribution along the backwall, the distribution width of one vehicle will be $7.67+6.00 / \cos (15)+7.67=21.55$ feet. For two vehicles separated by 6 feet the distribution width will be $7.67+3(6.00) / \cos (15)+7.67=33.97$ feet. For three vehicles each separated by 6 feet, the distribution width will be $7.67+$ $5(6.00) / \cos (15)+7.67=46.40$ feet. The resulting live load moment including the multiple presence factor is:

$$
\begin{aligned}
& \mathrm{M}=[32.00(1)(1.20) / 21.55](1.00)=-1.78 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
& \mathrm{M}=[32.00(2)(1.00) / 33.97](1.00)=-1.88 \mathrm{ft}-\mathrm{kt} \\
& \mathrm{M}=[32.00(3)(0.85) / 46.40](1.00)=-1.76 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

IM Load: the dynamic load allowance of 33 percent applies to the backwall.

$$
\mathrm{M}=(-1.88)(0.33)=-0.62 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

Reduction due to Earth Pressure [3.11.7]

BR Load: The effect of braking vehicles on the top of the backwall and approach slab must be included. The 18 kip braking force previously calculated for the footing design is intended to reflect the braking force applied to the superstructure. Since only an axle can react on the backwall, the force may be proportioned as follows: $\mathrm{BR}=18.00[(32) /(32+32+8)]=8.00 \mathrm{kips}$. Assume that the backwall resists the entire force without assistance from the approach slab.

$$
\begin{aligned}
& \mathrm{V}=[8.00(1)(1.20) / 21.55]=0.445 \mathrm{k} / \mathrm{ft} \\
& \mathrm{~V}=[8.00(2)(1.00) / 33.97]=0.471 \mathrm{k} / \mathrm{ft} \\
& \mathrm{~V}=[8.00(3)(0.85) / 46.40]=0.440 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}=(0.445)(7.67)=3.41 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
& \mathrm{M}=(0.471)(7.67)=3.61 \mathrm{ft}-\mathrm{kt} / \mathrm{ft} \\
& \mathrm{M}=(0.440)(7.67)=3.37 \mathrm{ft} \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

Since wind is not a design force for the backwall design, Strength I will be the controlling limit state. Since all the forces are not in the same direction, use the minimum load factor when the force reduces the magnitude of the final result.

For horizontal earth pressure, use half the value when opposing the primary direction of loads. This reduction need not be combined with the minimum load factor specified in Table 3.4.1-2.

A summary of unfactored backwall shears and moments follows:

| Load | $\mathrm{V}_{\text {pos }}$ | $\mathrm{V}_{\text {neg }}$ | $\mathrm{M}_{\text {pos }}$ | $\mathrm{M}_{\text {neg }}$ |
| :--- | :---: | :---: | :---: | :---: |
|  | $\mathrm{k} / \mathrm{ft}$ | $\mathrm{k} / \mathrm{ft}$ | $\mathrm{ft}-\mathrm{k} / \mathrm{ft}$ | $\mathrm{ft}-\mathrm{k} / \mathrm{ft}$ |
| $\mathrm{DC}_{\text {backwall }}$ | 0 | 0 | 0 | -0.21 |
| DC $_{\text {Appr slab }}$ | 0 | 0 | 0 | -1.09 |
| DW | 0 | 0 | 0 | -0.18 |
| EH | 1.041 | 0 | 2.66 | 0 |
| LL | 0 | 0 | 0 | -1.88 |
| IM | 0 | 0 | 0 | -0.62 |
| BR | 0.471 | -0.471 | 3.61 | -3.61 |

Standard ADOT practice is to use a minimum \#6 @ 12 inches for the vertical reinforcing in both faces unless calculations require additional reinforcing.
[3.4.1]
[5.7.3.1.1-4]
[5.7.2.1]
[5.7.2.2]
[C5.7.2.1]
[C5.5.4.2.1-1]
[5.5.4.2.1]
[5.7.3.2.2-1]

Maximum
Reinforcing
[5.7.3.3.1]

Strength I Limit State

$$
\begin{aligned}
\mathrm{M}_{\mathrm{u}}= & 1.25(-0.21-1.09)+1.50(-0.18)+0.50(2.66) \\
& +1.75[-1.88-0.62-3.61]=\underline{-11.26} \mathrm{ft}-\mathrm{k} / \mathrm{ft}<=\text { Critical }
\end{aligned}
$$

Assuming the approach slab is supported on soil and $\mathrm{DC}_{\text {Appr slab }}=0$.

$$
\mathrm{M}_{\mathrm{u}}=0.90(-0.21)+1.50(2.66)+1.75(3.61)=10.12 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

Try \# 6 @ 12 inches

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{s}}=0.44 \mathrm{in}^{2} / \mathrm{ft} \\
& \mathrm{~d}_{\mathrm{s}}=12.00-2.00 \text { clear }-0.75 / 2=9.63 \text { in }
\end{aligned}
$$

$$
c=\frac{A_{s} f_{y}}{0.85 f_{c}^{\prime} \beta_{1} b}=\frac{(0.44) \cdot(60)}{(0.85) \cdot(3.5) \cdot(0.85) \cdot(12)}=0.870 \mathrm{in}
$$

$$
\frac{c}{d_{s}}=\frac{0.870}{9.63}=0.090<0.6
$$

Therefore, $\mathrm{f}_{\mathrm{s}}$ in Equation 5.7.3.1.1-4 may be replaced by $\mathrm{f}_{\mathrm{y}}$.

$$
a=\beta_{1} c=(0.85) \cdot(0.870)=0.74 \mathrm{in}
$$

The net tensile strain in the reinforcing is:

$$
\varepsilon_{t}=0.003\left(\frac{d_{t}}{c}-1\right)=0.003\left(\frac{9.63}{0.870}-1\right)=0.030
$$

Since the net tensile strain, $\varepsilon_{\mathrm{t}}=0.030>0.005$, the section is tension-controlled. Since the section is tension-controlled, the reduction factor $\varphi=0.90$.

$$
\varphi M_{n}=(0.90) \cdot(0.44) \cdot(60) \cdot\left[9.63-\frac{0.74}{2}\right] \div 12=18.33 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

Since the factored flexural resistance is greater than the factored load, the section is adequate for flexure.

The provision that limited the amount of reinforcing in a section was deleted in 2005.

Minimum
Reinforcing
[5.7.3.3.2]
[5.4.2.6]

## Control of Cracking

 [5.7.3.4][5.4.2.6]

## [3.4.1]

Check section for minimum reinforcing criteria:

$$
\begin{aligned}
& S_{c}=\frac{b h^{2}}{6}=\frac{(12) \cdot(12)^{2}}{6}=288 \mathrm{in}^{3} / \mathrm{ft} \\
& f_{r}=0.37 \sqrt{f_{c}^{\prime}}=0.37 \sqrt{3.5}=0.692 \mathrm{ksi}
\end{aligned}
$$

The amount of reinforcing shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

$$
\begin{aligned}
& 1.2 \mathrm{M}_{\mathrm{cr}}=1.2(0.692)(288) \div 12=19.93 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
& 1.33 \mathrm{M}_{\mathrm{u}}=1.33(11.26)=14.98 \mathrm{ft}-\mathrm{k} / \mathrm{ft}<=\text { Critical }
\end{aligned}
$$

Since the flexural resistance, $\varphi \mathrm{M}_{\mathrm{n}}=18.33 \mathrm{ft}-\mathrm{k} / \mathrm{ft}>14.98 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$, the minimum reinforcing criteria is satisfied.

This section applies to all members in which tension in the cross-section exceeds 80 percent of the modulus of rupture at service limit state.

