

**SUBSTRUCTURE  
EXAMPLE****2-Column Pier  
On Drilled Shafts**

This example illustrates the design of a two column pier with circular columns supported on individual drilled shafts. The bridge has spans of 118 feet and 130 feet with zero skew. Standard ADOT 42-inch f-shape barriers will be used resulting in a bridge configuration of 1'-7" barrier, 12'-0" outside shoulder, two 12'-0" lanes, a 6'-0" inside shoulder and a 1'-7" barrier. The overall out-to-out width of the bridge is 45'-2". A plan view and typical section of the bridge are shown in Figures 1 and 2. The typical pier section is shown in Figure 3.

The following legend is used for the references shown in the left-hand column:

- [2.2.2] LRFD Specification Article Number
- [2.2.2-1] LRFD Specification Table or Equation Number
- [C2.2.2] LRFD Specification Commentary
- [A2.2.2] LRFD Specification Appendix
- [BDG] ADOT Bridge Design Guideline

**Superstructure**

Design Example 2 demonstrates basic design features for design of the superstructure using LRFD. Critical dimensions and loads are repeated here for ease of reference.

**Bridge Geometry**

Span lengths	118.00, 130.00 ft
Bridge width	45.17 ft
Roadway width	42.00 ft
Superstructure depth	5.50 ft

**Loads**

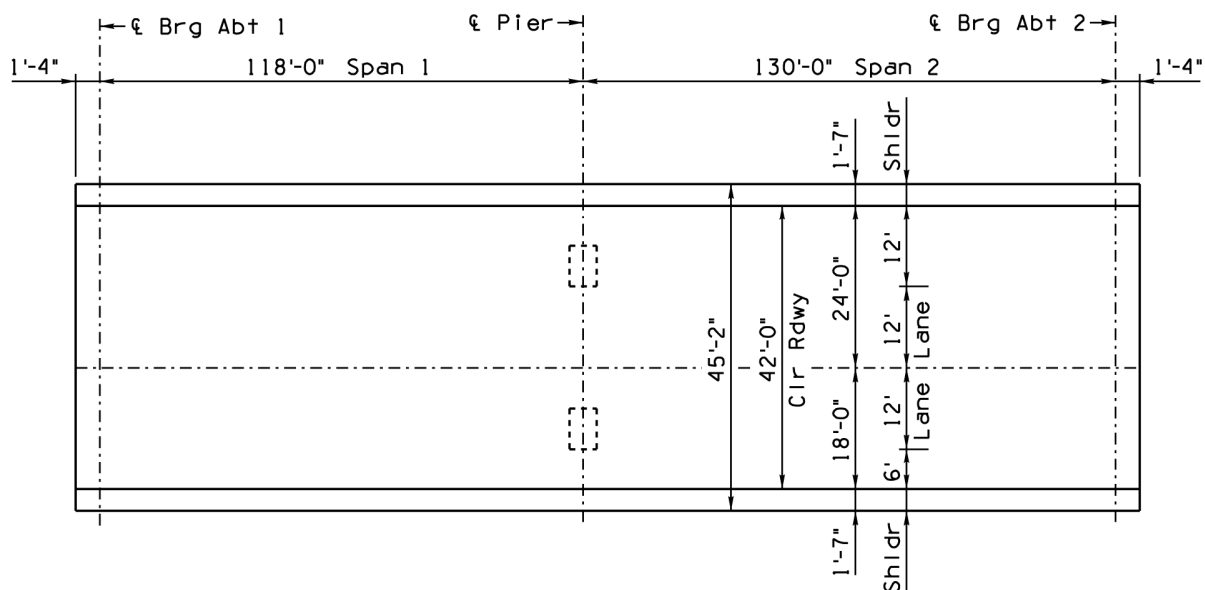
DC Superstructure	1986.4 kips
DC Barriers	167.1 kips
DW Superstructure	163.0 kips

**Substructure**

This example demonstrates basic design features for design of a pier consisting of a concrete pier cap with rectangular columns supported on individual drilled shafts. The substructure has been analyzed in accordance with the AASHTO LRFD Bridge Design Specifications, 4<sup>th</sup> Edition, 2007 and the 2008 Interim Revisions.

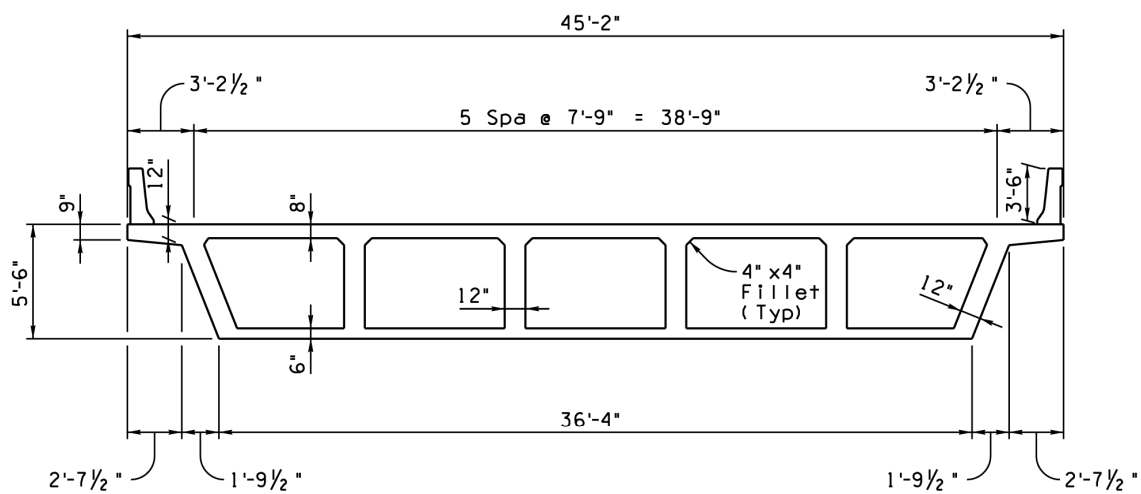
**Geotechnical**

The soil profile used in this example is the one used for the Geotechnical Policy Memo Number 3: "Development of Drilled Shaft Axial Resistance Charts for Use by Bridge Engineers". This memo should be read for a more in-depth discussion of the geotechnical aspects of drilled shafts and use of geotechnical recommendations by a bridge engineer.



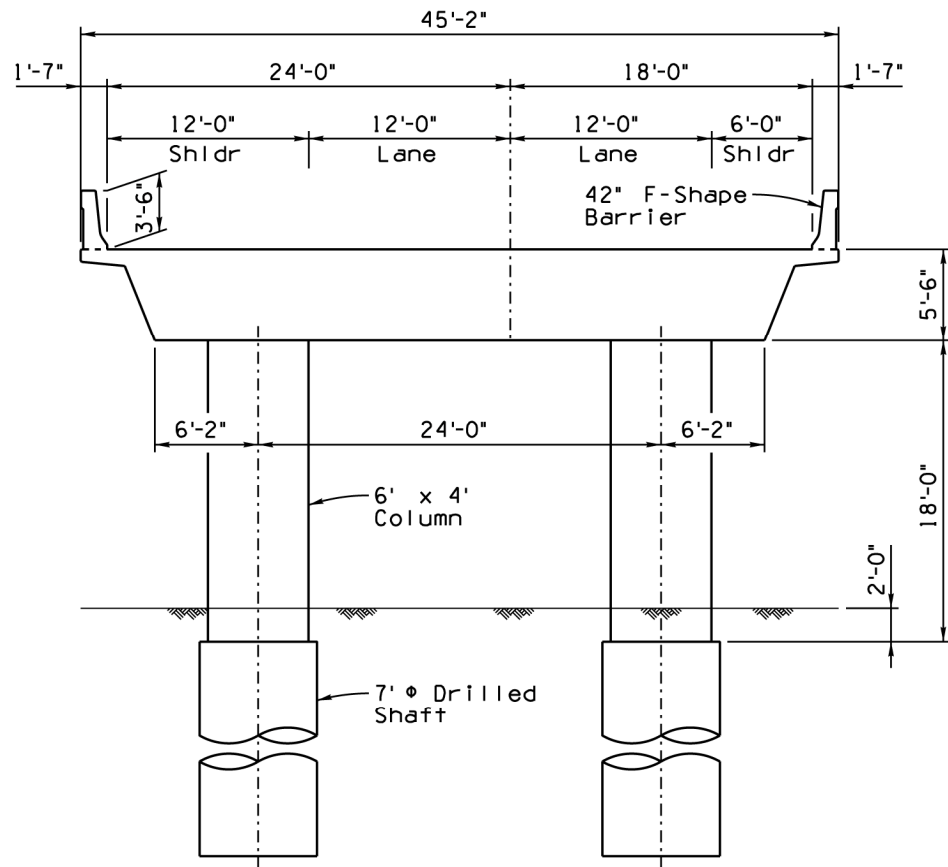
LOCATION PLAN

Figure 1



TYPICAL SECTION

Figure 2



TYPICAL SECTION

Figure 3

**Material Properties****Reinforcing Steel****[5.4.3.2]**

Yield Strength  $f_y = 60$  ksi  
 Modulus of Elasticity  $E_s = 29,000$  ksi

**Concrete**

$f'_c = 4.5$  ksi Pier Cap and Superstructure  
 $f'_c = 3.5$  ksi Columns and Drilled Shafts

**[3.5.1-1]****[C3.5.1]**

Unit weight for normal weight concrete is listed below. The unit weight for reinforced concrete increased 0.005 kcf greater than plain concrete.

Unit weight for computing  $E_c = 0.145$  kcf  
 Unit weight for DL calculation = 0.150 kcf

**[C5.4.2.4]**

The modulus of elasticity for normal weight concrete where the unit weight is 0.145 kcf may be taken as shown below:

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{4.5} = 3861 \text{ ksi, Pier Cap and Superstructure}$$

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{3.5} = 3405 \text{ ksi, Columns and Drilled Shafts}$$

**[5.7.1]**

The modular ratio of reinforcing to concrete should be rounded to the nearest whole number.

$$n = \frac{29000}{3861} = 7.51 \text{ Use } n = 8, \text{ Pier Cap and Superstructure}$$

$$n = \frac{29000}{3405} = 8.52 \text{ Use } n = 9, \text{ Columns and Drilled Shafts}$$

**[5.7.2.2]**

$\beta_1$  = the ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone stress block. For concrete strengths not exceeding 4.0 ksi,  $\beta_1 = 0.85$ .

**Modulus of Rupture**  
**[5.4.2.6]**

The modulus of rupture for normal weight concrete has several values. When used to calculate service level cracking, as specified in Article 5.7.3.4 for side reinforcing or in Article 5.7.3.6.2 for determination of deflections, the following equation should be used:

$$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{3.5} = 0.449 \text{ ksi}$$

When the modulus of rupture is used to calculate the cracking moment of a member for determination of the minimum reinforcing requirement as specified in Article 5.7.3.3.2, the following equation should be used:

$$f_r = 0.37\sqrt{f'_c} = 0.37\sqrt{3.5} = 0.692 \text{ ksi}$$

**Existing Soil**

The existing soil has the following properties:

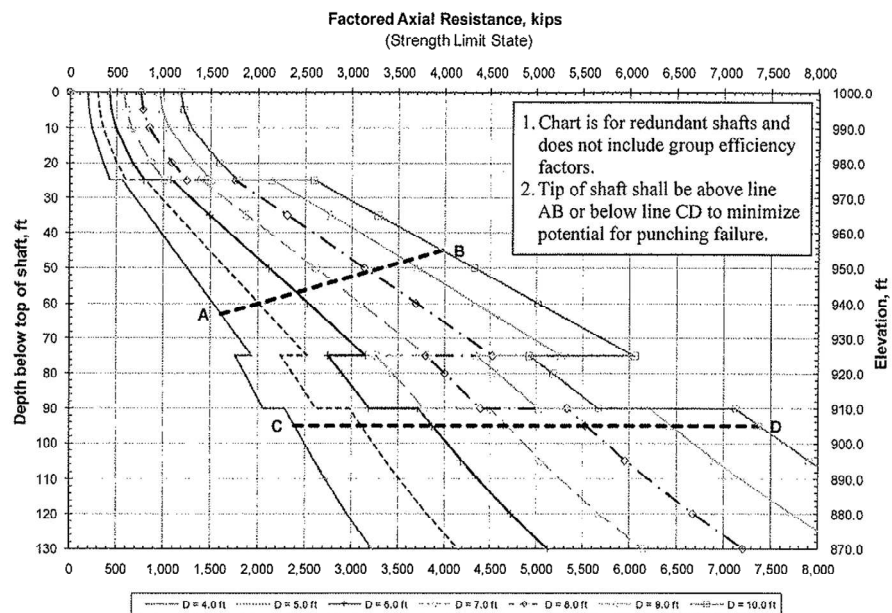
Depth ft	Soil Type	Total unit weight, $\gamma_s$ Pcf	$\phi'$ degrees
0-25	Fine to coarse sands	120	30
25-75	Gravely sands	125	36
75-90	Fine to coarse sands	120	30
90-130	Gravels	125	38

The following assumptions have been made:

No groundwater is present.

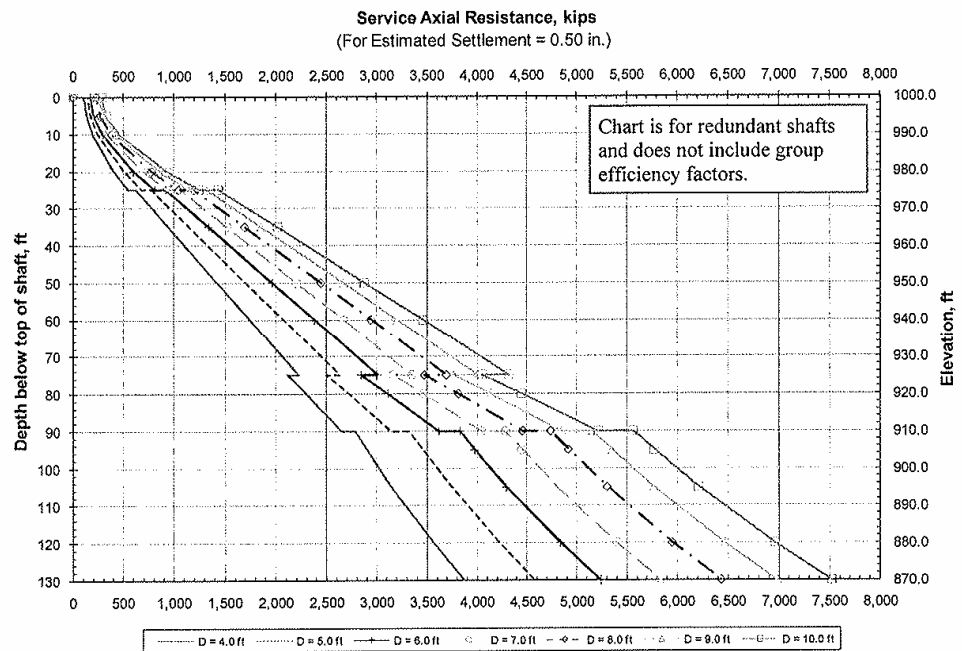
The soils will not experience any long-term (consolidation or creep) settlement.

Design Chart 1 is a plot of factored axial resistance (Strength Limit States) versus depth of embedment for various shaft diameters.



Design Chart 1

Chart 2 is a plot of factored axial resistance (Service Limit States) for a given vertical displacement at the top of the shaft versus depth of embedment for various shaft diameters. Geotechnical Policy Memo 3 represents Chart 2 for vertical displacements of 0.1 inch, 0.25 inch, 0.50 inch, 0.75 inch, 1.0 inch and 2.0 inches. An example design chart for 0.5 inch is shown below. The reader should refer to Geotechnical Policy Memo 3 for other charts.