For this requirement the modulus of rupture is:

$$
\begin{aligned}
& f_{r}=0.24 \sqrt{f_{c}^{\prime}}=0.24 \sqrt{3.5}=0.449 \mathrm{ksi} \\
& 0.80 f_{r}=(0.80)(0.449)=0.359 \mathrm{ksi}
\end{aligned}
$$

Service I Limit State controls as follows:

$$
\begin{aligned}
\mathrm{M}_{\mathrm{s}}= & 1.0(-0.21-1.09)+1.0(-0.18)+0.5(2.66) \\
& +1.0(-1.88-0.62-3.61)=-\underline{-6.26} \mathrm{ft}-\mathrm{k} / \mathrm{ft}<=\text { Critical } \\
\mathrm{M}_{\mathrm{s}}= & 1.0(-0.21)+1.0(2.66)+1.0(3.61)=6.06 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

The stress in the uncracked section of the backwall under service loads, where the section modulus was previously calculated, is as follows:

$$
f_{s}=\frac{M_{s}}{S_{c}}=\frac{(6.26) \cdot(12)}{288}=0.261 \mathrm{ksi}
$$

Since the service limit stress, $\mathrm{f}_{\mathrm{s}}=0.261 \mathrm{ksi}$, in the section is less than 80 percent of the cracking stress, $0.80 \mathrm{f}_{\mathrm{r}}=0.359$, the provisions of this section need not be satisfied.

## Shrinkage \& Temperature Reinforcement [5.10.8]

## Development of Reinforcement [5.11.2]

[5.11.2.1.1]
[5.11.2.1.3]

Splice Length
[5.11.5.3.1-1]

## [5.11.5.3.1]

Reinforcing shall be distributed equally on both faces in both directions with a minimum area of reinforcement satisfying:

$$
A_{s} \geq \frac{1.30 b h}{2(b+h) f_{y}}=\frac{(1.30) \cdot(92.0) \cdot(12.0)}{(2) \cdot(92.0+12.0) \cdot(60)}=0.115 \mathrm{in}^{2}
$$

and $0.11<\mathrm{A}_{\mathrm{s}}<0.60$
The spacing shall not exceed:
3.0 times the thickness $=(3.0)(12.0)=36$ in, or 18.0 in

Use \#5 @ 12 inches for horizontal temperature and shrinkage reinforcement in the backwall.

The vertical reinforcing must be developed on each side of the critical section for its full development length.

Required development length for \#6 bars:
For \#11 bar and smaller $\frac{1.25 A_{b} f_{y}}{\sqrt{f^{\prime}{ }_{c}}}=\frac{(1.25) \cdot(0.44) \cdot(60)}{\sqrt{3.5}}=17.6$ in
but not less than $0.4 \mathrm{~d}_{\mathrm{b}} \mathrm{f}_{\mathrm{y}}=0.4(0.75)(60)=18.0$ in

Modification Factors that decrease $l_{d}$ :
Spacing not less than 6 inch $=0.8$
Excess reinforcing $=11.26 / 18.33=0.614$
Required development:

$$
1_{d}=(0.8)(0.614)(18.0)=8.8 \text { in } \Rightarrow \text { Use } 1^{\prime}-0 \prime \text { " minimum }
$$

A Class C splice is required for the vertical bars in the backwall since all the bars are spliced at the same location and the area of reinforcing required divided by the area provided is less than 2. A Class C splice requires a minimum length of $1.7 l_{d}$.

$$
\text { Splice }=1.7(18.0)(0.8)(0.614)=15.0 \text { in } \Rightarrow \text { Use } 1^{\prime}-3 "
$$

Diagonal Shear
[5.8]
[3.4.1]
[5.8.2.9]
[C5.8.2.9-1]

Simplified
Procedure
[5.8.3.4.1]
[5.8.3.3-3]
[5.8.3.3-1]
[5.8.3.3-2]
[5.8.2.4-1]
[5.8.2.1-2]

The diagonal tension shear may be determined a distance $\mathrm{d}_{\mathrm{v}}$ from the face of the support. For simplicity determine the factored shear at the face. If the section does not have adequate shear resistance the calculation should be refined to use the factored shear at a distance $\mathrm{d}_{\mathrm{v}}$ from the support.

## Strength I Limit State

$$
\mathrm{V}_{\mathrm{u}}=1.50(1.041)+1.75(0.471)=2.39 \mathrm{k} / \mathrm{ft}
$$

The value for $d_{v}$ is the greater of the following:

$$
\begin{aligned}
& \mathrm{d}_{\mathrm{v}}=\mathrm{d}_{\mathrm{e}}-\mathrm{a} / 2=9.63-0.74 / 2=9.26 \mathrm{in}<=\text { Critical } \\
& \mathrm{d}_{\mathrm{v}}=0.9 \mathrm{~d}_{\mathrm{e}}=(0.9)(9.63)=8.67 \mathrm{in} \\
& \mathrm{~d}_{\mathrm{v}}=0.72 \mathrm{~h}=(0.72)(12.00)=8.64 \mathrm{in}
\end{aligned}
$$

For members having an overall depth of less than 16.0 inches the Simplified Procedure may be used and $\beta=2.0$.

The concrete resistance is as follows:

$$
\begin{aligned}
& V_{c}=0.0316 \beta \sqrt{f_{c}^{\prime}} b_{v} d_{v} \\
& V_{c}=(0.0316) \cdot(2.0) \cdot \sqrt{3.5} \cdot(12.0) \cdot(9.26)=13.14 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

Since the section will be checked as an unreinforced section for shear, $\mathrm{V}_{\mathrm{s}}$ will be zero.

The nominal shear resistance is the lesser of:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{n} 1}=\mathrm{V}_{\mathrm{c}}+\mathrm{V}_{\mathrm{s}}=13.14 \mathrm{k} / \mathrm{ft}<=\text { Critical } \\
& \mathrm{V}_{\mathrm{n} 2}=0.25 \mathrm{f}^{\prime}{ }_{\mathrm{c}} \mathrm{~b}_{\mathrm{v}} \mathrm{~d}_{\mathrm{v}}=(0.25)(3.5)(12.0)(9.26)=97.23 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

Since the shear behavior of the backwall is similar to that of a slab, Equation 5.8.2.4-1 need not be satisfied. Therefore shear reinforcing can be omitted when the factored shear is less than the factored resistance.

$$
\mathrm{V}_{\mathrm{r}}=\varphi \mathrm{V}_{\mathrm{n}}=(0.90)(13.14)=11.83 \mathrm{k} / \mathrm{ft}>2.39 \mathrm{k} / \mathrm{ft} \mathrm{ok}
$$

Interface Shear [5.8.4]
[5.8.4.1-4]
[5.8.4.1-5]
[5.8.4.3]
[5.8.4.1-3]
[5.8.4.1-1]
[5.8.4.1-2]
Minimum
Reinforcement [5.8.4.4-1]

Interface shear transfer shall be considered across a given plane at an interface between two concretes cast at different times such as the construction joint at the base of the backwall.

The factored load was determined above to be $\mathrm{V}_{\mathrm{u}}=2.39 \mathrm{k} / \mathrm{ft}$.
The nominal shear resistance, $\mathrm{V}_{\mathrm{ni}}$ used in the design shall not be greater than the lesser of:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{ni}} \leq \mathrm{K}_{1} \mathrm{f}^{\prime}{ }_{\mathrm{c}} \mathrm{~A}_{\mathrm{cv}} \text {, or } \\
& \mathrm{V}_{\mathrm{ni}} \leq \mathrm{K}_{2} \mathrm{~A}_{\mathrm{cv}}
\end{aligned}
$$

Where $\mathrm{A}_{\mathrm{cv}}=$ area of concrete considered to be engaged in interface shear transfer.

For concrete placed against a clean concrete surface, free of laitance with surface intentionally roughened to an amplitude of 0.25 inch:

$$
\begin{aligned}
& \mathrm{c}=0.24 \mathrm{ksi} \\
& \mu=1.0 \\
& \mathrm{~K}_{1}=0.25 \\
& \mathrm{~K}_{2}=1.5 \mathrm{ksi}
\end{aligned}
$$

The nominal shear resistance of the interface plans shall be taken as:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{ni}}=\mathrm{c} \mathrm{~A}_{\mathrm{cv}}+\mu\left(\mathrm{A}_{\mathrm{vf}} \mathrm{f}_{\mathrm{y}}+\mathrm{P}_{\mathrm{c}}\right) \\
& \mathrm{V}_{\mathrm{ni}}=(0.24)(12)(9.26)+1.0(0.44)(60+0)=53.07 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

But not greater than the lesser of:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{ni}}=(0.25)(3.5)(12.0)(9.26)=97.23 \mathrm{k} / \mathrm{ft} \\
& \mathrm{~V}_{\mathrm{ni}}=(1.5)(12.0)(9.26)=166.68 \mathrm{k} / \mathrm{ft} \\
& \mathrm{~V}_{\mathrm{ri}}=\varphi \mathrm{V}_{\mathrm{ni}}=(0.90)(53.07)=47.76 \mathrm{k} / \mathrm{ft} \geq \mathrm{V}_{\mathrm{ui}}=2.39 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

The minimum area of interface shear reinforcement shall satisfy:

$$
A_{v f} \geq \frac{0.05 A_{c v}}{f_{y}}=\frac{(0.050) \cdot(12.0) \cdot(9.26)}{60}=0.09 \mathrm{in}^{2} / \mathrm{ft}
$$

Since \#6 at 12 inches is provided across the interface, the criteria is satisfied.