Design Chart 2

**Limit States****[1.3.2]****[1.3.2.1-1]**

In the LRFD Specification, the general equation for design is shown below:

$$\sum \eta_i \gamma_i Q_i \leq \phi R_n = R_r$$

For loads for which a maximum value of  $\gamma_i$  is appropriate:

**[1.3.2.1-2]**

$$\eta_i = \eta_D \eta_R \eta_I \geq 0.95$$

For loads for which a minimum value of  $\gamma_i$  is appropriate:

**[1.3.2.1-3]**

$$\eta_i = \frac{1}{\eta_D \eta_R \eta_I} \leq 1.0$$

**[1.3.3]****Ductility**

For strength limit state for conventional design and details complying with the LRFD Specifications and for all other limit states:

$$\eta_D = 1.0$$

**[1.3.4]****Redundancy**

For the strength limit state for conventional levels of redundancy and for all other limit states:

$$\eta_R = 1.0$$

**[1.3.5]****Operational Importance**

For the strength limit state for typical bridges and for all other limit states:

$$\eta_I = 1.0$$

For an ordinary structure with conventional design and details and conventional levels of ductility, redundancy, and operational importance, it can be seen that  $\eta_i = 1.0$  for all cases. Since multiplying by 1.0 will not change any answers, the load modifier  $\eta_i$  has not been included in this example.

For actual designs, the importance factor may be a value other than one. The importance factor should be selected in accordance with the ADOT Bridge Design Guidelines.

**[BDG]**

**LONGITUDINAL  
FRAME**

The superstructure section properties have been calculated subtracting the ½ inch wearing surface from the top slab thickness. However, this wearing surface has been included in weight calculations. The bridge has a uniform cross section except where the web flares from 12 inches to 18 inches starting 16 feet from the face of the abutment diaphragms. A summary of section properties follows:

**Section Properties**

Superstructure Section Properties:

	12" Web	18" Web	
$Y_b$	36.63	35.93	in
$Y_t$	28.87	29.57	in
Inertia	6,596,207	7,063,707	in <sup>4</sup>
Area	10,741	12,660	in <sup>2</sup>

To properly model bridges with deep foundation elements such as drilled shafts, an analysis that considers the interaction of structural frame with the foundation system including soil is required. This is an involved iterative process that needs to be started with a certain set of loads (moments, lateral and vertical loads) which in turn requires a structural frame analysis. To aid in starting the analytical process, an initial simplification is often made wherein the bridge structure is analyzed separately in the longitudinal and transverse direction using an equivalent frame that models the drilled shaft foundation as fixed at a certain depth below the finished ground line and neglects the presence of soil. The depth at which the shaft is assumed to be fixed is often referred to as the “depth to fixity”. The equivalent length of the shaft (or depth to fixity) is the length of the shaft which, when fixed at the base, produces the same deflection and rotation at the level where the load effects are applied. Additionally, the depth to fixity must be such that the buckling load is equivalent to that for the actual conditions.

In the literature, there are a number of different methods to determine the equivalent length of the drilled shaft. Since the depth to fixity is meant to just facilitate initiation of the overall structural analysis, an initial rational estimate will get the analytical process started. For this example problem, an estimate of the depth to fixity for a single shaft was made from the following equation that is applicable to cohesionless soils (sands) and prismatic members:

[10.7.3.13.4-2]

$$L_e = 1.8T = 1.8 \left[ \frac{E_c I_g}{n_h} \right]^{0.2}$$

where,  $T$  is the relative stiffness factor, that considers the stiffness ( $E_c I_g$ ) of the column/shaft with respect to the stiffness of the soil represented by parameter  $n_h$ . The parameter  $n_h$  is the constant of horizontal subgrade reaction and its value should be obtained from the geotechnical engineer.



The assumption in the above formula is that the minimum depth of the drilled shaft is at least 3 times the value of  $T$ . For this example problem, the depth to fixity is computed below for a 7-ft diameter drilled shaft. A value of  $n_h = 0.200$  kci has been assumed as a fictitious value since the primary goal of the example problem is to demonstrate structural aspects of substructure design. During actual design, the geotechnical engineer should be contacted for the value of  $n_h$  that is representative of actual soil conditions.

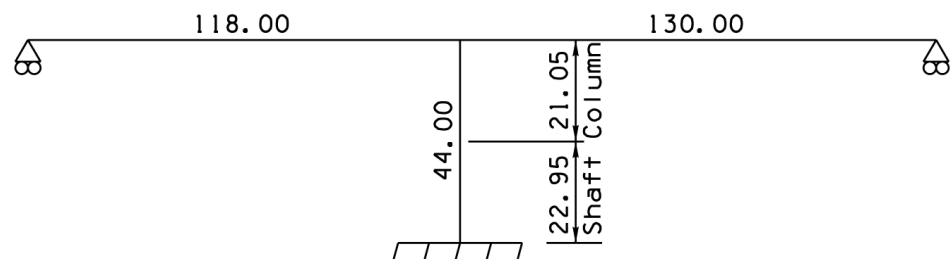
$$E_c I_g = (3405) \cdot \left( \frac{\pi \cdot (84)^4}{64} \right) = 8,321,500,000 \text{ k-in}^2$$

$$T = \left( \frac{E_c I_g}{n_h} \right)^{\frac{1}{5}} = \left( \frac{8,321,500,000}{0.200} \right)^{\frac{1}{5}} \div 12 = 11.08 \text{ ft}$$

Using the above values, the equivalent depth to fixity for this example problem is computed as follows:

$$L_e = 1.8T = 1.8(11.08) = 19.95 \text{ ft}$$

The above estimation of  $L_e$  assumes that the shaft has a proper lateral support from the finished ground line, i.e. the soils are not softened due to soil structure interaction or other factors. In practice, due to cyclic effects of loading, the upper portion of the shaft may not provide proper lateral support. Therefore, a depth of 5-ft below finished ground line is generally neglected. Since in this example the shaft is buried 2 feet below the finished ground line, the lateral support for the shaft is neglected for 3.0 feet and an equivalent depth to fixity of  $19.95 + 3.00 = 22.95$  feet is assumed as shown in Figure 4. The column length equals  $18.00 + 36.63 / 12 = 21.05$  feet.



LONGITUDINAL FRAME

Figure 4

Using the equivalent depth to fixity, a structural frame analysis is performed to determine the initial set of loads (shear, axial and moment) and deformations. Then the equivalent length is verified using a computer program such as L-Pile by comparing the resulting deflection from L-Pile due to the initial set of loads to that from the frame analysis using the equivalent length of the shaft. This process is repeated until convergence between loads and deformations is obtained between structural frame analysis and L-Pile analysis. To aid in rapid convergence of the analysis, the following is a brief discussion for prismatic and non-prismatic members.

### Prismatic

For a prismatic member fixed at the base and free at the top the equivalent length can be directly solved by using the following deflection formula:

$$\Delta = \frac{Pl^3}{3EI} \Rightarrow \text{solving for } l \text{ yields: } l = \sqrt[3]{\frac{3EI\Delta}{P}}$$

In the above formula, P is the applied load at the column top and  $\Delta$  is the resulting deflection from the L-Pile analysis.

### Non-Prismatic

For a non prismatic stepped member fixed at the base and free at the top the equivalent length of the shaft can be determined by iteration. The formula for a stepped column/shaft where the column has a different size or shape than the shaft is:

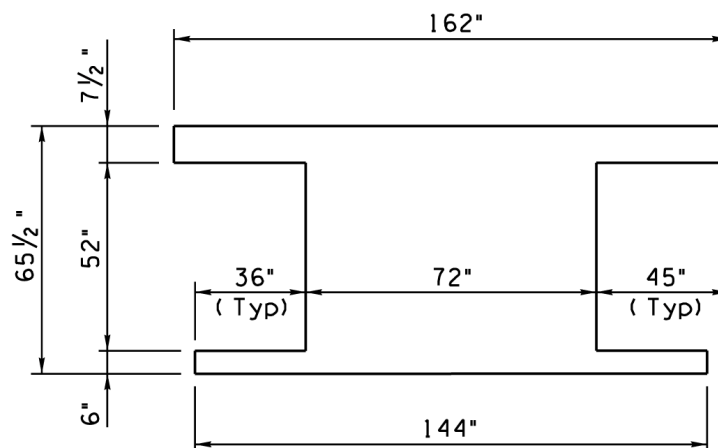
$$\Delta = \frac{P}{E} \left[ \frac{l_1^3}{3I_1} + \frac{l_1^2 l_2}{I_2} + \frac{l_1 l_2^2}{I_2} + \frac{l_2^3}{3I_2} \right]$$

Where: P = lateral load applied at top of column  
 $\Delta$  = deflection at top of column  
 $l_1$  = length of column  
 $I_1$  = moment of inertia of column  
 $l_2$  = equivalent length of shaft  
 $I_2$  = moment of inertia of shaft

Regardless of the type of member (prismatic or non-prismatic), L-pile should be run for the given lateral load, P. The above equation can then be solved by iteration for the equivalent shaft length  $l_2$ . The resulting equivalent length can then be used in the frame analysis to determine moments and shears at the top of the column. Values at the base from this analysis will overestimate the true magnitude of the loads. However, the values at the base may be used in the design if the forces are low and the 1 percent reinforcing requirement controls. Otherwise, the top moments and shears should be used as loads applied to an L-Pile problem.

**TRANSVERSE  
FRAME****Effective Flange  
Width  
[4.6.2.6]**

The transverse frame is shown in Figure 6. The pier cap is an integral part of the superstructure. To adequately capture its true stiffness, the cap should be modeled as a flanged member. The top and bottom flanges should be extended 6 times their thickness beyond the 6 foot wide cap on each side as shown in Figure 5.



TYPICAL SECTION

Pier Cap

Figure 5

Transverse section properties for the cap, column and shaft are shown below:

	Pier Cap	Column	Shaft	Units
Area	40.44	24.00	38.48	ft <sup>2</sup>
Inertia	126.97	72.00	117.86	ft <sup>4</sup>
y <sub>b</sub>	2.83	3.00	3.50	ft
E <sub>c</sub>	3861	3405	3405	ksi

In determining the equivalent length of the column/drilled shaft, several items must be considered. As noted earlier the shaft can be considered laterally supported at a depth of 5.0 feet below the ground surface. Since the shaft is buried 2 feet below the ground the depth of support for the shaft is 3.0 feet. However, laterally supported does not mean fixed. Fixity occurs below this location and depends on the stiffness of the supporting soil. For sands an estimate of the depth to fixity for a single shaft can be made from the following equation:

$$L_e = 1.8 \cdot \left[ \frac{E_c I_g}{n_h} \right]^{0.2}$$

$n_h$  = rate of increase of soil modulus with depth. See Foundation Report for site specific data. For this problem  $n_h = 0.200$  kcf

**[10.7.2.4.1]****[C10.7.2.4]**

The effect of group action must also be considered. The p-multipliers are used to reduce the effective stiffness of the soil for closely spaced drilled shafts. The method actually requires that a different multiplier be applied for each shaft creating a different effective length for each shaft. With the iteration required in the process this method complicates the analysis effort greatly. The LRFD Commentary states that “The multipliers on the pile rows are a topic of current research and may change in the future.” While modeling each column for a different length may be more theoretically correct, the extra work is not required for ordinary bridges considering the inexactness of the soil properties being used. Using an average multiplier for the group and applying this value to every column is an accepted alternative. Iteration is still required but the problem is greatly simplified without a major change in results.

For the transverse frame in the direction of loading, the ratio of shaft spacing to diameter =  $24.00 / 7.00 = 3.43$ . For drilled shaft spacing greater than 5 times the diameter, there will be no affect on the soil modulus. However, at a shaft spacing of 3 times the shaft diameter the p-multiplier is 0.7 for row 1 and 0.5 for row 2 and at a shaft spacing of 5 times the shaft diameter the p-multiplier is 1.0 for row 1 and 0.85 for row 2. For an intermediate spacing, modulus values may be estimated by interpolation as follows:

$$\text{Row 1} = 0.70 + (1.00 - 0.70)(3.43 - 3.0) / (5.0 - 3.0) = 0.765$$

$$\text{Row 2} = 0.50 + (0.85 - 0.50)(3.43 - 3.0) / (5.0 - 3.0) = 0.575$$

The equivalent length of each column is determined as follows:

Row 1

$$L_e = 1.8 \cdot \left[ \frac{(3405) \cdot (117.86) \cdot (12)^4}{(0.765) \cdot (0.200)} \right]^{0.2} \div 12 = 21.05$$

Row 2

$$L_e = 1.8 \cdot \left[ \frac{(3405) \cdot (117.86) \cdot (12)^4}{(0.575) \cdot (0.200)} \right]^{0.2} \div 12 = 22.28$$

For two shafts the average p-multiplier is  $(0.767 + 0.575) / 2 = 0.671$

For  $n_h = 0.200$  kci, the effective value becomes  $(0.671)(0.200) = 0.134$  kci. The effective length considering group effects is calculated as follows:

$$L_e = 1.8 \cdot \left[ \frac{(3405) \cdot (117.86) \cdot (12)^4}{0.134} \right]^{0.2} \div 12 = 21.61 \text{ ft}$$

The difference in lengths of the column in this example are quite small and normal practice would be to use the average of the columns for design of the transverse frame.

The transverse frame column length then equals the sum of center of gravity of the pier cap plus the column length or  $2.83 + 18.00 = 20.83$  feet. The effective equivalent length of the shaft equals the depth to lateral support plus the equivalent shaft length or  $3.00 + 21.61 = 24.61$  feet. The total combination column/shaft length equals  $20.83 + 24.61 = 45.44$  feet.

The example problem was originally analyzed using different group effect modifiers than shown above producing an effective length of drilled shaft of 28.14 feet as shown below. The remainder of this example uses the column shaft length of 48.97 feet as shown below.

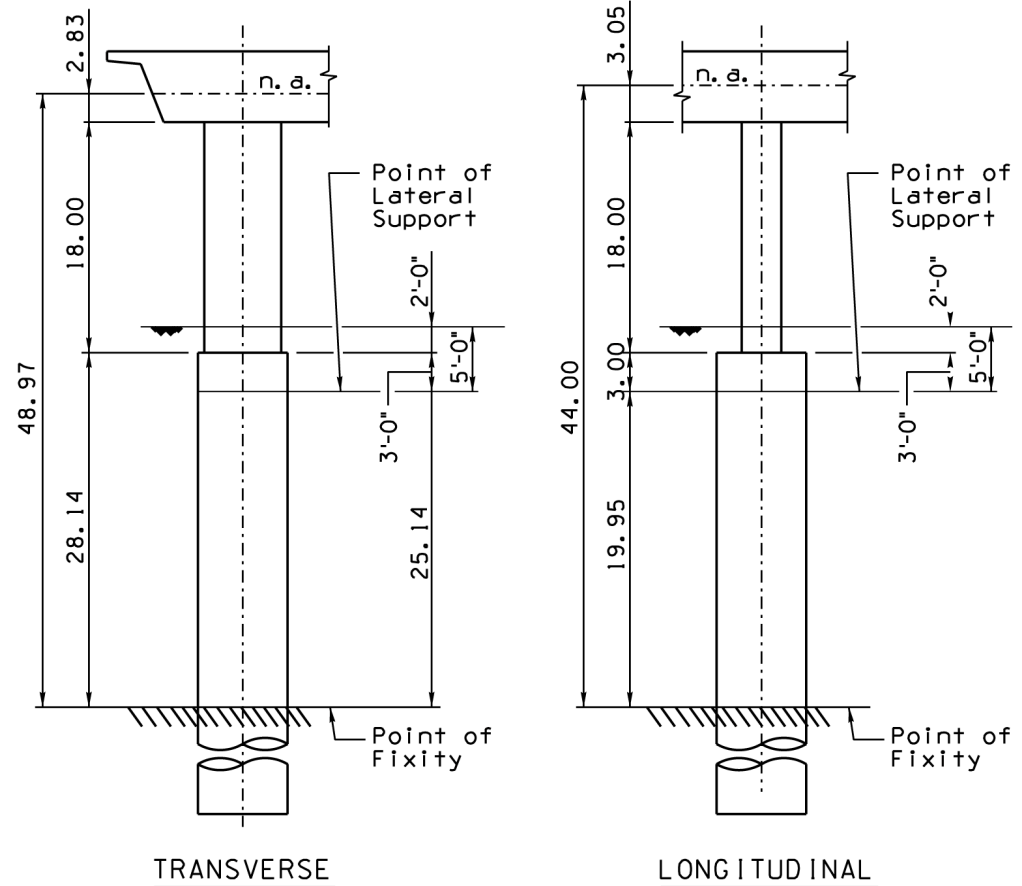


Figure 6

**SUBSTRUCTURE****Loads****Section 3****Loads**

There are several major changes and some minor changes concerning the determination of loads. The DC loads must be kept separate from the DW loads since different load factors apply. The live load is new as seen in the superstructure design. The dynamic load allowance is a constant rather than a function of the span. The Longitudinal Force in the Standard Specifications has been modified and replaced by the Braking Force. A vehicle collision force relating to protection of piers or abutments has been added. The wind and wind on live load is similar but has a modification factor for elevations above 30 feet. The vertical wind pressure is the same but the specification clarifies how to apply the force to the proper load group.

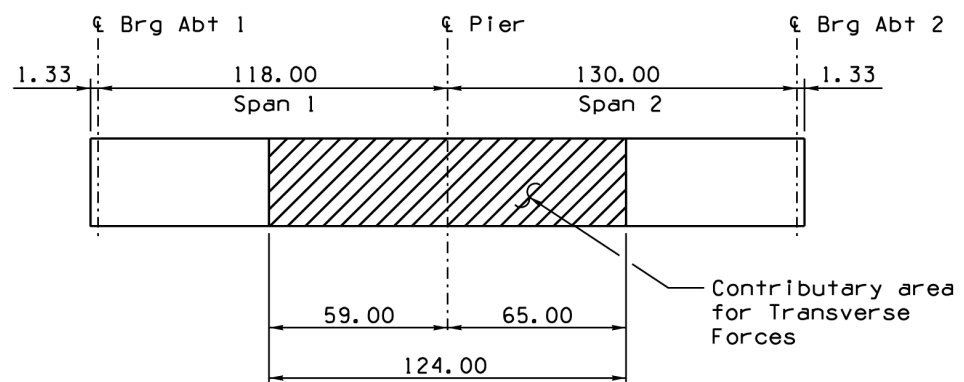
PLAN

Figure 7

Both abutments are expansion with the pier resisting all the longitudinal forces. The distribution of transverse forces is a complex problem involving the flexibility of the abutments, piers and the superstructure. For design of the pier, a conservative assumption is to design for the contributory area for transverse forces as shown in Figure 7.

**[10.8.2]****Limit States**

For substructure design, foundation design at the service limit state includes settlement and lateral displacement.

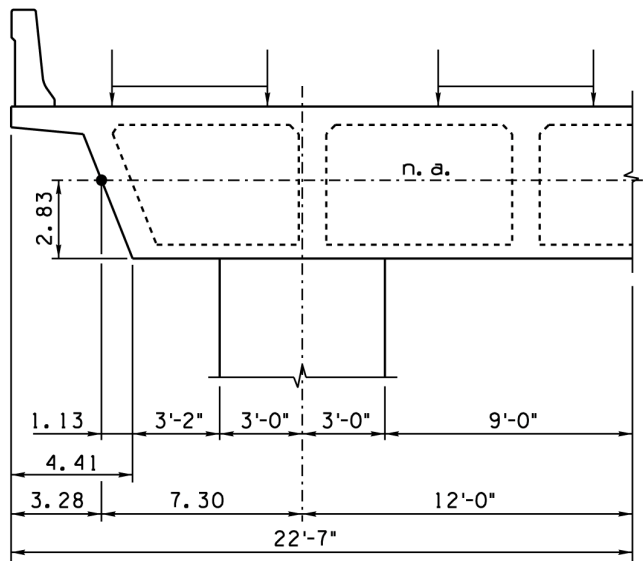
**[10.8.3]**

Foundation design at the strength limit state includes bearing resistance and structural capacity. For substructure design, three strength limit states require investigation. Strength I is the basic load combination without wind. Strength III is the load combination including wind exceeding 55 mph. Strength V is the load combination combining normal vehicular use with a wind of 55 mph.

**[3.6.5]**

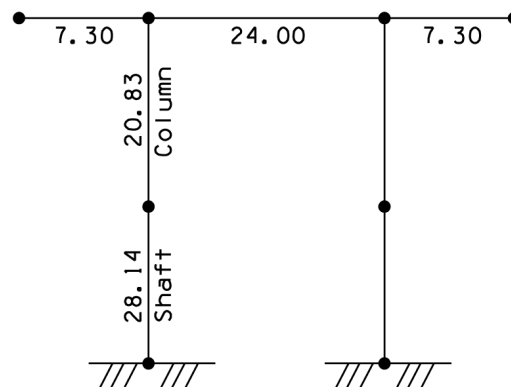
For substructure design, Extreme Event II is the load combination for collision of substructure units by vehicles. This limit state is not considered in this example.

A partial section showing the neutral axis and column centerline with dimensions in feet is shown in Figure 8. The resulting transverse frame with dimensions in feet is shown in Figure 9.



PARTIAL SECTION

Figure 8



TRANSVERSE FRAME

Figure 9

[3.5.1]

**PERMANENT LOADS****DC – Dead Load Structural Components**

The dead load reaction from the barriers of 167.1 kips is distributed equally to all webs. The reaction per web equals  $167.1 \div 6 = 27.85$  kips

The dead load reaction from the longitudinal frame analysis for self-weight and diaphragms equals 1986.4 kips. The pier cap weight equals  $(0.150)(5.50)(6.00)(38.60) = 191.1$  kips. The superstructure dead load is proportioned as follows:

Interior Web:

$$\text{Area} = (93.00)(8.00) + 2(0.5)(4)(4) + (12.00)(52.00) + (93.00)(6.00) \\ = 1942 \text{ in}^2$$

$$\text{Reaction per Web} = [(1942) / (10741)](1986.4 - 191.1) = 324.59 \text{ k}$$

$$\text{Barrier} = \underline{27.85 \text{ k}}$$

$$\text{DC} = 352.44 \text{ k}$$

Exterior Web:

$$\text{Area} = (10741 - 4(1942)) / 2 = 1486.5 \text{ in}^2$$

$$\text{Reaction per Web} = [(1486.5) / (10741)] (1986.4 - 191.1) = 248.46 \text{ k}$$

$$\text{Barrier} = \underline{27.85 \text{ k}}$$

$$\text{DC} = 276.31 \text{ k}$$

Pier Cap

$$w = 0.150(5.50)(6.00) = 4.950 \text{ k/ft}$$

**DW – Dead Load Wearing Surfaces and Utilities**

The DW superstructure load includes the future wearing surface and utility loads. This bridge has no utilities. The reaction from the future wearing surface equals 163.0 kips. The reaction per web equals  $163.0 \div 6 = 27.17$  kips.

[3.12.7]

**PS – Secondary Forces from Post-Tensioning**

The reaction from the secondary moment caused by post-tensioning equals the moment at the pier divided by the span length.

$$\text{Span 1} \quad (6243) / (118.00) = 52.91$$

$$\text{Span 2} \quad (7041) / (130.00) = \underline{54.16}$$

$$107.07 \text{ k}$$

The reaction per web equals  $-107.07 \div 6 = -17.85$  kips uplift.



**[3.6.1.1]****3.6.1.1.1]****[3.6.1.1.2]****TRANSIENT LOADS****LL – Vehicular Live Load**

The number of design lanes equals the integer part of the traffic opening divided by 12. For this bridge the number of design lanes equals  $w / 12 = 42 / 12 = 3$ . For the reaction, apply the multiple presence factor,  $m$ .

The reactions for the vehicle LL are: design lane = 99.33 k, design truck = 71.47 k, design tandem = 50.02 k, and two trucks 50 feet apart = 124.20 k. The resulting LL + IM reaction for one vehicle is the greater of is:

$$R = (99.33 + 1.33(71.47)) = 194.4 \text{ k}$$

$$R = (99.33 + 1.33(50.02)) = 165.9 \text{ k}$$

$$R = (0.90)(99.33 + 1.33(124.20)) = 238.0 \text{ k} \leq \text{Critical}$$

Each wheel line will be half that amount or 119.0 kips. While many combinations of possible live load locations exist, engineering judgment and experience can reduce the number to a few critical locations as shown in Figure 10 and described below.

**Live Load 1**

This live load is located 2 feet from the left side barrier producing maximum moments and shears on the cantilever and axial load, moments and shears on the column in one direction.

**Live Load 2**

This live load is located 6 feet from Live load 1 adding to the negative moment at the inside face of the pier cap. This load will add to the axial load on the column but will be in the opposite direction for moment and shear than Live Load 1.

**Live Load 3**

This live load is located 6 feet from Live Load 2 adding to the effects of Live Load 2. Live loads 1, 2 and 3 can be additive for negative moment but due to the Multiple Presence Factor,  $m$ , additional vehicles may not produce the maximum effects.

**Live Load 4**

This live load is located 2 feet from the right side barrier producing the maximum uplift on column 1 and adding to the column moment and shear from Live Load 1.

**Live Load 5**

This live load is centered in the pier cap span to produce maximum positive moments and shears and maximum column moments opposite to Live Load 1.

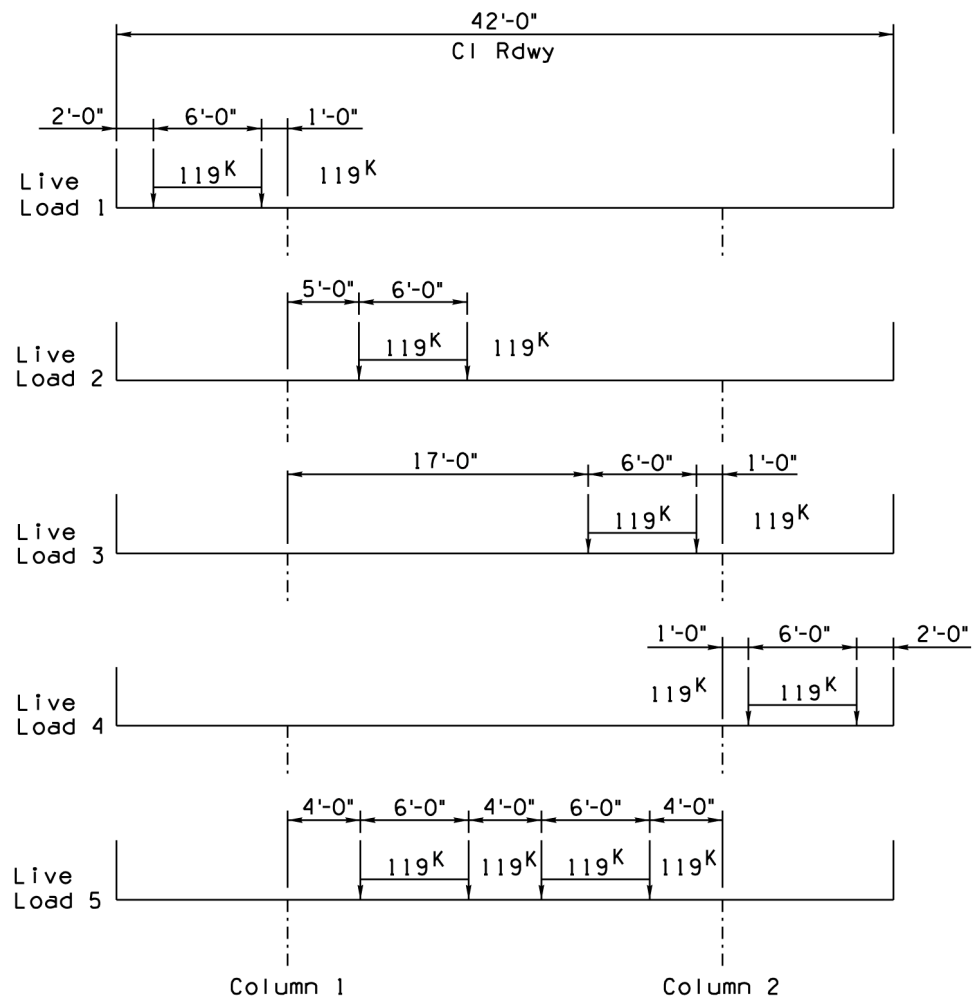


Figure 10

**[3.6.2]****IM – Dynamic Load Allowance**

Dynamic load allowance equal to 33% is applied to the design truck but not the design lane load for the design of the pier cap and column. The dynamic load allowance is not included for design of the drilled shafts. For the reaction the dynamic load allowance is:

$$IM = (0.90)(124.20)(0.33) = 36.9 \text{ k}$$

**[3.6.4]****BR – Vehicular Braking Force**

The braking force shall be taken as the greater of:

25 percent of the axle weights of the design truck or design tandem

$$V = (0.25)(32 + 32 + 8) = 18.00 \text{ k} \leq \text{Critical}$$

$$V = (0.25)(25 + 25) = 12.50 \text{ k}$$

5 percent of the design truck plus lane load or 5 percent of the design tandem plus lane load

$$V = (0.05)[32 + 32 + 8 + (248.00)(0.640)] = 11.54 \text{ k}$$

$$V = (0.05)[25 + 25 + (248.00)(0.640)] = 10.44 \text{ k}$$

The braking force shall be placed in all design lanes that are considered to be loaded which carry traffic in the same direction. The bridge is a one directional structure with all three design lanes headed in the same direction. The multiple presence factors shall apply.

$$BR = (18.00)(3)(0.85) = 45.90 \text{ k}$$

This load is to be applied 6 feet above the deck surface. However, standard analysis of a frame does not allow for a load to be applied other than at the center of gravity of the frame. To consider the higher load application location, the force can be increased by the ratio of the actual load location to the center of gravity of the frame. The column length to the soffit in the longitudinal direction is  $L = 18.00 + 3.00 + 19.95 = 40.95$  feet.

$$\text{Ratio}_{\text{long}} = (6.00 + 5.50 + 40.95) / (3.05 + 40.95) = 1.192$$

$$BR_{\text{adj}} = (1.192)(45.90) / 2 = 27.36 \text{ k per column}$$

Assuming the superstructure is rigid and both columns will deflect the same for forces applied in the longitudinal direction, each column will resist half the load.