## STEM DESIGN

[3.5.1]

## Stem Design

The major loads on the stem are the horizontal earth pressure and the loads transmitted from the superstructure to the substructure through the bearings or pinned connection. Design is based on a one foot wide strip. Dimensions are shown in feet.


STEM
Figure 18
DC Loads: The stem must resist the eccentric dead loads from the seat, backwall, the approach slab and the superstructure.

Seat $=0.15[(1.00)(1.00)(2.50)+0.50(1.00)(1.00)(2.33)]=-0.55 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
Appr Slab $=0.15(1.00) \cos (15)(7.50)(2.50)=-2.72 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
Backwall $=0.15(1.00)(7.67)(1.50)=-1.73 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
Super $=1229.30(0.75) / 48.48=19.02 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
$\mathrm{DC}=-0.55-2.72-1.73+19.02=14.02 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
DW Load: The approach slab could have a wearing surface added in the future.

FWS Super $=85.63(0.75) / 48.48=1.32 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
FWS Appr Slab $=0.025(1.00) \cos (15)(7.50)(2.50)=-0.45 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
DW $=1.32-0.45=0.87 \mathrm{k} / \mathrm{ft}$

## [3.11.5.3]

## [3.11.6.4]

[3.6.1]
[3.6.2]
[3.6.4]
[3.13]
[14.6.3.2-3]

## EH Load:

$$
\begin{aligned}
& \mathrm{V}=(0.295)(0.120)(21.00)^{2} / 2=7.81 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}=(7.81)(21.00) / 3=54.64 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

LS Load: The live load surcharge will be reduced by the presence of the approach slab. However, ignore this effect for the design of the stem.

$$
\begin{aligned}
& \mathrm{V}=(0.295)(0.120)(2.00)(21.00)=1.49 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}=(1.49)(21.00) / 2=15.61 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

LL Load: The live load will react through the bearing producing a moment in the stem. For a fully loaded structure, three vehicles will be present.

$$
\mathrm{M}=[(52.19+67.80)(3)(0.85) / 48.48](0.75)=4.73 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

IM Load: The dynamic load allowance of 33 percent applies only to the design truck portion of the live load.

$$
\mathrm{M}=0.33[(67.80)(3)(0.85) / 48.48](0.75)=0.88 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

BR Load: From foundation load calculations the longitudinal forces are:

$$
\begin{aligned}
& \mathrm{V}=(44.34) / 48.48=0.91 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}=(44.34)(13.33) / 48.48=12.19 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

FR Load:

$$
\begin{aligned}
& \mathrm{V}=(160.83) / 48.48=3.32 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}=(160.83)(13.33) / 48.48=44.22 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

Bearing Pad Rotation:

$$
\begin{aligned}
& \mathrm{V}=0 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}=(961) / 48.48=19.82 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

The FR Load and Bearing Pad Rotation forces are already factored and only apply to the strength limit states.

A summary of unfactored stem shears and moments unless otherwise noted follow:

| Load | $\mathrm{V}_{\text {long }}$ | $\mathrm{M}_{\text {long }}$ |
| :--- | ---: | ---: |
| $\mathrm{k} / \mathrm{ft}$ | ft <br> $\mathrm{k} / \mathrm{ft}$ |  |
| DC | 0 | 14.02 |
| DW | 0 | 0.87 |
| EH | 7.81 | 54.64 |
| LL | 0 | 4.73 |
| IM | 0 | 0.88 |
| BR | 0.91 | 12.19 |
| LS | 1.49 | 15.61 |
| FR* | 3.32 | 44.22 |
| Bearing <br> Rotation* | 0 | 19.82 |

*Factored loads for strength limit states only
The flexural resistance for the stem will be controlled by the Strength I Limit State.

Pinned Abutment

$$
\begin{aligned}
\mathrm{M}_{\mathrm{u}}= & 1.25(14.02)+1.50(0.87)+1.50(54.64) \\
& +1.75(4.73+0.88+12.19+15.61)+19.82=179.08 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

## Expansion Abutment

$$
\begin{aligned}
\mathrm{M}_{\mathrm{u}}= & 1.25(14.02)+1.50(0.87)+1.50(54.64) \\
& +1.75(4.73+0.88+0+15.61)+44.22+19.82 \\
= & \underline{201.97} \mathrm{ft}-\mathrm{k} / \mathrm{ft}<=\text { Critical }
\end{aligned}
$$

Try \#8 @ 7 inches

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{s}}=(0.79)(12 / 7)=1.35 \mathrm{in}^{2} / \mathrm{ft} \\
& \mathrm{~d}_{\mathrm{s}}=48.00-2.00 \text { clear }-1.00 / 2=45.50 \text { in }
\end{aligned}
$$

$$
c=\frac{A_{s} f_{y}}{0.85 f^{\prime}{ }_{c} \beta_{1} b}=\frac{(1.35) \cdot(60)}{0.85 \cdot(3.5) \cdot(0.85) \cdot(12)}=2.669 \mathrm{in}
$$

$$
\frac{c}{d_{s}}=\frac{2.669}{45.50}=0.059<0.6 \text { Therefore, } \mathrm{f}_{\mathrm{y}} \text { may be used in above equation }
$$

$$
a=\beta_{1} c=(0.85) \cdot(2.669)=2.27 \mathrm{in}
$$

The net tensile strain in the reinforcing is:

$$
\varepsilon_{t}=0.003\left(\frac{d_{t}}{c}-1\right)=0.003\left(\frac{45.50}{2.669}-1\right)=0.048
$$

[C5.5.4.2.1-1]
[5.5.4.2.1]
[5.7.3.2.2-1]

Maximum
Reinforcement
[5.7.3.3.1]

## Minimum

Reinforcement
[5.7.3.3.2]
[5.4.2.6]

## Control of Cracking

 [5.7.3.4][5.4.2.6]

Since the net tensile strain, $\varepsilon_{\mathrm{t}}=0.048>0.005$, the section is tension-controlled. Since the section is tension-controlled the reduction factor $\varphi=0.90$.

$$
\varphi M_{n}=(0.90) \cdot(1.35) \cdot(60) \cdot\left[45.50-\frac{2.27}{2}\right] \div 12=269.52 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

Since the factored flexural resistance is greater than the factored load, the section is adequate for flexure.

The provision that limited the amount of reinforcing in a section was deleted in 2005.

Check section for minimum reinforcing criteria:

$$
\begin{aligned}
& S_{c}=\frac{b h^{2}}{6}=\frac{(12) \cdot(48)^{2}}{6}=4608 \mathrm{in}^{3} / \mathrm{ft} \\
& f_{r}=0.37 \sqrt{f_{c}^{\prime}}=0.37 \sqrt{3.5}=0.692 \mathrm{ksi}
\end{aligned}
$$

The amount of reinforcing shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

$$
\begin{aligned}
& 1.2 \mathrm{M}_{\mathrm{cr}}=1.2(0.692)(4608) \div 12=318.87 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
& 1.33 \mathrm{M}_{\mathrm{u}}=1.33(201.97)=268.62 \mathrm{ft}-\mathrm{k} / \mathrm{ft}<=\text { Critical }
\end{aligned}
$$

Since the flexural resistance, $\varphi \mathrm{M}_{\mathrm{n}}=269.52 \mathrm{ft}-\mathrm{k} / \mathrm{ft}>268.62 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$, the minimum reinforcing criteria is satisfied.

This section applies to all members in which tension in the cross-section exceeds 80 percent of the modulus of rupture at service limit state.