**[3.8]****WS – Wind Load on Structure**

Wind pressures are based on a base design wind velocity of 100 mph. For structures with heights over 30 feet above the groundline, a formula is available to adjust the wind velocity. The wind is assumed to act uniformly on the area exposed to the wind. The exposed area is the sum of the areas of all components as seen in elevation taken perpendicular to the assumed wind direction.

$$\text{Long Area} = [5.50 + 3.50 + 0.02(45.17)][118 + 130] = 2456 \text{ ft}^2$$

$$\text{Trans Area} = [5.50 + 3.50 + 0.02(45.17)][59 + 65] = 1228 \text{ ft}^2$$

**[3.8.1.2-1]**

The base pressure for girder bridges corresponding to the 100 mph wind is 0.050 ksf. The minimum wind loading shall not be less than 0.30 klf. Since the wind exposure depth is greater than 6 feet this criteria is satisfied. The AASHTO LRFD 2008 Interim Revisions added a simplified wind procedure for usual girder and slab bridges having an individual span length of not more than 125 feet. Since the maximum span is 130 feet, this simplification may not be used. Therefore, various angles of attack should be investigated. The center of gravity of the loads is located  $[5.50 + 3.50 + 0.02(45.17)] / 2 = 4.95$  feet above the soffit of the bridge. The column height to the soffit for the transverse direction is  $L = 18.00 + 3.00 + 25.14 = 46.14$  feet. Wind pressures for various angles of attack are taken from Table 3.8.1.2.2-1.

$$\text{Ratio}_{\text{long}} = (4.95 + 40.95) / (3.05 + 40.95) = 1.043$$

$$\text{Ratio}_{\text{trans}} = (4.95 + 46.14) / (2.83 + 46.14) = 1.043$$

**0 Degree Skew Angle**

$$V_{\text{long}} = (2456)(0.000)(1.043) = 0.00 \text{ k}$$

$$V_{\text{trans}} = (1228)(0.050)(1.043) = 64.04 \text{ k} \Leftarrow \text{Critical}$$

**15 Degree Skew Angle**

$$V_{\text{long}} = (2456)(0.006)(1.043) = 15.37 \text{ k}$$

$$V_{\text{trans}} = (1228)(0.044)(1.043) = 56.36 \text{ k}$$

**30 Degree Skew Angle**

$$V_{\text{long}} = (2456)(0.012)(1.043) = 30.74 \text{ k}$$

$$V_{\text{trans}} = (1228)(0.041)(1.043) = 52.51 \text{ k}$$

**45 Degree Skew Angle**

$$V_{\text{long}} = (2456)(0.016)(1.043) = 40.99 \text{ k}$$

$$V_{\text{trans}} = (1228)(0.033)(1.043) = 42.27 \text{ k}$$

**60 Degree Skew Angle**

$$V_{\text{long}} = (2456)(0.019)(1.043) = 48.67 \text{ k} \Leftarrow \text{Critical}$$

$$V_{\text{trans}} = (1228)(0.017)(1.043) = 21.77 \text{ k}$$

A conservative answer can be achieved by using the maximum values from the above. If wind controls the design, the complexities of combining 5 wind combinations should be performed. A summary of wind forces used in the design follows:

$$V_{\text{long}} = 48.67 \text{ k}$$

$$V_{\text{trans}} = 64.04 \text{ k}$$

**[3.8.1.2.3]**

The transverse and longitudinal forces to be applied directly to the substructure are calculated from as assumed base wind pressure of 0.040 ksf.

$$V_{\text{long}} = 0.040(2)(6.00) = 0.480 \text{ k/ft}$$

$$V_{\text{trans}} = 0.040(4.00) = 0.160 \text{ k/ft/column}$$

**[3.8.1.3]****WL – Wind Pressure on Vehicles**

Wind pressure on vehicles is represented by a moving force of 0.10 klf acting normal to and 6.0 feet above the roadway. Loads normal to the span should be applied at a height of  $5.50 + 6.00 = 11.50$  feet above the soffit.

$$\text{Ratio}_{\text{long}} = (11.50 + 40.95) / (3.05 + 40.95) = 1.192$$

$$\text{Ratio}_{\text{trans}} = (11.50 + 46.14) / (2.83 + 46.14) = 1.177$$

**0 Degree Skew Angle**

$$V_{\text{long}} = 248.00(0.000)(1.192) = 0.00 \text{ k}$$

$$V_{\text{trans}} = 124.00(0.100)(1.177) = 14.59 \text{ k} \Leftarrow \text{Critical}$$

**15 Degree Skew Angle**

$$V_{\text{long}} = 248.00(0.012)(1.192) = 3.55 \text{ k}$$

$$V_{\text{trans}} = 124.00(0.088)(1.177) = 12.84 \text{ k}$$

**30 Degree Skew Angle**

$$V_{\text{long}} = 248.00(0.024)(1.192) = 7.09 \text{ k}$$

$$V_{\text{trans}} = 124.00(0.082)(1.177) = 11.97 \text{ k}$$

**45 Degree Skew Angle**

$$V_{\text{long}} = 248.00(0.032)(1.192) = 9.46 \text{ k}$$

$$V_{\text{trans}} = 124.00(0.066)(1.177) = 9.63 \text{ k}$$

**60 Degree Skew Angle**

$$V_{\text{long}} = 248.00(0.038)(1.192) = 11.23 \text{ k} \Leftarrow \text{Critical}$$

$$V_{\text{trans}} = 124.00(0.034)(1.177) = 4.96 \text{ k}$$

As with the wind load, a conservative answer for wind on live load can be achieved by using the maximum values from the above. If wind controls the design, the complexities of combining 5 wind directions should be performed.

$$V_{\text{long}} = 11.23 \text{ k}$$

$$V_{\text{trans}} = 14.59 \text{ k}$$

**[3.8.2]****Vertical Wind Pressure**

A vertical upward wind force of 0.020 ksf times the width of the deck shall be applied at the windward quarter point of the deck. This load is only applied for limit states which include wind but not wind on live load (Strength Limit State III) and only when the direction of wind is taken to be perpendicular to the longitudinal axis of the bridge. When applicable the wind loads are as shown:

$$P = (0.020)(45.17)(124) = -112.02 \text{ upward per pier}$$

$$M_{\text{trans}} = (112.02)(45.17 / 4) = 1265 \text{ ft-k}$$

$$P = (112.02) / 2 + (1265)(12.00) / [(2)(12.00)^2] = -108.72 \text{ k per column}$$

**[3.12]****TU, SH, CR – Superimposed Deformations**

Internal force effects that cause the pier cap to expand and contract shall be considered in the design. Due to the expansion joints at the abutments there are no longitudinal deformation forces in the pier column. However, internal forces will result from transverse deformation of the two column bent for temperature changes. For temperature deformations the member movement is taken to include the entire temperature range rather than the rise and fall from the mean temperature that was used in the past. The Bridge Design Guidelines provide the appropriate temperature range to use. For this low elevation site the design temperature range is 70 degrees

**[BDG]****TU**

$$TU = 0.000006(70)(12.00)(12) = 0.06048 \text{ in}$$

**SH**

From the Bridge Design Guidelines, design for a shrinkage factor of 0.0002 that includes a combination of the ultimate shrinkage and creep effects of the column.

$$SH = 0.0002(12.00)(12) = 0.0288 \text{ in}$$

**[3.12.6]****SE – Force Effects Due to Settlement**

Differential settlement between the pier and abutments will cause moments and shears in the superstructure that are transferred to the column and drilled shaft due to continuity between the superstructure and substructure. Based on a differential settlement of ½ inch, fixed end moments in the superstructure are determined as follows:

$$\begin{aligned} \text{Span 1: FEM} &= 6EI\Delta \div L^2 = 6(3861)(6,596,207)(0.5) \div (118)^2 / 1728 \\ &= 3175 \text{ ft-k} \end{aligned}$$

$$\begin{aligned} \text{Span 2: FEM} &= 6EI\Delta \div L^2 = 6(3861)(6,596,207)(0.5) \div (130)^2 / 1728 \\ &= 2616 \text{ ft-k} \end{aligned}$$

[3.4.1-3]

The substructure for this bridge was analyzed using the gross section properties for the column and drilled shaft. For forced deformations such as secondary moments due to prestressing, temperature, creep and shrinkage, a strength limit state load factor of 0.5 is specified. A summary of maximum and minimum load factors for the critical strength and service limit states follows:

[3.4.1-1]

### **LOAD COMBINATIONS STRENGTH I**

$$\text{Max} = 1.25\text{DC} + 1.50\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.75(\text{LL} + \text{IM} + \text{BR}) + 0.50\text{TU} + 1.00\text{SE}$$

$$\text{Min} = 0.90\text{DC} + 0.65\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.75(\text{LL} + \text{IM} + \text{BR}) + 0.50\text{TU} + 1.00\text{SE}$$

### **STRENGTH III**

$$\text{Max} = 1.25\text{DC} + 1.50\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) + 1.40\text{WS} \\ + 0.50\text{TU} + 1.00\text{SE}$$

$$\text{Min} = 0.90\text{DC} + 0.65\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) + 1.40\text{WS} \\ + 0.50\text{TU} + 1.00\text{SE}$$

### **STRENGTH V**

$$\text{Max} = 1.25\text{DC} + 1.50\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.35(\text{LL} + \text{IM} + \text{BR}) + 0.40\text{WS} + 1.00\text{WL} + 0.50\text{TU} + 1.00\text{SE}$$

$$\text{Min} = 0.90\text{DC} + 0.65\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.35(\text{LL} + \text{IM} + \text{BR}) + 0.40\text{WS} + 1.00\text{WL} + 0.50\text{TU} + 1.00\text{SE}$$

### **SERVICE I**

$$\text{Max} = 1.00(\text{DC} + \text{DW} + \text{PS} + \text{CR} + \text{SH}) + 1.00(\text{LL} + \text{IM} + \text{BR}) \\ + 0.30\text{WS} + 1.00\text{WL} + 1.00\text{TU} + 1.00\text{SE}$$

The load factor for settlement,  $\gamma_{\text{SE}}$ , should be determined on a project specific basis and is not defined in the specification. In lieu of project specific information to the contrary, the load factor for settlement may be taken as 1.0. Use of  $\gamma_{\text{SE}} = 1.0$  appears to be inconsistent with the 0.50 load factor used for CR, SH and PS. However, those forces are the result of the stiffness of the column. As the column is loaded and cracks the force effects will be relieved. For settlement, the prestressed superstructure member would have to crack to relieve the forces. If the superstructure were reinforced concrete and load factor of 0.50 might be appropriate for settlement forces.

**Pier Cap Design****Step 1 – Determine Structural Model****[3.6.3.1]**

The sectional model of analysis is appropriate for the design of a pier cap where the assumptions of traditional beam theory are valid. However, the strut-and-tie model should be considered for situations in which the distance between the centers of the applied load and the supporting reactions is less than twice the member thickness.

**[5.8.1.1]**

Components in which the distance from the point of zero shear to the face of the support is less than twice the depth or in which a load causing more than one-half of the shear at a support is closer than twice the depth from the face of the support may be considered to be deep components and require a strut-and-tie analysis. For this problem the webs are closer than twice the section depth so the strut-and-tie model is used.

A strut-and-tie model is used to determine the internal force effects near supports and the points of application of concentrated loads. A strut-and-tie analysis is a strength limit state analysis. As such no service limit state analysis is performed with the minimum reinforcing requirement satisfying that requirement.

The reaction due to differential settlement was not included in the pier cap design since the geotechnical aspects of the problem were not resolved at the time the example was created. The resulting 23.4 kip reaction at the pier due to differential settlement between the pier and abutments is small compared to the total reaction. In an actual design, this load would be included.

A section showing the relationship between the transverse frame model and the typical section with dimensions in feet is shown in Figure 11 below.

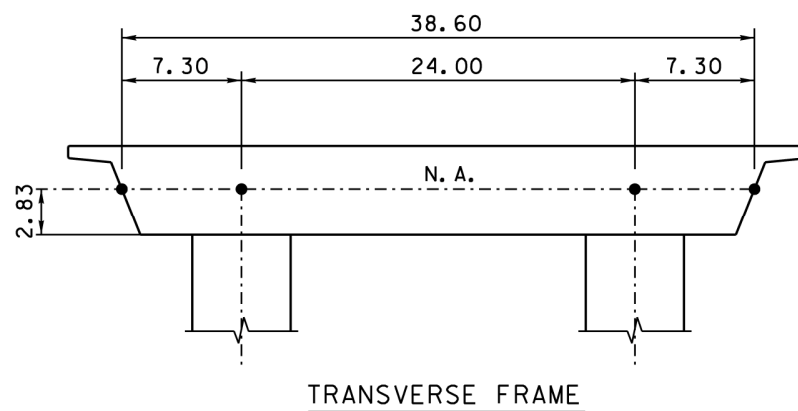


Figure 11



## Step 2 – Create Strut-and-Tie Model

A half section of the strut-and-tie model showing the nodes, angles and dimensions in inches is shown in Figure 12. In determining the model, consideration must be given to the required depth of struts, ties and the nodal zone. Establishing geometry requires iteration in which member sizes are assumed, the truss geometry is established, member forces are determined, and the assumed member sizes verified.

If a member is statically indeterminate such as in a rigid frame, a realistic strut-and-tie model will also be statically indeterminate. The support reactions and moments for the indeterminate truss can be found from an elastic analysis of the actual frame (Figure 11) with the internal forces in the truss determined from statics.

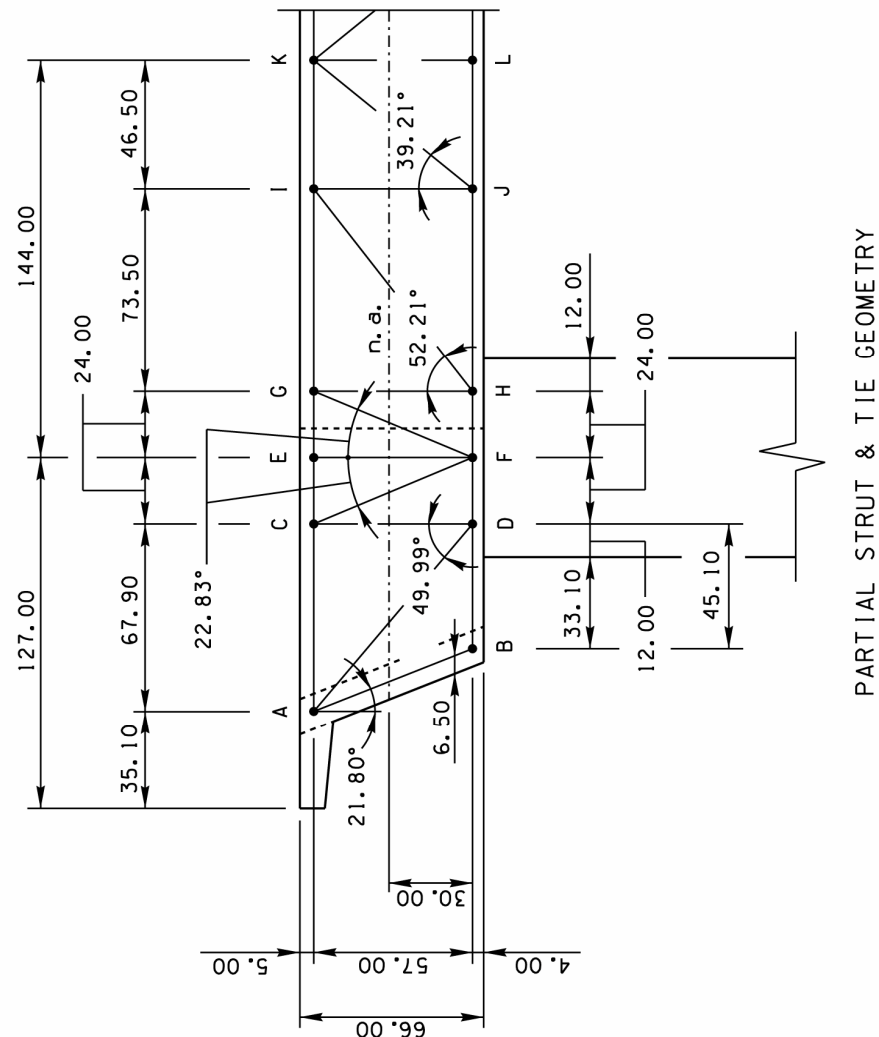


Figure 12

### Step 3 – Determine Loads

In a strut-and-tie analysis the location of the applied load affects the answer. For a post-tensioned concrete box girder bridge, the pier cap is integral with the superstructure and the applied loads from the web are uniformly distributed throughout the depth of the superstructure. Therefore half the load from the webs will be applied to the top nodes and half to the bottom nodes.

For this problem Strength I Limit State with two live load cases will be investigated to demonstrate the process. The first load case will be a combination of Live Load 1 and Live Load 2 to determine maximum tension in the top. The second load case will be Live Load 5 to determine maximum shear and maximum tension in the bottom. In an actual design additional load cases may require investigation.

To more accurately distribute the load, the pier cap weight is subtracted from the superstructure reaction and distributed to the nodes based on the contributing span of the nodes based on simple span reactions. Figure 13 shows the uniform load and the contributing length for each node. The factored node concentrated loads for the pier cap are shown below:

Node A/B:	$R = 1.25(4.950)(29.65 / 12) \div 2 = 7.64 \text{ k}$
Node C/D:	$R = 1.25(4.950)(45.95 / 12) \div 2 = 11.85 \text{ k}$
Node E/F:	$R = 1.25(4.950)(24.00 / 12) \div 2 = 6.19 \text{ k}$
Node G/H:	$R = 1.25(4.950)(48.75 / 12) \div 2 = 12.57 \text{ k}$
Node I/J:	$R = 1.25(4.950)(60.00 / 12) \div 2 = 15.47 \text{ k}$
Node K/L:	$R = 1.25(4.950)(46.50 / 12) \div 2 = 11.99 \text{ k}$

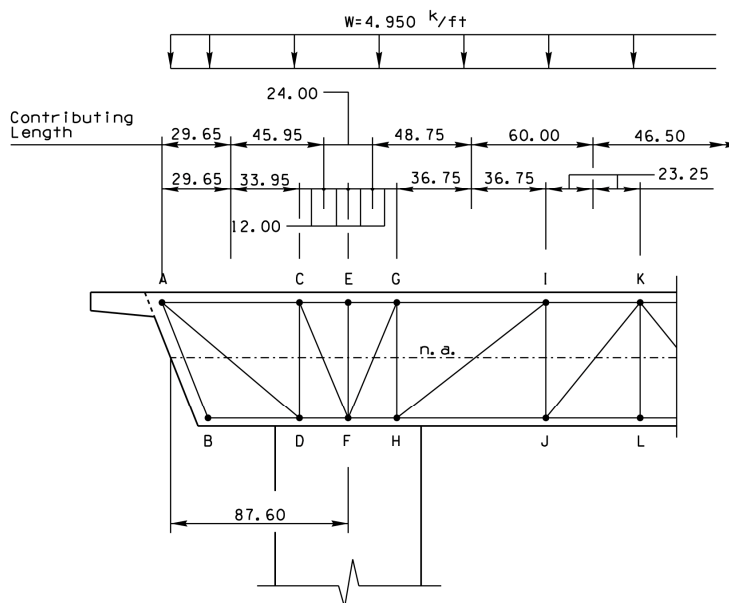


Figure 13

The dead load from the superstructure is concentrated in the webs. For the exterior and center webs the loads are directly over nodes and the reaction is divided between the top and bottom nodes. The first interior web reaction is located between nodes E and G and the reaction is proportioned as a simple span. See Figure 14.

$$\begin{aligned}
 \text{Node A/B:} \quad R &= [1.25(276.31) + 1.50(27.17) + 1.00(-17.85)] \div 2 \\
 &= 184.15 \text{ k} \\
 \text{Node E/F:} \quad R &= [1.25(352.44) + 1.50(27.17) + 1.00(-17.85)] \\
 &\quad (19.50 / 24.00) \div 2 = 188.28 \text{ k} \\
 \text{Node G/H:} \quad R &= [1.25(352.44) + 1.50(27.17) + 1.00(-17.85)] \\
 &\quad (4.50 / 24.00) \div 2 = 43.45 \text{ k} \\
 \text{Node I/J:} \quad R &= [1.25(352.44) + 1.50(27.17) + 1.00(-17.85)] \div 2 \\
 &= 231.73 \text{ k}
 \end{aligned}$$

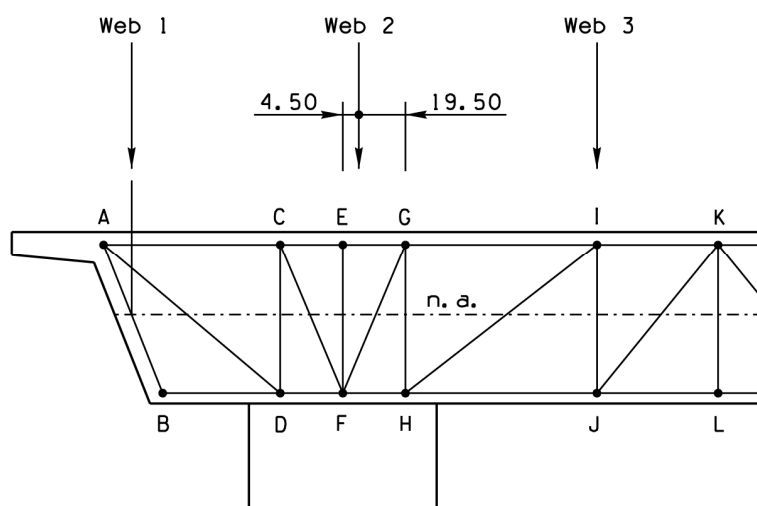


Figure 14

The live load for Case 1 as shown in Figure 15 is divided between the nodes as follows.

$$\begin{aligned}
 \text{Node A/B:} \quad R &= 1.75(119.00)(60.00 / 57.10) \div 2 = 109.41 \text{ k} \\
 \text{Node C/D:} \quad R &= 1.75(119.00)(-2.90 / 57.10 + 12.00/24.00) \div 2 \\
 &= 46.77 \text{ k} \\
 \text{Node E/F:} \quad R &= 1.75(119.00)(12.00 / 24.00) \div 2 = 52.06 \text{ k} \\
 \text{Node G/H:} \quad R &= 1.75(119.00)(37.50 / 73.50) \div 2 = 53.13 \text{ k} \\
 \text{Node I/J:} \quad R &= 1.75(119.00)(36.00 / 73.50 + 12.00 / 46.50) \div 2 \\
 &= 77.87 \text{ k} \\
 \text{Node K/L:} \quad R &= 1.75(119.00)(34.50 / 46.50) \div 2 = 77.25 \text{ k}
 \end{aligned}$$

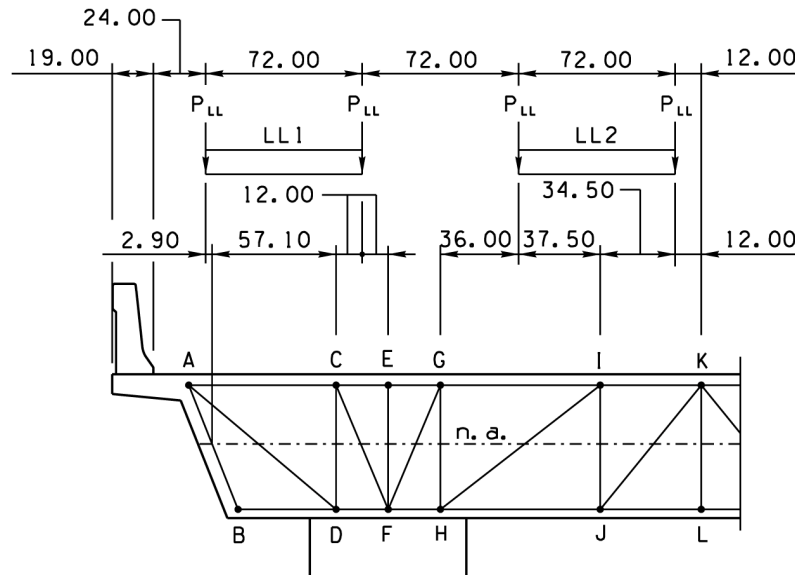


Figure 15

The live load for Case 2 as shown in Figure 16 is divided between the nodes as follows.

Node G/H:  $R = 1.75(119.00)(49.50 / 73.50) \div 2 = 70.13 \text{ k}$

Node I/J:  $R = 1.75(119.00)(24.00 / 73.50 + 24.00 / 46.50) \div 2 = 87.74 \text{ k}$

Node K/L:  $R = 1.75(119.00)(22.50 / 46.50 + 22.50 / 46.50) \div 2 = 100.77 \text{ k}$

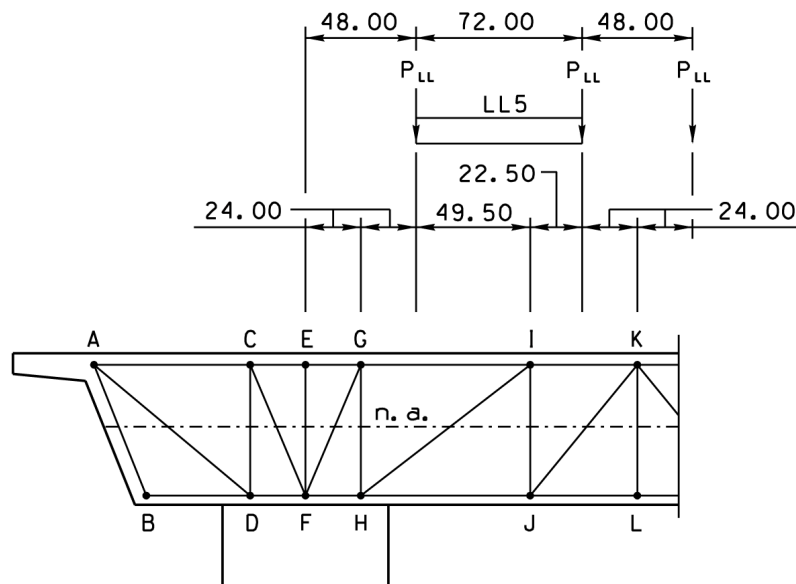


Figure 16

**Step 4 – Determine Reactions**

The pier cap in this problem is integral with the column. This restraint must be properly handled to preserve the equations of statics and compatibility. The column is divided into three equally spaced nodes. For the 72 inch wide column Node D is 12 inches from the face, Node F is at the center and Node H is 12 inches from the other face. For an applied uniform vertical load each node will have the same contributory area and same resulting reaction. The applied shear force will also be divided equally between the three nodes. The bending moment is handled with a couple between Node D and Node H resulting in axial loads at the nodes by dividing the moment by 4 feet.

Strength I Limit State also includes the restraint forces and moments from temperature and shrinkage of the frame. Since there are no externally applied loads, this load is considered by determining the moment, shear and axial load from a traditional transverse frame analysis. For Case I, temperature rise will be used since it causes tension in the top. For Case 2, temperature fall and shrinkage is used since it causes tension in the bottom. Differential settlement forces were not included only because this example was completed prior to settlement data being finalized. In an actual problem the differential settlement reaction would be included.

Case 1

A transverse frame analysis yielded the following reactions, shears and moments.

$$R = 1.25(1076.7) + 1.50(81.5) + 1.00(-53.6) + 1.75(275.7 + 159.4) \\ = 2176.0 \text{ k}$$

$$V = 1.25(-2.9) + 1.50(0.3) + 1.00(-0.2) + 1.75(5.3 - 6.6) + 0.5(-2.1) \\ = -6.7 \text{ k}$$

$$M = 1.25(-88) + 1.50(10) + 1.00(-6) + 1.75(188 - 211) + 0.5(-39) \\ = -161 \text{ ft-k}$$

The restraints for the strut-and-tie model are not located at the neutral axis where the moment is taken. The moment must be reduced to the moment at the bottom strut (2.50 feet from the column top). The reduced moment is:

$$M = -161 + 6.7(2.50) = -144 \text{ ft-k}$$

The reactions at the restraint nodes are as follows:

Node D:

$$R = 2176.0 / 3 - 144 / 4.00 = 689.33 \text{ ft-k}$$

$$V = 6.7 / 3 = 2.23 \text{ k}$$

Node F:

$$R = 2176.0 / 3 = 725.34 \text{ k}$$

$$V = 6.7 / 3 = 2.24 \text{ k}$$

Node H:

$$R = 2176.0 / 3 + 144 / 4.00 = 761.33 \text{ ft-k}$$

$$V = 6.7 / 3 = 2.23 \text{ k}$$

### Case 2

The transverse frame analysis produced the following reactions, shears and moments.

$$R = 1.25(1076.7) + 1.50(81.5) + 1.00(-53.6) + 1.75(238.0) = 1831.0 \text{ k}$$

$$V = 1.25(-2.9) + 1.50(0.3) + 1.00(-0.2) + 1.75(-12.2) + 0.5(5.2) = -22.1 \text{ k}$$

$$M = 1.25(-88) + 1.50(10) + 1.00(-6) + 1.75(-375) + 0.5(96) = -709 \text{ ft-k}$$

Since the restraint for the strut-and-tie model is not at the neutral axis, the moment must be reduced to the moment at the bottom strut location as follows:

$$M = -709 + 22.1(2.50) = -654 \text{ ft-k}$$

The reactions at the restraint nodes are as follows:

Node D:

$$R = 1831.0 / 3 - 654 / 4.00 = 446.83 \text{ ft-k}$$

$$V = 22.1 / 3 = 7.37 \text{ k}$$

Node F:

$$R = 1831.0 / 3 = 610.34 \text{ k}$$

$$V = 22.1 / 3 = 7.36 \text{ k}$$

Node H:

$$R = 1831.0 / 3 + 654 / 4.00 = 773.83 \text{ ft-k}$$

$$V = 22.1 / 3 = 7.37 \text{ k}$$

**Step 5 – Determine Member Forces**

A summation of Case 1 factored external loads applied at the various nodes follows:

Node	Uniform	Web	LL1 + LL2	Reaction	Total
A	7.64	184.15	109.41		301.20
B	7.64	184.15	109.41		301.20
C	11.85		46.77		58.62
D	11.85		46.77	-689.33	-630.71
E	6.19	188.28	52.06		246.53
F	6.19	188.28	52.06	-725.34	-478.81
G	12.57	43.45	53.13		109.15
H	12.57	43.45	53.13	-761.33	-652.18
I	15.47	231.73	77.87		325.07
J	15.47	213.73	77.87		325.07
K	11.99		77.25		89.24
L	11.99		77.25		89.24

With the geometry of the model established and the applied loads and reactions determined, the force in each member is calculated. Starting with Node B the vertical and horizontal forces at the node are summed and set equal to zero with the unknown forces in Members B-D and B-A calculated. The process is repeated for Node A where Members A-C and A-D are calculated. This process is continued until all the member forces are known.

When the last node is evaluated all the member forces are known. Summing the forces in both directions provides a magnitude for the error in the model. If the transverse frame model is not the same of that used for the strut-and-tie model errors will be introduced in addition to the expected rounding errors due the number of separate calculations.

The partial strut-and-tie model is shown in Figure 17 with the tension and compression values rounded to the nearest kip. The maximum tension of 593 kips occurs in Members C-E and E-G over the column.