For this requirement the modulus of rupture is:

$$
f_{r}=0.24 \sqrt{f_{c}^{\prime}}=0.24 \sqrt{3.5}=0.449 \mathrm{ksi}
$$

$$
0.80 f_{r}=(0.80)(0.449)=0.359 \mathrm{ksi}
$$

## [3.4.1]

## Shrinkage \& Temperature Reinforcement [5.10.8

Service I Limit State controls as follows:
Pinned Abutment

$$
\begin{aligned}
\mathrm{M}_{\mathrm{s}}= & 1.00(14.02)+1.00(0.87)+1.00(54.64) \\
& +1.0(4.73+0.88+12.19+15.61)=102.94 \mathrm{ft}-\mathrm{k} / \mathrm{ft}<=\text { Critical }
\end{aligned}
$$

Expansion Abutment

$$
\begin{aligned}
\mathrm{M}_{\mathrm{s}}= & 1.00(14.02)+1.00(0.87)+1.00(54.64) \\
& +1.00(4.73+0.88+0+15.61)=90.75 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

The stress in the uncracked section of the stem under service loads follows:

$$
f_{s}=\frac{M_{s}}{S_{c}}=\frac{(102.94) \cdot(12)}{4608}=0.268 \mathrm{ksi}
$$

Since this service limit stress is less than 80 percent of the cracking load stress, the crack control criteria is satisfied.

Reinforcing shall be distributed equally on both faces in both directions with a minimum area of reinforcement satisfying:

$$
A_{s} \geq \frac{1.30 b h}{2(b+h) f_{y}}=\frac{(1.30) \cdot(160.0) \cdot(48.0)}{(2) \cdot(160.0+48.0) \cdot(60)}=0.400 \mathrm{in}^{2}
$$

and $0.11<\mathrm{A}_{\mathrm{s}}<0.60$
The spacing shall not exceed:
3.0 times the thickness $=(3.0)(48.0)=144.0 \mathrm{in}$, or 18.0 in
12.0 inches for walls greater than 18 inch thick.

Use \#6 @ 12 inches for horizontal temperature and shrinkage reinforcement in the stem.

## Development of Reinforcement [5.11.2]

[5.11.2.1.1]

## [5.11.2.1.3]

The vertical reinforcing must be developed on each side of the critical section for its full development length.

Required development length for \#8 bars:
For \#11 bar and smaller $\frac{1.25 A_{b} f_{y}}{\sqrt{f^{\prime}{ }_{c}}}=\frac{(1.25) \cdot(0.79) \cdot(60)}{\sqrt{3.5}}=31.7 \mathrm{in}$
but not less than $0.4 \mathrm{~d}_{\mathrm{b}} \mathrm{f}_{\mathrm{y}}=0.4(1.00)(60)=24.0$ in

Modification Factors that decrease $l_{d}$ :
Spacing not less than 6 inches $\quad=0.8$
Excess reinforcing $=201.97 / 269.52=0.749$
Required development:

$$
1_{d}=(0.8)(0.749)(31.7)=19.0 \text { in } \Rightarrow \text { Use } 1^{\prime}-9 " \text { minimum }
$$

Adequate development length is available to embed straight bars into the footing. However, hook bars per usual practice for ease of construction where the hooked bars can be set on top of the bottom matt of reinforcing.

## Diagonal Shear [5.8]

[3.4.1]
[5.8.2.9]
[C5.8.2.9-1]
[5.8.3.4.1]
[5.8.3.4.2]
[5.8.3.4.2-5]

The diagonal tension shear may be determined a distance $d_{v}$ from the face of the support. For simplicity determine the factored shear at the face. If the section does not have adequate shear resistance the calculation should be refined to use the factored shear a distance $d_{v}$ from the support.

## Strength I Limit State

## Pinned Abutment

$$
\mathrm{V}_{\mathrm{u}}=1.50(7.81)+1.75(0.91+1.49)=15.92 \mathrm{k} / \mathrm{ft}
$$

## Expansion Abutment

$$
\mathrm{V}_{\mathrm{u}}=1.50(7.81)+1.75(1.49)+3.32=\underline{17.64} \mathrm{k} / \mathrm{ft}<=\text { Critical }
$$

The value for $\mathrm{d}_{\mathrm{v}}$ is the greater of the following:

$$
\begin{aligned}
& \mathrm{d}_{\mathrm{v}}=\mathrm{d}_{\mathrm{e}}-\mathrm{a} / 2=45.50-2.27 / 2=44.37 \text { in } \Leftarrow \text { Critical } \\
& \mathrm{d}_{\mathrm{v}}=0.9 \mathrm{~d}_{\mathrm{e}}=(0.9)(45.50)=40.95 \text { in } \\
& \mathrm{d}_{\mathrm{v}}=0.72 \mathrm{~h}=(0.72)(48.00)=34.56 \text { in }
\end{aligned}
$$

Based on the above, the shear depth, $\mathrm{d}_{\mathrm{v}}$, equals 44.37 inches.

## Design Procedure

Three methods are available to determine shear resistance. Since none of the criteria required to use the simplified procedure is satisfied, this simplified method may not be used. The second method described in the General Procedure does not require minimum transverse reinforcement and will be used.

## Determine Crack Width

Shear reinforcing is typically not used in a stem wall. For shear design with less than minimum transverse reinforcing the maximum expected crack width must be determined.

$$
s_{x e}=s_{x} \frac{1.38}{a_{g}+0.63} \leq 80 \text { in }
$$

$\mathrm{a}_{\mathrm{g}}=$ maximum aggregate size $=3 / 4$ inches.
$s_{x}=d_{v}$ for members without uniformly spaced reinforcing throughout the depth, where the primary reinforcing is lumped in one location as in the case for the stem design.

$$
s_{x e}=(44.37) \cdot \frac{1.38}{0.75+0.63}=44.37 \mathrm{in}
$$

## Calculate strain, $\varepsilon_{\mathrm{s}}$

The strain in nonprestressed longitudinal tension reinforcement may be determined by the following formula when $\varepsilon_{\mathrm{s}}$ is positive.

$$
\varepsilon_{s}=\left[\frac{\frac{\left|M_{u}\right|}{d_{v}}+0.5 N_{u}+\left|V_{u}-V_{p}\right|-A_{p s} f_{p o}}{E_{s} A_{s}+E_{p} A_{p s}}\right]
$$

where:
$\mathrm{A}_{\mathrm{ps}}=$ area of prestressing steel on the flexural tension side of the member.
$\mathrm{A}_{\mathrm{ps}}=0$ in $^{2}$
$\mathrm{A}_{\mathrm{s}}=$ area of nonprestressed steel on the flexural tension side of the member.
$\mathrm{A}_{\mathrm{s}}=1.35 \mathrm{in}^{2}$.
$\mathrm{f}_{\mathrm{po}}=0 \mathrm{ksi}$
$\mathrm{N}_{\mathrm{u}}=$ factored axial force taken as positive if tensile. This load is only
used for permanent loads that will always be present. This value can be conservatively ignored for compressive loads as is done in this example.
$\mathrm{N}_{\mathrm{u}}=0 \mathrm{kips}$
$\mathrm{V}_{\mathrm{u}}=$ factored shear force.
$\mathrm{V}_{\mathrm{u}}=17.64$ kips
$\mathrm{V}_{\mathrm{p}}=0 \mathrm{kips}$
$\mathrm{M}_{\mathrm{u}}=$ factored moment but not to be taken less than $\mathrm{V}_{\mathrm{u}} \mathrm{d}_{\mathrm{v}}$.
$\mathrm{M}_{\mathrm{u}}=201.97 \mathrm{ft}-\mathrm{k}>(17.64)(44.37) / 12=65.22 \mathrm{ft}-\mathrm{k}$

$$
\begin{aligned}
& \varepsilon_{s}=\left[\frac{\frac{|(201.97) \cdot(12)|}{44.37}+0.5 \cdot(0)+|17.64-0|-0}{(29000) \cdot(1.35)+0}\right] \\
& \varepsilon_{\mathrm{s}}=0.00185
\end{aligned}
$$

Prior to the 2008 Interim Revisions, the General Procedure for shear design was iterative and required the use of tables for the evaluation of $\beta$ and $\theta$. With the 2008 Revisions, this design procedure was modified to be non-iterative and algebraic equations were introduced for the evaluation of $\beta$ and $\theta$. When sections do not contain at least the minimum amount of shear reinforcement:

$$
\begin{aligned}
& \beta=\frac{4.8}{\left(1+750 \varepsilon_{s}\right)} \frac{51}{\left(39+s_{x e}\right)} \\
& \beta=\frac{4.8}{(1+750 \cdot(0.00185))} \frac{51}{(39+44.37)}=1.23 \\
& \theta=29+3500 \varepsilon_{s}=29+(3500) \cdot(0.00185)=35.5 \text { degrees }
\end{aligned}
$$