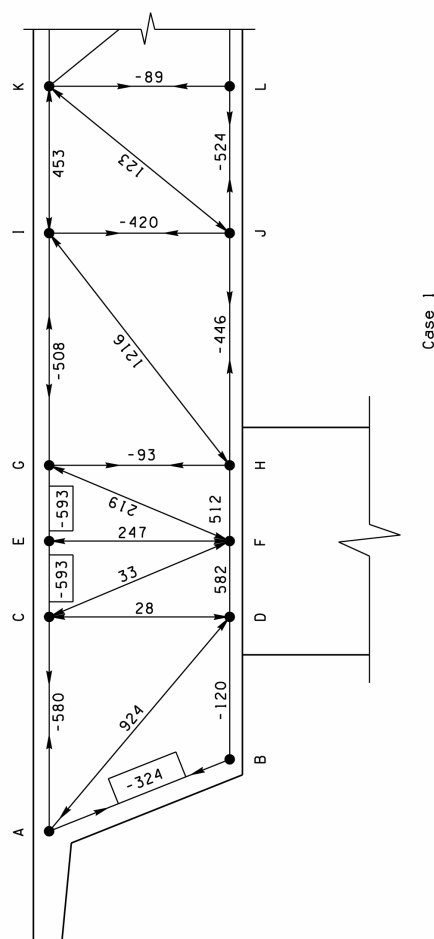


Figure 17

A summation of Case 2 external loads applied at the various nodes follows:

Node	Uniform	Web	LL5	Reaction	Total
A	7.64	184.15			191.79
B	7.64	184.15			191.79
C	11.85				11.85
D	11.85			-446.83	-434.98
E	6.19	188.28			194.47
F	6.19	188.28		-610.34	-415.87
G	12.57	43.45	70.13		126.15
H	12.57	43.45	70.13	-773.83	-647.68
I	15.47	231.73	87.74		334.94
J	15.47	213.73	87.74		334.94
K	11.99		100.77		112.76
L	11.99		100.77		112.76



With the geometry of the model established and the applied loads and reactions determined, the force in each member is calculated.

A partial strut-and-tie model is shown in Figure 18 with the tension and compression values rounded to the nearest kip. The maximum tension of 739 kips occurs in Member J-L at the midpoint between the columns. The maximum shear of 448 kips occurs in Member I-J. The maximum compression force of 1277 kips occurs in Member H-I.

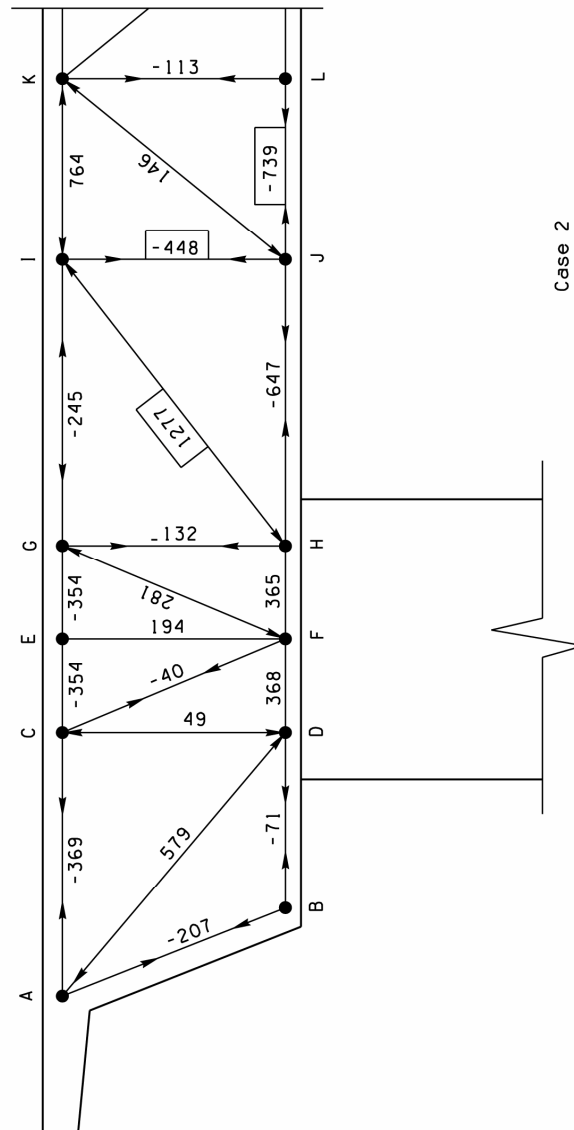


Figure 18

**Step 6 – Capacity of Tension Ties****(a) Top Reinforcement over column**

The maximum tension over the column occurs in Members C-E and E-G for Load Case 1. The required area of tension tie reinforcement,  $A_{st}$ , is calculated as follows:

**[5.6.3.4.1-1]**

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{593}{(0.90) \cdot (60)} = 10.98 \text{ in}^2$$

Use 14 - #8 bars with an  $A_s = 11.06 \text{ in}^2$

**(b) Bottom Reinforcement at midspan**

The maximum tension at midspan occurs in Member J-L for Load Case 2. The required area of tension tie reinforcement,  $A_{st}$ , is calculated as follows:

**[5.6.3.4.1-1]**

$$A_{st} = \frac{P_u}{\phi f_y} = \frac{739}{(0.90) \cdot (60)} = 13.69 \text{ in}^2$$

Use 14 - #9 bars with an  $A_s = 14.00 \text{ in}^2$

**(c) Vertical Stirrups**

The maximum vertical tension force of 448 kips occurs in Member I-J for Load Case 2. This tension force can be resisted by stirrups within a certain length of the beam as indicated by the stirrup bands. Using #6 stirrups with 6 legs, the number of stirrups,  $n$ , required in the band is:

$$n = \frac{P_u}{\phi A_{st} f_y} = \frac{448}{(0.90) \cdot (6) \cdot (0.44) \cdot (60)} = 3.1 \text{ Use 4 stirrups}$$

The required spacing,  $s$ , within the 46.5 inch band is:

$$s \leq \frac{46.50}{3.1} = 15.0 \text{ in}$$

Use #6 stirrups with 6 legs spaced at 12 inches.

**(d) Sloping Web**

The maximum tension force in Tie A-B is 324 kips. The required number of two legged #5 stirrups is:

$$n = \frac{324}{(0.9) \cdot (2) \cdot (0.31) \cdot (60)} = 9.7 \text{ Use 10 stirrups at 7 inch spacing}$$

**Step 7 – Check Anchorage of Tension Tie****(a) Top Longitudinal Reinforcement****[5.6.3.4.2]**

The top 14 - #8 rebar must be developed at the inner edge of Node A. The available embedment length is  $35.10 + 6.50 - 2 \text{ clear} = 39.60$  inches. The required development length is:

**[5.11.2.1.1]**

$$l_d = \frac{1.25 A_b f_y}{\sqrt{f'_c}} = \frac{(1.25) \cdot (0.79) \cdot (60)}{\sqrt{4.5}} = 27.9 \text{ in}$$

$$\text{but not less than } 0.4 d_b f_y = 0.4(1.00)(60) = 24.0 \text{ in}$$

Since a horizontal construction joint is located 12 inches below the deck, the top bars are not considered top bars for development length determination. Therefore the bars are adequately developed.

**(b) Vertical Stirrups**

The vertical stirrups must be developed beyond the inner edge of the nodes. The required development length is:

**[5.11.2.1.1]**

$$l_d = \frac{1.25 A_b f_y}{\sqrt{f'_c}} = \frac{(1.25) \cdot (0.44) \cdot (60)}{\sqrt{4.5}} = 15.6 \text{ in}$$

$$\text{but not less than } 0.4 d_b f_y = 0.4(0.75)(60) = 18.0 \text{ in}$$

The available embedment length is 8 inches, the width of the bottom strut, minus 2 inches clear or 6 inches. Therefore the stirrups must be hooked. The basic development length of a hooked #6 rebar is:

**[5.11.2.4.1]**

$$l_{dh} = \frac{38.0 d_b}{\sqrt{f'_c}} = \frac{(38.0)(0.75)}{\sqrt{4.5}} = 13.4 \text{ inches}$$

Where side cover normal to the plane of the hook is not less than 2.5 inches and for a 90 degree hook the cover on the bar extension beyond the hook is not less than 2.0 inches a modification factor of 0.7 may be applied. The modified development length is  $(0.7)(13.4) = 9.4$  inches.

To properly develop the vertical stirrups, the struts and ties must be 12 inches thick. The strut-and-tie model was constructed on the assumption that the top strut would be 10 inches deep and the bottom strut 8 inches deep resulting in a vertical dimension of 57 inches between the top and bottom members. This model must be revised to reflect the embedment requirement resulting in a dimension between the top and bottom members of 54 inches. For this problem the model will not be reevaluated based on this new depth.

### Step 8 – Capacity of Struts

Strut H-I has the highest compression force of 1277 kips. Since this strut is anchored at Joint I, that also anchors tension Ties G-I and I-J as shown in Figure 18, this is the most critical strut.

The effective cross sectional area of a strut is determined by considering the available concrete area and the anchorage conditions at the ends of the strut. When a strut is anchored by reinforcement, the effective concrete area may be considered to extend a distance of up to six bar diameters from the anchored bar where  $d_{ba}$  = diameter of the longitudinal reinforcing. See Figure 19 for determination of the depth and width of the strut.

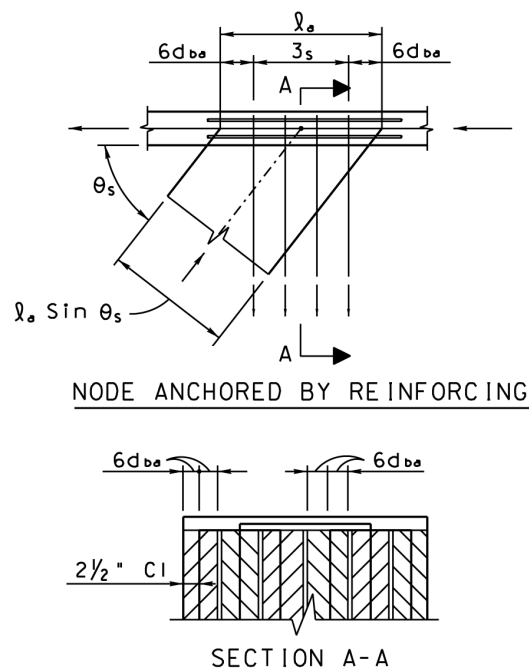


Figure 19

The effective cross sectional area is:

$$l_a \sin \theta_s = [6(1.00)(2) + 3(12.00)] \sin(37.79) = 29.41 \text{ inches}$$

$$w_{\text{effective}} = 2[2.50 + 6(1.00)] + 4[(2)(6)(1.00)] = 65.00 \text{ inches} \leq 72.0 \text{ inches}$$

$$A_{cs} = (29.41)(65.00) = 1912 \text{ in}^2$$

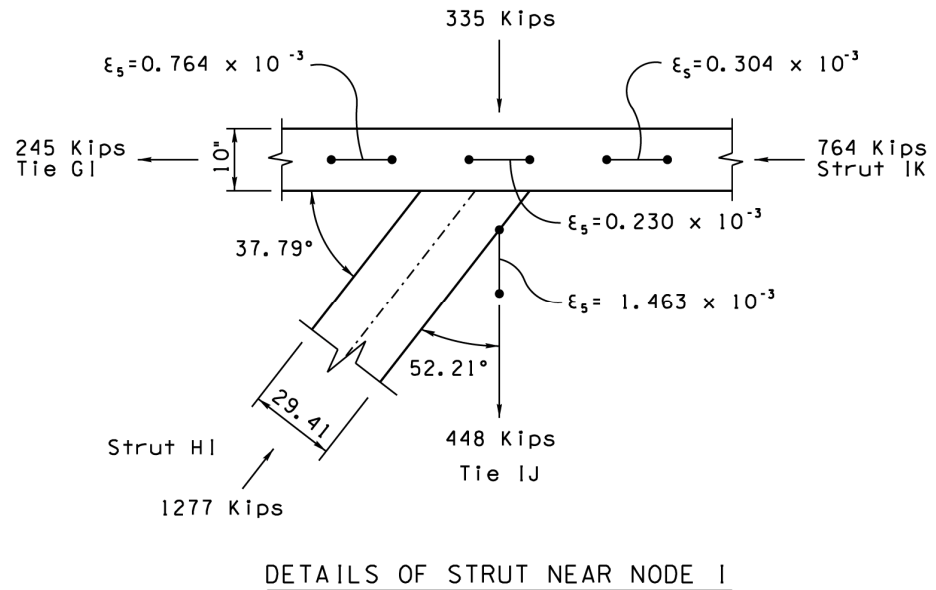


Figure 20

The limiting compressive stress in the strut,  $f_{cu}$ , is usually controlled by the tensile strain in the tie that is at the smallest angle to the strut. However, for this problem both tensile strains will be investigated. From the geometry of the truss, the angle between tension Tie G-I and Strut H-I is 37.79 degrees. The tensile strain in Tie G-I is:

$$\epsilon_s = \frac{P_u}{A_{st} E_s} = \frac{245}{(11.06) \cdot (29000)} = 0.764 \times 10^{-3}$$

The compressive strain the horizontal Strut I-K is:

$$\epsilon_s = \frac{P_u}{A_c E_c} = \frac{764}{(65.00) \cdot (10.00) \cdot (3861)} = 0.304 \times 10^{-3}$$

At the center of the node the horizontal strain will be the average strain for the two horizontal members joining at the node. The average strain is:

$$\epsilon_s = (0.764 \times 10^{-3} - 0.304 \times 10^{-3}) / 2 = 0.230 \times 10^{-3}$$

[5.6.3.3.3-2]

The principal strain,  $\epsilon_1$ , is determined as follows:

$$\epsilon_1 = \epsilon_s + (\epsilon_s + 0.002) \cot^2 \alpha_s$$

$$\epsilon_1 = 0.230 \times 10^{-3} + (0.230 \times 10^{-3} + 0.002) \cot^2(37.79) = 3.94 \times 10^{-3}$$

The limiting compressive stress,  $f_{cu}$ , in the Strut H-I is:

[5.6.3.3.3-1]

$$f_{cu} = \frac{f'_c}{0.8 + 170\epsilon_1} \leq 0.85f'_c$$

$$f_{cu} = \frac{4.5}{0.8 + (170) \cdot (3.94 \times 10^{-3})} = 3.06 \text{ ksi}$$

If the vertical Tie I-J is considered, the tension strain is:

$$\epsilon_s = \frac{P_u}{A_{st} E_s} = \frac{448}{(6) \cdot (0.44) \cdot (4) \cdot (29000)} = 1.463 \times 10^{-3}$$

From this tie the principle strain is:

$$\epsilon_1 = 1.463 \times 10^{-3} + (1.463 \times 10^{-3} + 0.002) \cot^2(52.21) = 3.54 \times 10^{-3}$$

Since this is less than the previously calculated value for principle strain, this tie does not govern the compressive capacity of the strut.

The nominal resistance of the strut is based on the limiting stress,  $f_{cu}$ , and the strut dimensions. The nominal resistance is:

$$P_n = f_{cu} A_{cs} = (3.06)(1912) = 5851 \text{ kips}$$

[5.6.3.5]

Since the strut is anchored in tension on 2 sides,  $\phi = 0.65$  and the factored resistance of the strut is:

[5.6.3.3.1-1]

$$P_r = \phi P_n = (0.70)(5851) = 4096 \text{ kips} > P_u = 1277 \text{ kips}$$

**Step 9 – Node Regions****[5.6.3.5]**

The concrete compression stress in the node region of the strut shall not exceed  $0.65 \phi f'_c$  for node regions anchoring tension ties in more than one direction. For Node I:

$$f_c = 0.65 \phi f'_c = (0.65) \cdot (0.70) \cdot (4.5) = 2.05 \text{ ksi}$$

The nodal zone compressive stress is:

$$f_c = \frac{1277}{1912} = 0.67 \text{ ksi}$$

The tension tie reinforcing shall be uniformly distributed over an effective area of concrete. Check to ensure that the tension ties are sufficiently spread out in the effective anchorage area. The effective anchorage area is equal to twice the depth to the top tie of 12 inches.

The nodal tensile zone stress to anchor the tension tie is:

$$f_c = \frac{580}{(12.00) \cdot (65.00)} = 0.744 \text{ ksi}$$

Since the stresses are less than the limiting stress the criteria is satisfied.

**Step 10 – Provide Crack Control Reinforcing**

Due to the presence of concentrated loads at Nodes A and I within a distance less than the member depth from the face of the column support, these zones will be considered D-regions.

In D-regions crack control reinforcing in the form of orthogonal grid on both faces is required. The minimum ratio of reinforcing to gross concrete area is 0.003. The required area of reinforcing per foot of depth is:

$$A_s = (0.003)(12.00)(72.00) = 2.59 \text{ in}^2$$

Use 6 - #6 with an  $A_s = (6)(0.44) = 2.64 \text{ in}^2$ .

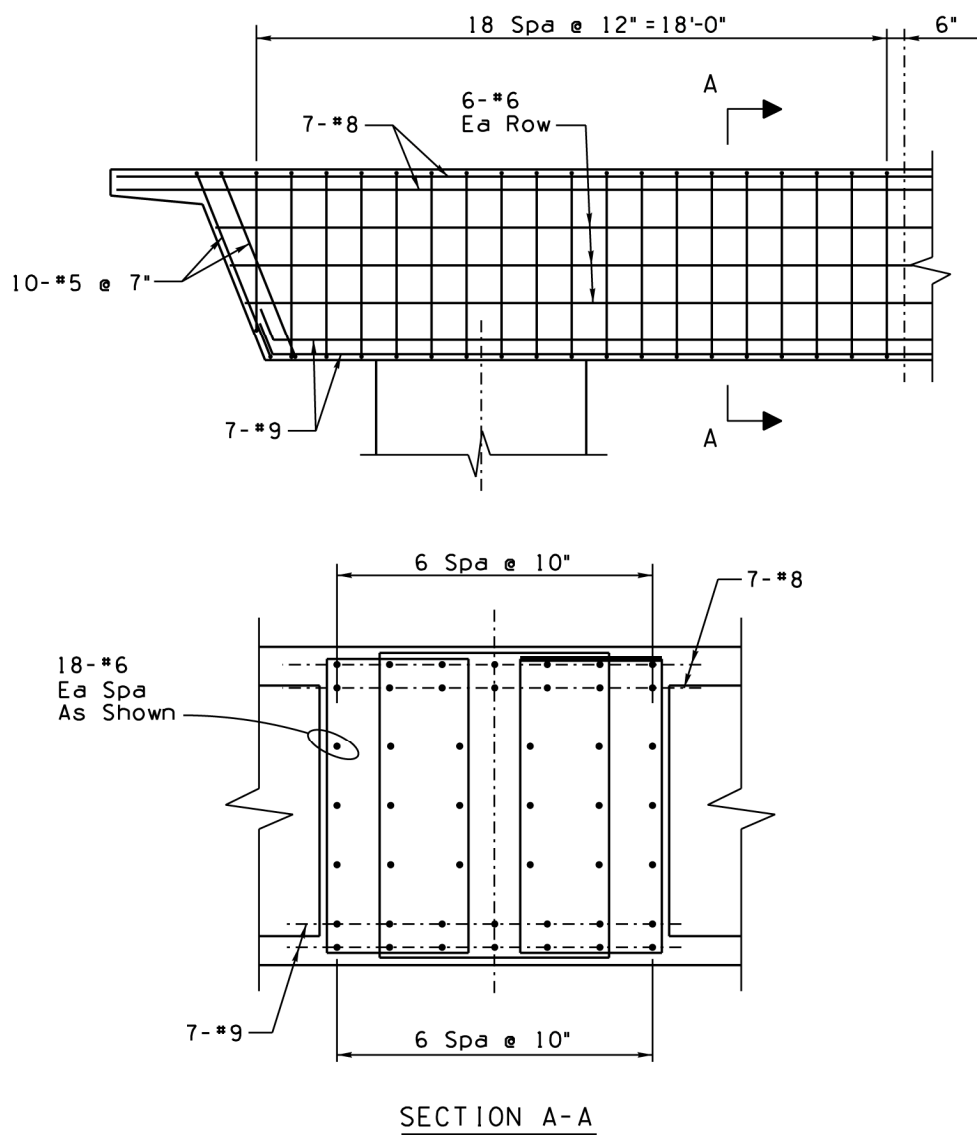
**Step 11 – Sketch Required Reinforcing**

Figure 21



**Column Design**

The column is designed as a member under axial load and moment. A summary of moments and axial loads at the top and bottom of the column in both the longitudinal and transverse directions is required. A summary of values is shown below:

**Top of Column – Unfactored Axial Forces and Moments**

Load	P <sub>max</sub>	P <sub>min</sub>	M <sub>trans</sub>	M <sub>long</sub>
DC	1076.7	1076.7	-88	-283
DW	81.5	0	10	-23
PS	-53.6	-53.6	-6	399
LL 1	275.7		188	
LL 2	159.4		-211	
LL 3	38.9		-112	
LL 4		-37.7	140	
LL 5	238.0		-375	
LL+IM Pos	435.1	-45.2	328	1472
LL+IM Neg	435.1	-45.2	-375	-1695
BR	0.3	-0.3	0	±378
WS super	54.5	-54.5	±654	±336
WS sub	2.5	-2.5	±30	±25
WS	57.0	-57.0	±684	±361
WS vert	0	-108.7	0	0
WL	12.4	-12.4	±149	±78
TU	0	0	±333	0
CR + SH	0	0	158	0
SE	11.7	-11.7	0	±17

The transverse live loads (LL1 – LL5) can be combined with the multiple presence factor included with the following LL+IM Pos and Neg results:

$$1 \text{ vehicle } P_{\max} = (275.7)(1.2) = 330.8 \text{ k}$$

$$2 \text{ vehicles } P_{\max} = (275.7 + 159.4)(1.0) = \underline{435.1 \text{ k}} \leq \text{Critical}$$

$$3 \text{ vehicles } P_{\max} = (275.7 + 159.4 + 38.9)(0.85) = 402.9 \text{ k}$$

$$P_{\min} = (-37.7)(1.2) = -45.2 \text{ k}$$

$$M_{\text{pos}} = (188 + 140)(1.00) = 328 \text{ ft-k}$$

$$M_{\text{neg}} = (-211 - 112)(1.00) = -323 \text{ ft-k}$$

$$M_{\text{neg}} = (-375)(1.00) = \underline{-375 \text{ ft-k}} \leq \text{Critical}$$

The reaction at the base of the column =  $0.15(6.0)(4.0)(18.0) = 64.8 \text{ k}$   
 increasing the reaction at the base to  $1076.7 + 64.8 = 1141.5 \text{ k}$ .

**Bottom of Column – Unfactored Axial Forces and Moments**

Load	P <sub>max</sub>	P <sub>min</sub>	M <sub>trans</sub>	M <sub>long</sub>
DC	1141.5	1141.5	-29	-283
DW	81.5	0	3	-23
PS	-53.6	-53.6	-2	399
LL 1	275.7		77	
LL 2	159.4		-74	
LL 3	38.9		-30	
LL 4		-37.7	29	
LL 5	238.0		-121	
LL+IM Pos	435.1	-45.2	106	150
LL+IM Neg	435.1	-45.2	-121	-174
BR	0.3	-0.3	0	±134
WS super	54.5	-54.5	±13	±119
WS sub	2.5	-2.5	±5	±11
WS	57.0	-57.0	±18	±130
WS vert	0	-108.7	0	0
WL	12.4	-12.4	±3	±27
TU	0	0	±42	0
CR + SH	0	0	-20	0
SE	11.7	-11.7	0	±17

The column must be designed for the strength limit state. Strength I, III and V Limit States will be investigated to determine the critical one.

**STRENGTH I**

$$\text{Max} = 1.25\text{DC} + 1.50\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.75(\text{LL} + \text{IM} + \text{BR}) + 0.50\text{TU} + 1.00\text{SE}$$

$$\text{Min} = 0.90\text{DC} + 0.65\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.75(\text{LL} + \text{IM} + \text{BR}) + 0.50\text{TU} + 1.00\text{SE}$$

Top of Column

$$P_{\text{max}} = 1.25(1076.7) + 1.50(81.5) + 0.50(-53.6) + 1.75(435.1 + 0.3) \\ + 1.00(11.7) = \underline{2215.0} \text{ k} \leq \text{Critical}$$

$$P_{\text{min}} = 0.90(1076.7) + 0.65(0) + 0.50(-53.6) + 1.75(-45.2 - 0.3) \\ + 1.00(-11.7) = \underline{850.9} \text{ k}$$

$$M_{\text{trans}} = 0.9(-88) + 1.50(10) + 0.50(-6) + 0.50(158) + 1.75(328 + 0) \\ + 0.50(333) = 737 \text{ ft-k}$$

$$M_{\text{trans}} = 1.25(-88) + 0.65(0) + 0.50(-6) + 0.50(0) + 1.75(-375 + 0) \\ + 0.50(-333) = \underline{-936} \text{ ft-k} \leq \text{Critical}$$

$$M_{\text{long}} = 0.90(-283) + 0.65(0) + 0.50(399) + 1.75(1472 + 378) + 1.00(17) \\ = 3199 \text{ ft-k}$$

$$M_{\text{long}} = 1.25(-283) + 1.50(-23) + 0.50(399) + 1.75(-1695 - 378) \\ + 1.00(-17) = \underline{-3834} \text{ ft-k} \leq \text{Critical}$$

Bottom of Column

$$P_{\text{max}} = 1.25(1141.5) + 1.50(81.5) + 0.50(-53.6) + 1.75(435.1 + 0.3) \\ + 1.00(11.7) = 2296.0 \text{ k}$$

$$P_{\text{min}} = 0.90(1141.5) + 0.65(0) + 0.50(-53.6) + 1.75(-45.2 - 0.3) \\ + 1.00(-11.7) = 909.2 \text{ k}$$

$$M_{\text{trans}} = 0.90(-29) + 1.50(3) + 0.50(-2) + 0.50(0) + 1.75(106 + 0) \\ + 0.50(42) = 184 \text{ ft-k}$$

$$M_{\text{trans}} = 1.25(-29) + 0.65(0) + 0.50(-2) + 0.50(-20) + 1.75(-121 + 0) \\ + 0.50(-42) = -280 \text{ ft-k}$$

$$M_{\text{long}} = 0.90(-283) + 0.65(0) + 0.50(399) + 1.75(150 + 134) + 1.00(17) \\ = 459 \text{ ft-k}$$

$$M_{\text{long}} = 1.25(-283) + 1.50(-23) + 0.50(399) + 1.75(-174 - 134) + 1.00(-17) \\ = -745 \text{ ft-k}$$

**STRENGTH III**

$$\text{Max} = 1.25\text{DC} + 1.50\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) + 1.40\text{WS} \\ + 0.50\text{TU} + 1.00\text{SE}$$

$$\text{Min} = 0.90\text{DC} + 0.65\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) + 1.40\text{WS} \\ + 0.50\text{TU} + 1.00\text{SE}$$

Top of Column

$$P_{\text{max}} = 1.25(1076.7) + 1.50(81.5) + 0.50(-53.6) + 1.40(57.0) + 1.00(11.7) \\ = \underline{1532.8 \text{ k}}$$

$$P_{\text{min}} = 0.90(1076.7) + 0.65(0) + 0.50(-53.6) + 1.40(-57.0 - 108.7) \\ + 1.00(-11.7) = \underline{698.6 \text{ k}}$$

$$M_{\text{trans}} = 0.90(-88) + 1.50(10) + 0.50(-6) + 0.50(158) + 1.40(684) \\ + 0.50(333) = \underline{1136 \text{ ft-k}}$$

$$M_{\text{trans}} = 1.25(-88) + 0.65(0) + 0.50(-6) + 0.50(0) + 1.40(-684) \\ + 0.50(-333) = -1237 \text{ ft-k}$$

$$M_{\text{long}} = 0.90(-283) + 0.65(0) + 0.50(399) + 1.40(361) + 1.00(17) \\ = 467 \text{ ft-k}$$

$$M_{\text{long}} = 1.25(-283) + 1.50(-23) + 0.50(399) + 1.40(-361) + 1.00(-17) \\ = \underline{-711 \text{ ft-k}}$$

Bottom of Column

$$P_{\text{max}} = 1.25(1141.5) + 1.50(81.5) + 0.50(-53.6) + 1.40(57.0) + 1.00(11.7) \\ = 1613.8 \text{ k}$$

$$P_{\text{min}} = 0.90(1141.5) + 0.65(0) + 0.50(-53.6) + 1.40(-57.0 - 108.7) \\ + 1.00(-11.7) = 756.9 \text{ k}$$

$$M_{\text{trans}} = 0.90(-29) + 1.50(3) + 0.50(-2) + 1.40(18) + 0.50(42) = 24 \text{ ft-k}$$

$$M_{\text{trans}} = 1.25(-29) + 0.65(0) + 0.50(-2) + 0.50(-20) + 1.40(-18) + 0.50(-42) \\ = -93 \text{ ft-k}$$

$$M_{\text{long}} = 0.90(-283) + 0.65(0) + 0.50(399) + 1.40(130) + 1.00(17) = 144 \text{ ft-k}$$

$$M_{\text{long}} = 1.25(-283) + 1.50(-23) + 0.50(399) + 1.40(-130) + 1.00(-17) \\ = -388 \text{ ft-k}$$

**STRENGTH V**

$$\text{Max} = 1.25\text{DC} + 1.50\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.35(\text{LL} + \text{IM} + \text{BR}) + 0.40\text{WS} + 1.00\text{WL} + 0.50\text{TU} + 1.00\text{SE}$$

$$\text{Min} = 0.90\text{DC} + 0.65\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.35(\text{LL} + \text{IM} + \text{BR}) + 0.40\text{WS} + 1.00\text{WL} + 0.50\text{TU} + 1.00\text{SE}$$

Top of Column

$$P_{\max} = 1.25(1076.7) + 1.50(81.5) + 0.50(-53.6) + 1.35(435.1 + 0.3) \\ + 0.40(57.0) + 1.00(12.4) + 1.00(11.7) = \underline{2076.0 \text{ k}}$$

$$P_{\min} = 0.90(1076.7) + 0.65(0) + 0.50(-53.6) + 1.35(-45.2 - 0.3) \\ + 0.40(-57.0) + 1.00(-12.4) + 1.00(-11.7) = \underline{833.9 \text{ k}}$$

$$M_{\text{trans}} = 0.90(-88) + 1.50(10) + 0.50(-6) + 0.50(158) + 1.35(328 + 0) \\ + 0.40(684) + 1.00(149) + 0.50(333) = 1044 \text{ ft-k}$$

$$M_{\text{trans}} = 1.25(-88) + 0.65(0) + 0.50(-6) + 0.50(0) + 1.35(-375 - 0) \\ + 0.40(-684) + 1.00(-149) + 0.50(-333) = \underline{-1208 \text{ ft-k}}$$

$$M_{\text{long}} = 0.90(-283) + 0.65(0) + 0.50(399) + 1.35(1472 + 378) \\ + 0.40(361) + 1.00(78) + 1.00(17) = 2682 \text{ ft-k}$$

$$M_{\text{long}} = 1.25(-283) + 1.50(-23) + 0.50(399) + 1.35(-1695 - 378) \\ + 0.40(-361) + 1.00(-78) + 1.00(-17) = \underline{-3227 \text{ ft-k}}$$

Bottom of Column

$$P_{\max} = 1.25(1141.5) + 1.50(81.5) + 0.50(-53.6) + 1.35(435.1 + 0.3) \\ + 0.40(57.0) + 1.00(12.4) + 1.00(11.7) = 2157.0 \text{ k}$$

$$P_{\min} = 0.90(1141.5) + 0.65(0) + 0.50(-53.6) + 1.35(-45.2 - 0.3) \\ + 0.40(-57.0) + 1.00(-12.4) + 1.00(-11.7) = 892.2 \text{ k}$$

$$M_{\text{trans}} = 0.90(-29) + 1.50(3) + 0.50(-2) + 0.50(0) + 1.35(106 + 0) \\ + 0.40(18) + 1.00(3) + 0.50(42) = 152 \text{ ft-k}$$

$$M_{\text{trans}} = 1.25(-29) + 0.65(0) + 0.50(-2) + 0.50(-20) + 1.35(-121 - 0) \\ + 0.40(-18) + 1.00(-3) + 0.50(-42) = -242 \text{ ft-k}$$

$$M_{\text{long}} = 0.90(-283) + 0.65(0) + 0.50(399) + 1.35(150 + 134) + 0.40(130) \\ + 1.00(27) + 1.00(17) = 424 \text{ ft-k}$$

$$M_{\text{long}} = 1.25(-283) + 1.50(-23) + 0.50(399) + 1.35(-174 - 134) \\ + 0.40(-130) + 1.00(-27) + 1.00(-17) = -701 \text{ ft-k}$$

**Slenderness Effects**

A review of the group load combinations indicates that Strength I Limit State for the top of the column is critical. The slenderness effects will be considered for this load. The design requirements are contained in Section 4.5.3.2.2 (Moment Magnification - Beam Columns), Section 4.5.5 (Equivalent Members), Section 4.6.2.5 (Effective length factor, K) and Section 5.7.4.3 (Approximate Evaluation of Slenderness Effects).