## Calculate Concrete Shear Strength, $\mathbf{V}_{\mathrm{c}}$

The nominal shear resistance from concrete, $\mathrm{V}_{\mathrm{c}}$, is calculated as follows:

$$
\begin{aligned}
& V_{c}=0.0316 \beta \sqrt{f^{\prime}{ }_{c}} b_{v} d_{v} \\
& V_{c}=0.0316 \cdot(1.23) \cdot \sqrt{3.5} \cdot(12.00) \cdot(44.37)=38.72 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

The nominal shear resistance is the lesser of:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{n} 1}=\mathrm{V}_{\mathrm{c}}+\mathrm{V}_{\mathrm{s}}+\mathrm{V}_{\mathrm{p}}=[38.72+0+0]=38.72 \mathrm{k} / \mathrm{ft} \\
& \mathrm{~V}_{\mathrm{n} 2}=0.25 \mathrm{f}^{\prime}{ }_{\mathrm{c}} \mathrm{~b}_{\mathrm{v}} \mathrm{~d}_{\mathrm{v}}+\mathrm{V}_{\mathrm{p}}=0.25(3.5)(12.00)(44.37)+0=465.89 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

Except for slabs, footings and culverts, transverse reinforcing shall be provided where $V_{u}>0.5 \varphi\left(V_{c}+V_{p}\right)$. Since the stem may be treated as a slab, this equation need not be satisfied.

$$
\mathrm{V}_{\mathrm{r}}=\varphi \mathrm{V}_{\mathrm{n}}=(0.90)(38.72)=34.85 \mathrm{k} / \mathrm{ft}>\mathrm{V}_{\mathrm{u}}=17.64 \mathrm{k} / \mathrm{ft}
$$

[5.8.2.1-2]

Interface Shear [5.8.4]
[5.8.4.1-4]
[5.8.4.1-5]
[5.8.4.3]
[5.8.4.1-3]
[5.8.4.1-1]
[5.8.4.1-2]
Minimum
Reinforcement [5.8.4.4-1]

Interface shear transfer shall be considered across a given plane at an interface between two concretes cast at different times such as the construction joint at the base of the stem.

The factored load was determined above to be $\mathrm{V}_{\mathrm{u}}=17.64 \mathrm{k} / \mathrm{ft}$.
The nominal shear resistance, $\mathrm{V}_{\mathrm{ni}}$ used in the design shall not be greater than the lesser of:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{ni}} \leq \mathrm{K}_{1} \mathrm{f}^{\prime}{ }_{\mathrm{c}} \mathrm{~A}_{\mathrm{cv}} \text {, or } \\
& \mathrm{V}_{\mathrm{ni}} \leq \mathrm{K}_{2} \mathrm{~A}_{\mathrm{cv}}
\end{aligned}
$$

Where $\mathrm{A}_{\mathrm{cv}}=$ area of concrete considered to be engaged in interface shear transfer.

For concrete placed against a clean concrete surface, free of laitance with surface intentionally roughened to an amplitude of 0.25 inch:

$$
\begin{aligned}
& \mathrm{c}=0.24 \mathrm{ksi} \\
& \mu=1.0 \\
& \mathrm{~K}_{1}=0.25 \\
& \mathrm{~K}_{2}=1.5 \mathrm{ksi}
\end{aligned}
$$

The nominal shear resistance of the interface plans shall be taken as:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{ni}}=\mathrm{c} \mathrm{~A}_{\mathrm{cv}}+\mu\left(\mathrm{A}_{\mathrm{vf}} \mathrm{f}_{\mathrm{y}}+\mathrm{P}_{\mathrm{c}}\right) \\
& \mathrm{V}_{\mathrm{ni}}=(0.24)(12)(44.37)+1.0(1.35)(60+0)=208.79 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

But not greater than the lesser of:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{ni}}=(0.25)(3.5)(12.0)(44.37)=465.89 \mathrm{k} / \mathrm{ft} \\
& \mathrm{~V}_{\mathrm{ni}}=(1.5)(12.0)(44.37)=798.66 \mathrm{k} / \mathrm{ft} \\
& \mathrm{~V}_{\mathrm{ri}}=\varphi \mathrm{V}_{\mathrm{ni}}=(0.90)(208.79)=187.91 \mathrm{k} / \mathrm{ft} \geq \mathrm{V}_{\mathrm{ui}}=17.64 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

The minimum area of interface shear reinforcement shall satisfy:

$$
A_{v f} \geq \frac{0.05 A_{c v}}{f_{y}}=\frac{(0.050) \cdot(12.0) \cdot(44.37)}{60}=0.44 \mathrm{in}^{2}
$$

Since \#8 at 7 inches is provided across the interface, the criteria is satisfied.

## TOE DESIGN

[3.4.1]
[C3.4.1]

## Footing Design

The major loads to be used in the footing design were determined for the stability analysis. For the footing design, loads and moments are reduced to a one-foot wide strip analysis.

## Load Combinations

A discussion on load factors is in order since an understanding of this issue is critical to proper design of footings. A brief summary of key items discussed in the Specification follows.

The load factors shall be selected to produce the total extreme factored force effect. For each load combination both positive and negative extremes shall be investigated. In load combinations where one force effect decreases another effect, the minimum value shall be applied to the load reducing the force effect. For permanent force effects the load factor that produces the more critical combination shall be selected from Table 3.4.1-4 since the actual magnitude of permanent loads may be less than the nominal value. It is unnecessary to assume that one type of load varies by span, length or component within a bridge. For example, when investigating uplift at a bearing in a continuous beam, it would not be appropriate to use the maximum load factor for permanent loads in spans that produce a negative reaction and a minimum load factor in spans that produce a positive reaction. For each force effect, both extreme combinations may need to be investigated by applying either the high or the low load factor as appropriate. The algebraic sums of these products are the total force effects for which the bridge and its components should be designed.

From the previous results of this example, Strength I Limit State using maximum load factors for the expansion abutment produces the maximum soil stress in the toe. Since a single load factor should be used for each load type the toe, heel and stem should all have the same factor. Use of a maximum load factor for the DC and EV loads produces the maximum soil pressure but also produces the maximum resisting moment and shear since the overburden soil and footing toe resist the soil pressure. Use of a minimum load factor for the DC and EV loads reduces soil pressure but produces the minimum resisting moment or shear. Whether a maximum or minimum load factor produces the maximum moment and shear is not obvious, resulting in the need to analyze each possible combination of maximum and minimum load factors for all the loads.

This problem does not exist when considering the effects of Service I Limit State since all critical loads have a load factor of 1.0.

## Toe Design

A simplified method of analysis that is used in this example is to determine maximum moments and shears for the toe of a footing based on use of load factors that produce the maximum soil pressure and minimum resisting loads even when those different load factors are used for the same component. This method is neither consistent nor in strict adherence with the LRFD Specification but is conservative and eliminates the need for multiple combinations. For the toe design in this example, minimum load factors are used for the opposing forces from the overburden and footing self- weight. A more rigorous analysis that includes consistent use of load factors for all possible combinations is always acceptable and should be used when the simplified method becomes too conservative.

A summary of dimensions in feet for the footing toe and loads is shown below.


TOE

Figure 19

Unlike settlement and bearing resistance checks where the average uniform bearing stress is determined, for the design of structural elements a triangular or trapezoidal shaped soil stress distribution is assumed. This assumption will provide the maximum moments and shears in the footing.

$$
\begin{aligned}
& \mathrm{P}_{\max }=4659 / 49.52=94.08 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}_{\text {long }}=11044 / 49.52=223.02 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
& \mathrm{e}_{\text {long }}=223.02 / 94.08=2.370 \mathrm{ft} \\
& \mathrm{q}_{\max }=\mathrm{P} / \mathrm{B}(1+6 \mathrm{e} / \mathrm{B})=[94.08 / 16][1+6(2.370) / 16]=11.106 \mathrm{ksf} \\
& \mathrm{q}_{\min }=\mathrm{P} / \mathrm{B}(1-6 \mathrm{e} / \mathrm{B})=[94.08 / 16][1-6(2.370) / 16]=0.654 \mathrm{ksf} \\
& \\
& \mathrm{q}_{\text {toe }}=0.654+(11.106-0.654)(10.50) / 16=7.513 \mathrm{ksf} \\
& \mathrm{q}_{\mathrm{d}}=0.654+(11.106-0.654)(16.00-2.37) / 16=9.558 \mathrm{ksf}
\end{aligned}
$$

Strict adherence to LRFD Specifications would require that the same load factors used in the determination for the soil stress would also be used for determining shears and moments requiring several combinations of maximum and minimum load factors to be studied. The simplified method is demonstrated below.

DC Moment $=0.90[0.15(3.50)(5.50)](2.75)=7.15 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
EV Moment $=1.00[0.12(3.00)(5.50)](2.75)=5.45 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
$\mathrm{M}_{\mathrm{u}}=7.513(5.50)^{2} \div 2+(11.106-7.513)(5.50)^{2} \div 3-7.15-5.45=137.26 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
If the maximum load factors were used the moment would decrease by $(7.15)(1.25) / 0.90+(5.45)(1.35) / 1.00-7.15-5.45=4.69 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$ or less than 4 percent of the total.

Try \#8 @ 8 inches

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{s}}=(0.79)(12 / 8)=1.185 \mathrm{in}^{2} / \mathrm{ft} \\
& \mathrm{~d}_{\mathrm{s}}=42.00-3.00 \text { clear }-1.00 / 2=38.50 \text { in }
\end{aligned}
$$

[5.7.3.1.1-4]
[5.7.2.1]
[5.7.2.2]

## Strength I <br> Limit State <br> [3.4.1]

Flexural
Resistance
[5.7.3.2]
-

$$
c=\frac{A_{s} f_{y}}{0.85 f^{\prime}{ }_{c} \beta_{1} b}=\frac{(1.185) \cdot(60)}{0.85 \cdot(3.5) \cdot(0.85) \cdot(12)}=2.343 \mathrm{in}
$$

$$
\frac{c}{d_{s}}=\frac{2.343}{38.50}=0.061<0.6 \text { Therefore, } \mathrm{f}_{\mathrm{y}} \text { may be used in the above equation }
$$

$$
a=\beta_{1} c=(0.85) \cdot(2.343)=1.99 \text { in }
$$

[C5.7.2.1]
[C5.5.4.2.1-1]
[5.5.4.2.1]
[5.7.3.2.2-1]

Maximum
Reinforcing
[5.7.3.3.1]
Minimum
Reinforcing
[5.7.3.3.2]
[5.4.2.6]

Controlling Cracking
[5.7.3.4]
[5.7.2.6]

The net tensile strain in the reinforcing is:

$$
\varepsilon_{t}=0.003\left(\frac{d_{t}}{c}-1\right)=0.003\left(\frac{38.50}{2.343}-1\right)=0.046
$$

Since the net tensile strain, $\varepsilon_{\mathrm{t}}=0.046>0.005$, the section is tension-controlled and the reduction factor $\varphi=0.90$.

$$
\varphi M_{n}=(0.90) \cdot(1.185) \cdot(60) \cdot\left[38.50-\frac{1.99}{2}\right] \div 12=200.00 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

Since the factored strength, $\varphi \mathrm{M}_{\mathrm{n}}=200.00 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$, is greater than the factored load, $\mathrm{M}_{\mathrm{u}}=137.26 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$, the section is adequate for flexural strength.

The provision that limited the amount of reinforcing in a section was deleted in 2005.

Check section for minimum reinforcing criteria:

$$
\begin{aligned}
& S_{c}=\frac{b h^{2}}{6}=\frac{(12) \cdot(42)^{2}}{6}=3528 \mathrm{in}^{3} / \mathrm{ft} \\
& f_{r}=0.37 \sqrt{f_{c}^{\prime}}=0.37 \sqrt{3.5}=0.692 \mathrm{ksi}
\end{aligned}
$$

The amount of reinforcing shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

$$
\begin{aligned}
& 1.2 \mathrm{M}_{\mathrm{cr}}=1.2(0.692)(3528) \div 12=244.14 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
& 1.33 \mathrm{Mu}=1.33(137.26)=182.56 \mathrm{ft}-\mathrm{k} / \mathrm{ft}<=\text { Critical }
\end{aligned}
$$

Since the flexural resistance, $\varphi \mathrm{M}_{\mathrm{n}}=200.00 \mathrm{ft}-\mathrm{k} / \mathrm{ft}>182.56 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$, the minimum reinforcing criteria is satisfied.

This section applies to all members in which tension in the cross-section exceeds 80 percent of the modulus of rupture at service limit state.

For this requirement the modulus of rupture is:

$$
\begin{aligned}
& f_{r}=0.24 \sqrt{f_{c}^{\prime}}=0.24 \sqrt{3.5}=0.449 \mathrm{ksi} \\
& 0.80 f_{r}=(0.80)(0.449)=0.359 \mathrm{ksi}
\end{aligned}
$$

The Service I Limit State applies with the pinned abutment critical. The pinned abutment controls because the longitudinal forces are transmitted thru this abutment and the large friction and rotation load from the bearing pads is not considered for service limit states. The service limit state design moment follows:

$$
\mathrm{P}_{\mathrm{s}}=1.0(2177+86+850)+1.0(306+72)=3491 \mathrm{k}
$$

Pinned Abutment

$$
\begin{aligned}
\mathrm{M}_{\mathrm{s}}= & 1.0(1489+107+4027-3080)+1.0(382+746+986) \\
& +0.3(634)+1.0(125)=4972 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

Expansion Abutment

$$
\begin{aligned}
\mathrm{M}_{\mathrm{s}}= & 1.0(1489+107+4027-3080)+1.0(382+968)+0.3(256) \\
& +1.0(63)=4033 \mathrm{ft}-\mathrm{k}
\end{aligned}
$$

For the pinned abutment:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{s}}=3491 / 49.52=70.50 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}_{\mathrm{s}}=4972 / 49.52=100.40 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
& \mathrm{e}_{\mathrm{B}}=100.40 / 70.50=1.424 \mathrm{ft} \\
& \\
& \mathrm{q}_{\max }=70.50 / 16.00(1+6(1.424) / 16)=6.759 \mathrm{ksf} \\
& \mathrm{q}_{\min }=70.50 / 16.00(1-6(1.424) / 16)=2.053 \mathrm{ksf} \\
& \mathrm{q}_{\mathrm{toe}}=2.053+(6.759-2.053)(10.50) / 16=5.141 \mathrm{ksf}
\end{aligned}
$$

Figure 20 shows the service load soil stress distribution.


TOE SOIL STRESS

Figure 20

## Shrinkage \& Temperature Reinforcement [5.10.8]

Development of Reinforcement [5.11.2]
[5.11.2.1.1]

## [5.11.2.1.3]

The toe moment is the result of the upward soil pressure reduced by the weight of the footing and cover soil taken at the face of the abutment stem.

$$
\begin{aligned}
\mathrm{M}_{\mathrm{s}}= & 5.141(5.50)^{2} \div 2+(6.759-5.141)(5.50)^{2} \div 3 \\
& -[0.15(3.50)+0.12(3.00)](5.50)^{2} \div 2=80.69 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
f_{s}= & \frac{M_{s}}{S_{c}}=\frac{(80.69) \cdot(12)}{3528}=0.274 \mathrm{ksi}
\end{aligned}
$$

Since the service limit stress is less than 80 percent of the modulus of rupture the crack control criteria need not be applied.

Reinforcing shall be distributed equally on both faces in both directions with a minimum area of reinforcement satisfying:

$$
A_{s} \geq \frac{1.30 b h}{2(b+h) f_{y}}=\frac{(1.30) \cdot(66.0) \cdot(42.0)}{(2) \cdot(66.0+42.0) \cdot(60)}=0.278 \mathrm{in}^{2}
$$

and $0.11<\mathrm{A}_{\mathrm{s}}<0.60$
The spacing shall not exceed:
3.0 times the thickness $=126 \mathrm{in}$, or 18.0 in
12.0 inches for walls greater than 18 inch thick

Use \#5 @ 12 inches for temperature and shrinkage reinforcement in the toe.
The reinforcing must be developed on each side of the critical section for its full development length. For the toe design the critical available embedment length is $(5.50)(12)-2.00$ clear $=64.00$ inches.

Required development length for \#8 bars:
For \#11 bar and smaller $\frac{1.25 A_{b} f_{y}}{\sqrt{{f^{\prime}}_{c}}}=\frac{(1.25) \cdot(0.79) \cdot(60)}{\sqrt{3.5}}=31.7 \mathrm{in}$
but not less than $0.4 \mathrm{~d}_{\mathrm{b}} \mathrm{f}_{\mathrm{y}}=0.4(1.00)(60)=24.0$ in
Modification Factors that Decrease $1_{d}$ :
Spacing not less than 6 inch $=0.8$
Excess reinforcing $=137.26 / 200.00=0.686$
Required development:
$1_{d}=(0.8)(0.686)(31.7)=17.4$ in
Since the available development length is greater than the required, the development length criteria is satisfactory.

Shear
[5.8]
[5.8.2.9]
[C5.8.2.9-1]
[3.4.1]
[5.8.3.4.1]
[5.8.3.3-3]
[5.8.3.3-1]
[5.8.3.3-2]
[5.8.2.4-1]
[5.8.2.1-2]

The critical shear is located a distance $\mathrm{d}_{\mathrm{v}}$ from the face of the stem.
The value for $d_{v}$ is the greater of the following:

$$
\begin{aligned}
& \mathrm{d}_{\mathrm{v}}=\mathrm{d}_{\mathrm{e}}-\mathrm{a} / 2=38.50-1.99 / 2=37.51 \text { in }<=\text { Critical } \\
& \mathrm{d}_{\mathrm{v}}=0.9 \mathrm{~d}_{\mathrm{e}}=(0.9)(38.50)=34.65 \text { in } \\
& \mathrm{d}_{\mathrm{v}}=0.72 \mathrm{~h}=(0.72)(42.00)=30.24 \text { in }
\end{aligned}
$$

The distance from the toe to a distance $\mathrm{d}_{\mathrm{v}}$ from the face of support equals 5.50 $-37.51 / 12=2.37$ feet.

## Strength I Limit State

$$
\begin{aligned}
\mathrm{V}_{\mathrm{u}}= & (2.37)(11.106+9.558) / 2-0.90(0.15)(3.50)(2.37) \\
& -1.00(0.12)(3.00)(2.37)=22.51 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

## Simplified Procedure

For concrete footings in which the distance from the point of zero shear (toe) to the face of the wall $(5.50$ feet $)$ is less than $3 \mathrm{~d}_{\mathrm{v}}=3(37.51) / 12=9.38$ feet:

$$
\begin{aligned}
& \beta=2.0 \\
& V_{c}=0.0316 \beta \sqrt{f_{c}^{\prime}} b_{v} d_{v} \\
& V_{c}=0.0316 \cdot(2.0) \cdot \sqrt{3.5} \cdot(12.0) \cdot(37.51)=53.22 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

Since the section will be checked as an unreinforced section for shear, $\mathrm{V}_{\mathrm{s}}$ will be zero.

The nominal shear resistance is the lesser of:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{n} 1}=\mathrm{V}_{\mathrm{c}}+\mathrm{V}_{\mathrm{s}}=53.22 \mathrm{k} / \mathrm{ft}<=\text { Critical } \\
& \mathrm{V}_{\mathrm{n} 2}=0.25 \mathrm{f}^{\prime}{ }_{\mathrm{c}} \mathrm{~b}_{\mathrm{v}} \mathrm{~d}_{\mathrm{v}}=(0.25)(3.5)(12.0)(37.51)=393.86 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

For footings:

$$
\mathrm{V}_{\mathrm{r}}=\varphi \mathrm{V}_{\mathrm{n}}=(0.90)(53.22)=47.90 \mathrm{k} / \mathrm{ft}>\mathrm{V}_{\mathrm{u}}=22.51 \mathrm{k} / \mathrm{ft}
$$

Since the factored shear resistance exceeds the factored load, the shear criteria is satisfied.

## HEEL DESIGN

## Heel Design

The simplest method for design of the heel is to ignore the soil reaction producing a very conservative design for both shear and moment. For this example the effects of the soil stress will be included. The loads that will act directly on the heel are the self-weight, soil, live load surcharge and the resisting soil stress. The simplified method discussed for the toe design will be used for the heel also. The critical group combination for the heel design will be the load factors producing the minimum axial loads with maximum eccentricities resulting in the minimum soil pressure. The critical combination is Strength I with minimum axial load for the expansion abutment.


HEEL
Figure 21
$\mathrm{P}_{\text {min }}=2809 / 49.52=56.72 \mathrm{k} / \mathrm{ft}$
$\mathrm{M}_{\text {long }}=11044 / 49.52=223.02 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$
$\mathrm{e}_{\text {long }}=223.02 / 56.72=3.932 \mathrm{ft}$
Since the strength limit state eccentricity is greater than one-sixth the footing width there will be some uplift for a triangular soil stress distribution.

Length of stress region $=(8.000-3.932)(3)=12.204 \mathrm{ft}$
$\mathrm{q}_{\max }=[(56.72)(2)] / 12.204=9.295 \mathrm{ksf}$
$\mathrm{q}_{\text {heel }}=[(6.500-3.800) /(12.204)](9.295)=2.056 \mathrm{ksf}$

Strength I Limit State [3.4.1]

Flexural
Resistance
[5.7.3.2]
[5.7.3.1.1-4]
[5.7.2.1]
[5.7.2.2]
[C5.7.2.1]
[C5.5.4.2.1-1]
[5.5.4.2.1]
[5.7.3.2.2-1]

Maximum
Reinforcing
[5.7.3.3.1]

The location of the critical moment and shear will be at the back face of the stem.

$$
\begin{aligned}
\mathrm{M}_{\mathrm{u}}= & 1.25(0.150)(3.50)(6.50)(3.25)+1.35(0.120)(21.00)(6.50)(3.25) \\
& +1.75(0.120)(2.0)(6.50)(3.25)-(2.056)(2.70)^{2} \div 6=92.10 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

Try \#7 @ 10 inches

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{s}}=(0.60)(12 / 10)=0.720 \mathrm{in}^{2} / \mathrm{ft} \\
& \mathrm{~d}_{\mathrm{s}}=42.00-2.00 \text { clear }-0.875 / 2-1.00 \text { bottom }=38.56 \text { in }
\end{aligned}
$$

For footings it is common practice to ignore the bottom inch in strength calculations since this concrete is cast directly on the soil. The varying soil level and moisture content may affect the strength of the contact layer.

$$
\begin{aligned}
& c=\frac{A_{s} f_{y}}{0.85 f^{\prime}{ }_{c} \beta_{1} b}=\frac{(0.720) \cdot(60)}{0.85 \cdot(3.5) \cdot(0.85) \cdot(12)}=1.424 \mathrm{in} \\
& \frac{c}{d_{s}}=\frac{1.424}{38.56}=0.037<0.6 \text { Therefore, } \mathrm{f}_{\mathrm{y}} \text { may be used in above equation } \\
& a=\beta_{1} c=(0.85) \cdot(1.424)=1.21 \mathrm{in}
\end{aligned}
$$

The net tensile strain in the reinforcing is:

$$
\varepsilon_{t}=0.003\left(\frac{d_{t}}{c}-1\right)=0.003\left(\frac{38.56}{1.424}-1\right)=0.078
$$

Since the net tensile strain, $\varepsilon_{\mathrm{t}}=0.078>0.005$, the section is tension-controlled and the reduction factor $\varphi=0.90$.

$$
\varphi M_{n}=(0.90) \cdot(0.72) \cdot(60) \cdot\left[38.56-\frac{1.21}{2}\right] \div 12=122.97 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
$$

Since the flexural resistance, $\varphi \mathrm{M}_{\mathrm{n}}=122.97 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$, is greater than the factored load, $\mathrm{M}_{\mathrm{u}}=92.10 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$, the section is adequate for flexural resistance.

The provision that limited the amount of reinforcing in a section was deleted in 2005.

Minimum Reinforcement
[5.7.3.3.2]
[5.4.2.6]

## Control of Cracking

 [5.7.3.4][5.4.6.2]

## Service I Limit State [3.4.1]

Check section for minimum reinforcing criteria:

$$
\begin{aligned}
& S_{c}=\frac{b h^{2}}{6}=\frac{(12) \cdot(42)^{2}}{6}=3528 \mathrm{in}^{3} / \mathrm{ft} \\
& f_{r}=0.37 \sqrt{f_{c}^{\prime}}=0.37 \sqrt{3.5}=0.692 \mathrm{ksi}
\end{aligned}
$$

The amount of reinforcing shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

$$
\begin{aligned}
& 1.2 \mathrm{M}_{\mathrm{cr}}=1.2(0.692)(3528) \div 12=244.14 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
& 1.33 \mathrm{M}_{\mathrm{u}}=1.33(92.10)=122.49 \mathrm{ft}-\mathrm{k} / \mathrm{ft}
\end{aligned}
$$

Since the flexural resistance, $\varphi \mathrm{M}_{\mathrm{n}}=122.97 \mathrm{ft}-\mathrm{k} / \mathrm{ft}>122.49 \mathrm{ft}-\mathrm{k} / \mathrm{ft}$, the minimum reinforcing criteria is satisfied.

This section applies to all members in which tension in the cross-section exceeds 80 percent of the modulus of rupture at service limit state.

For this requirement the modulus of rupture is:

$$
\begin{aligned}
& f_{r}=0.24 \sqrt{f_{c}^{\prime}}=0.24 \sqrt{3.5}=0.449 \mathrm{ksi} \\
& 0.80 f_{r}=(0.80)(0.449)=0.359 \mathrm{ksi} \\
& \mathrm{P}_{\min }=[1.0(2177)+1.0(850)] / 49.52=61.13 \mathrm{k} / \mathrm{ft} \\
& \mathrm{M}_{\text {long }}=4972 / 49.52=100.40 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
& \mathrm{e}_{\text {long }}=100.40 / 61.13=1.642 \mathrm{ft}
\end{aligned}
$$

The heel moment is the result of the weight of the footing, soil on the heel and the weight of the live load surcharge reduced by the upward soil pressure taken at the face of the abutment stem.

$$
\begin{aligned}
\mathrm{q}_{\max } & =(61.13 / 16.00)(1+6(1.642) / 16.00)=6.173 \mathrm{ksf} \\
\mathrm{q}_{\min } & =(61.13 / 16.00)(1-6(1.642) / 16.00)=1.468 \mathrm{ksf} \\
\mathrm{q}_{\text {heel }} & =1.468+(6.173-1.468)(6.50) / 16.00=3.379 \mathrm{ksf} \\
\mathrm{M}_{\mathrm{s}}= & {[0.15(3.50)+0.12(21.00)+0.12(2.00)](6.50)(3.25) } \\
& -(1.468)(6.50)(3.25)-(3.379-1.468)(6.50)(6.50 / 6)=24.93 \mathrm{ft}-\mathrm{k} / \mathrm{ft} \\
f_{s}= & \frac{M_{s}}{S_{c}}=\frac{(24.93) \cdot(12)}{3528}=0.085 \mathrm{ksi}
\end{aligned}
$$

Since the service limit stress is less than 80 percent of the modulus of rupture the crack control criteria need not be applied.

Shrinkage \& Temperature Reinforcement [5.10.8]

## Development of Reinforcement [5.11.2]

## [5.11.2.1.1]

[5.11.2.1.2]
[5.11.2.1.3]

Reinforcing shall be distributed equally on both faces in both directions with a minimum area of reinforcement satisfying:

$$
A_{s} \geq \frac{1.30 b h}{2(b+h) f_{y}}=\frac{(1.30) \cdot(78.0) \cdot(42.0)}{(2) \cdot(78.0+42.0) \cdot(60)}=0.296 \mathrm{in}^{2}
$$

and $0.11<\mathrm{A}_{\mathrm{s}}<0.60$
The spacing shall not exceed:
3.0 times the thickness $=(3)(42.0)=126.0 \mathrm{in}$, or 18.0 in
12.0 inches for walls greater than 18 inch thick

Use \#5 @ 12 inches for temperature and shrinkage reinforcement in the heel.

The reinforcing must be developed on each side of the critical section. For the heel design the critical available embedment length is $(6.50)(12)-2.00$ clear $=$ 76.00 inches.

Required development length for \#7 bars:
For \#11 bar and smaller $\frac{1.25 A_{b} f_{y}}{\sqrt{{f^{\prime}}_{c}}}=\frac{1.25 \cdot(0.60) \cdot(60)}{\sqrt{3.5}}=24.1 \mathrm{in}$
but not less than $0.4 \mathrm{~d}_{\mathrm{b}} \mathrm{f}_{\mathrm{y}}=0.4(0.875)(60)=21.0$ in

Modification Factors that Increase $1_{\mathrm{d}}$ :
Top bars $=1.40$
Modification Factors that Decrease $1_{d}$ :
Spacing not to exceed 6 inch $=0.8$
Excess reinforcing $=92.10 / 122.97=0.749$
Required development:

$$
l_{d}=(24.1)(1.40)(0.8)(0.749)=20.2 \text { in }
$$

Since the available development length is greater than the required, the development length criteria is satisfied.

## Shear <br> [5.8] <br> Strength I <br> Limit State <br> [3.4.1]

[5.8.2.9]
[C5.8.2.9-1]
[5.8.3.4.1]
[5.8.3.3-3]
[5.8.3.3-1]
[5.8.3.3-2]
[5.8.3.4-1]
[5.8.2.1-2]

The critical shear occurs at the back face of the stem.

$$
\begin{aligned}
\mathrm{V}_{\mathrm{u}}= & 1.25(0.150)(3.50)(6.50)+1.35(0.120)(21.00)(6.50) \\
& +1.75(0.120)(2.00)(6.50)-(2.056)(2.70) / 2=26.33 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

The value for $\mathrm{d}_{\mathrm{v}}$ is the greater of the following:

$$
\begin{aligned}
& \mathrm{d}_{\mathrm{v}}=\mathrm{d}_{\mathrm{e}}-\mathrm{a} / 2=38.56-1.21 / 2=37.96 \text { in }<=\text { Critical } \\
& \mathrm{d}_{\mathrm{v}}=0.9 \mathrm{~d}_{\mathrm{e}}=(0.9)(38.56)=34.70 \text { in } \\
& \mathrm{d}_{\mathrm{v}}=0.72 \mathrm{~h}=(0.72)(42.00)=30.24 \text { in }
\end{aligned}
$$

## Simplified Procedure

For concrete footings in which the distance from the point of zero shear (heel) to the face of the stem $(6.50$ feet $)$ is less than $3 \mathrm{~d}_{\mathrm{v}}=3(37.96) / 12=9.49$ feet, the simplified procedure for shear may be used.

$$
\beta=2.0
$$

## Concrete Strength

$$
\begin{aligned}
& V_{c}=0.0316 \beta \sqrt{f_{c}^{\prime}} b_{v} d_{v} \\
& V_{c}=0.0316 \cdot(2.0) \cdot \sqrt{3.5} \cdot(12.0) \cdot(37.96)=53.86 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

Since the heel will be checked as an unreinforced section for shear, $\mathrm{V}_{\mathrm{s}}$ will be zero.

The nominal shear resistance is the lesser of:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{n} 1}=\mathrm{V}_{\mathrm{c}}+\mathrm{V}_{\mathrm{s}}=53.86 \mathrm{k} / \mathrm{ft} \\
& \mathrm{~V}_{\mathrm{n} 2}=0.25 \mathrm{f}^{\prime}{ }_{\mathrm{c}} \mathrm{~b}_{\mathrm{v}} \mathrm{~d}_{\mathrm{v}}=(0.25)(3.5)(12.0)(37.96)=398.58 \mathrm{k} / \mathrm{ft}
\end{aligned}
$$

For footings:

$$
\mathrm{V}_{\mathrm{r}}=\varphi \mathrm{V}_{\mathrm{n}}=(0.90)(53.86)=48.47 \mathrm{k} / \mathrm{ft}>\mathrm{V}_{\mathrm{u}}=26.33 \mathrm{k} / \mathrm{ft}
$$

Since the factored shear resistance is greater than the factored load, the section is adequate for shear.


TYPICAL SECTION
Figure 22