**Longitudinal Slenderness**

For members not braced against sidesway, the effects of slenderness shall be considered where the slenderness ratio  $Kl_u/r$  is greater than 22. The effects of slenderness must be considered separately in each direction as the moment of inertia of the column and the degree of restraint at the top varies with each direction. The longitudinal direction will be considered for the following Strength I Limit State loads:  $P_u = 2215.0$  kips and  $M_u = 3834$  ft-k.

**[4.5.5]**

For a prismatic compression member, the calculations are straight forward. However, for a non-prismatic member the problem is more complex. One way to simplify the problem is to find the equivalent length of a prismatic compression member that has the same buckling capacity. The following equation applies to a stepped member that is free at the top and fixed at the base:

$$\frac{k_2}{k_1} \tan k_1 l_1 \tan k_2 l_2 = 1 \text{ where } k_1 = \sqrt{\frac{P}{EI_1}} \text{ and } k_2 = \sqrt{\frac{P}{EI_2}}$$

$$\begin{array}{lll} I_1 = 32.00 \text{ ft}^4 & L_1 = 18.00 \text{ ft} & E = 490,320 \text{ ksf} \\ I_2 = 117.86 \text{ ft}^4 & L_2 = 22.95 \text{ ft} & E = 490,320 \text{ ksf} \end{array}$$

An iterative process is used to solve the equation. After several iterations the value for the critical buckling load  $P$  is determined to equal 61,019 kips. This answer is verified below:

$$k_1 = \sqrt{\frac{61,019}{(490,320) \cdot (32.00)}} = 0.062362$$

$$k_2 = \sqrt{\frac{61,019}{(490,320) \cdot (117.86)}} = 0.032494$$

$$\frac{0.032494}{0.062362} \tan[(0.062362) \cdot (18.00)] \tan[(0.032494) \cdot (22.95)] = 1.001 \text{ ok}$$

Knowing the critical load for the stepped column/shaft allows solving for the equivalent column length of a prismatic column. Using the section properties for the top of the column, the general equation for the buckling load follows:

$$P_{cr} = \frac{\pi^2 EI}{(Kl_u)^2}$$

Solve the above equation for  $l_u$  using the previously calculated critical load for the stepped member for  $P_{cr}$ .

$$l_u = \frac{\pi}{K} \sqrt{\frac{EI}{P_{cr}}} = \frac{\pi}{2} \sqrt{\frac{(490,320) \cdot (32.00)}{61,019}} = 25.19 \text{ ft}$$

The radius of gyration for a rectangular section may be taken as 0.30 times the overall dimension in the direction in which stability is being considered. For the longitudinal direction:  $r = 0.30(4.00) = 1.20 \text{ ft}$ .

The effective length factor,  $K$ , is used to modify the length according to the restraint at the ends of the column against rotation and translation. For members not braced against sidesway,  $K$  is determined with due consideration for the effects of cracking and reinforcement on the relative stiffness and may not be taken less than 1.0. Because the compression member is integral with the superstructure, the relative stiffness of the superstructure and compression member must be determined. For slenderness ratios less than 60 sufficiently accurate values of  $K$  are obtained using the gross moment of inertia for columns and one-half the gross moment of inertia for restraining beams.

For restraining members or beam

$$0.5I_g = (0.5)(6,596,207) \div (12)^4 = 159.05 \text{ ft}^4$$

$$E_g = (3861)(144) = 555,984 \text{ ksf}$$

For each column

$$I_c = 32.00 \text{ ft}^4$$

$$E_c = (3405)(144) = 490,320 \text{ ksf}$$

$$G_{top} = \frac{\sum \left( \frac{E_c I_c}{L_c} \right)}{\sum \left( \frac{E_g I_g}{L_g} \right)} = \frac{2 \cdot \left( \frac{(490,320) \cdot (32.00)}{25.19} \right)}{\left( \frac{(555,984) \cdot (159.05)}{118.00} + \frac{(555,984) \cdot (159.05)}{130.00} \right)} = 0.871$$

The specification does not discuss the stiffness of drilled shafts. Therefore, judgment is required.

$G_{bottom} = 1.00$  for the equivalent length to fixity.

The equivalent length factor may be determined by solution of the following equation or by use of the alignment chart in Figure 22.

For unbraced frames:

[C4.6.2.5-2]

$$\frac{G_A G_B \left( \frac{\pi}{K} \right)^2 - 36}{6(G_A + G_B)} = \frac{\frac{\pi}{K}}{\tan \left( \frac{\pi}{K} \right)}$$

$G_A$  and  $G_B$  are determined above for the two ends of the column. The problem is solved by an iterative method yielding  $K = 1.298$ . The results are verified below:

$$\text{Since } \frac{(0.871) \cdot (1.00) \cdot \left( \frac{\pi}{1.298} \right)^2 - 36}{6 \cdot (0.871 + 1.00)} = -2.752 \text{ and } \frac{\frac{\pi}{1.298}}{\tan \left( \frac{\pi}{1.298} \right)} = -2.752$$

The value of  $K = 1.298$  is verified.

From the alignment chart  $K = 1.30$ . Due to the various assumptions made in this analysis a value of  $K = 1.30$  will be used.

[C4.6.2.5-2]

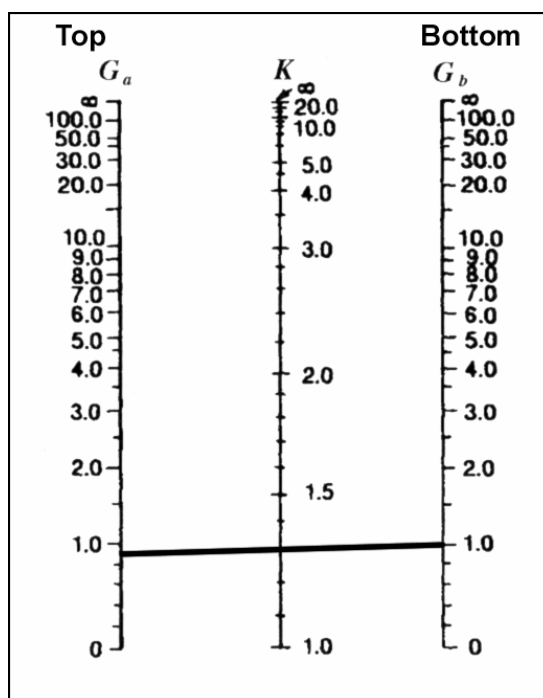


Figure 22



The slenderness ratio is calculated as follows:

$$\frac{Kl_u}{r} = \frac{(1.30) \cdot (25.19)}{1.20} = 27.3$$

[5.7.4.3]

Since the slenderness ratio is greater than 22, slenderness effects must be considered. Since the slenderness ratio is less than 100 the approximate moment magnifier method may be used.

In lieu of a more precise method, EI shall be taken as the greater of:

[5.7.4.3-1]

$$EI = \frac{\frac{E_c I_g}{5} + E_s I_s}{1 + \beta_d}$$

[5.7.4.3-2]

$$EI = \frac{\frac{E_c I_g}{2.5}}{1 + \beta_d}$$

The first equation will control for large percentages of reinforcement. The second equation will control when the reinforcing percentage is small. At this stage the reinforcing pattern is usually not known requiring that  $I_s$  be ignored. See Figure 24 for the reinforcing pattern for this problem.

$$I_s = \sum A_s d^2 = 1.56[(7(20.67)^2 + 2(13.78)^2 + 2(6.89)^2](2) = 10,812 \text{ in}^4$$

The permanent moment on the column includes the total dead load and secondary moment from prestress.

Strength I Limit State

$$M_u = 1.25(-283) + 1.50(-23) + 0.50(399) = -189 \text{ ft-k}$$

$$\beta_d = (189) / (3834) = 0.049$$

$$EI = \frac{\frac{(490,320) \cdot (32.00)}{5} + (10,812) \cdot (29000) \div 144}{1 + 0.049} = 5,067,000 \text{ k-ft}^2$$

$$EI = \frac{\frac{(490,320) \cdot (32.00)}{2.5}}{1 + 0.049} = 5,983,000 \text{ k-ft}^2 \leq \text{Critical}$$

**[4.5.2.2b]**

For moment magnification, assume half the column/shaft weight is applied at the top. For this example use the weight of the column of 64.8 kips.

For the braced condition, the rigid frame has a rotational restraint between conditions (a) and (b) shown in Table C4.6.2.5-1. To be conservative use the higher value of  $K = 0.80$ .

**[4.5.3.2.2b-5]**

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (5,983,000)}{[(0.80) \cdot (25.19)]^2} = 145,406 \text{ k}$$

$$P_u = 2215.0 + 1.25(64.8) = 2296 \text{ k}$$

$$\delta_b = \frac{C_m}{1 - \frac{P_u}{\phi_K P_e}} = \frac{1.0}{1 - \frac{2296}{(0.75) \cdot (145,406)}} = 1.022 \geq 1.0$$

The moment on the compression member,  $M_{2b}$ , is equal to the factored gravity loads that result in no appreciable sidesway. For this problem the dead load and prestress secondary moment result in sidesway due to the unequal spans so  $M_{2b} = 0$ .

For the unbraced condition, use the previously calculated value of  $K = 1.30$ .

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (5,983,000)}{[(1.30) \cdot (25.19)]^2} = 55,065 \text{ k}$$

**[4.5.3.2.2b]**

To determine the sum of factored loads, one must be careful not to add just double the maximum load on a single column. The live load plus dynamic impact must be determined for the total number of live load vehicles that can be placed on the roadway including the multiple presence factor. Lateral loads that add a reaction to one column but subtract from another are not included.

$$\begin{aligned} \sum P_u &= 1.25(2)(1076.7 + 64.8) + 1.50(2)(81.5) + 0.50(2)(-53.6) \\ &\quad + 1.75(238)(3)(0.85) + 1.00(11.7)(2) = 4130 \text{ k} \end{aligned}$$

**[4.5.3.2.2b-1]**

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{\phi_K \sum P_e}} = \frac{1}{1 - \frac{(4130)}{(0.75) \cdot (55,065) \cdot (2)}} = 1.053$$

The moment on the compression member,  $M_{2s}$  is equal to the factored lateral or gravity load that results in sidesway. For this problem in the longitudinal direction, all moments are included.

The magnified moments are increased to reflect effects of deformation as follows:

$$M_c = \delta_b M_{2b} + \delta_s M_{2s}$$

$$M_c = 1.022(0) + 1.053(3834) = 4037 \text{ ft-k}$$

This magnified moment is used with the maximum and minimum factored axial loads to design the compression member.

### Strength III Limit State

$$M_u = 1.25(-283) + 1.50(-23) + 0.50(399) = 189 \text{ ft-k}$$

$$\beta_d = (189) / (711) = 0.266$$

$$EI = \frac{\frac{(490,320) \cdot (32.00)}{2.5}}{1 + 0.266} = 4,957,000 \text{ k-ft}^2$$

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (4,957,000)}{[(1.30) \cdot (25.19)]^2} = 45,622 \text{ k}$$

$$\begin{aligned} \sum P_u &= 1.25(2)(1076.7 + 64.8) + 1.50(2)(81.5) + 0.50(2)(-53.6) \\ &\quad + 1.00(11.7)(2) = 3068 \text{ k} \end{aligned}$$

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{\phi_K \sum P_e}} = \frac{1}{1 - \frac{(3068)}{(0.75) \cdot (45,622) \cdot (2)}} = 1.047$$

$$M_c = 1.047(711) = 744 \text{ ft-k}$$

## Strength V Limit State

$$M_u = 1.25(-283) + 1.50(-23) + 0.50(399) = -189 \text{ ft-k}$$

$$\beta_d = (189) / (3227) = 0.059$$

$$EI = \frac{(490,320) \cdot (32.00)}{\frac{2.5}{1 + 0.059}} = 5,926,000 \text{ k-ft}^2$$

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (5,926,000)}{[(1.30) \cdot (25.19)]^2} = 54,540 \text{ k}$$

$$\begin{aligned} \Sigma P_u &= 1.25(2)(1076.7 + 64.8) + 1.50(2)(81.5) + 0.50(2)(-53.6) \\ &\quad + 1.35(238)(3)(0.85) + 1.00(11.7)(2) = 3887 \text{ k} \end{aligned}$$

$$\delta_s = \frac{1}{1 - \frac{\Sigma P_u}{\phi_K \Sigma P_e}} = \frac{1}{1 - \frac{(3887)}{(0.75) \cdot (54,540) \cdot (2)}} = 1.050$$

$$M_c = 1.050(3227) = 3388 \text{ ft-k}$$

A precise analysis using a refined second order analysis is complex and not recommended nor required for conventional bridges. However, the approximate moment magnifier method provides magnified moments at the ends of a member but not between the ends. Moments at the top of the column and bottom of the shaft can be estimated using this method. The moment at the base of the column can not be determined using the moment magnifier method. In this example the top of the column controls the design but if the column/shaft member were fixed at the base and free at the top the designer would have to use more refined methods of analysis to determine the moment at the interface.

**Transverse  
Slenderness**

The effects of deformation on the moments in the transverse direction will now be considered. For the non-prismatic member the following equation applies for a member that is free at the top and fixed at the base:

$$\frac{k_2}{k_1} \tan k_1 l_1 \tan k_2 l_2 = 1 \text{ where } k_1 = \sqrt{\frac{P}{EI_1}} \text{ and } k_2 = \sqrt{\frac{P}{EI_2}}$$

$$\begin{array}{lll} I_1 = 72.00 \text{ ft}^4 & L_1 = 18.00 \text{ ft} & E = 490,320 \text{ ksf} \\ I_2 = 117.86 \text{ ft}^4 & L_2 = 28.14 \text{ ft} & E = 490,320 \text{ ksf} \end{array}$$

An iterative process is used to solve the above equation. After several iterations the value for the critical buckling load P is determined to equal 63,180 kips. This answer will be verified below:

$$k_1 = \sqrt{\frac{63,180}{(490,320) \cdot (72.00)}} = 0.042304$$

$$k_2 = \sqrt{\frac{63,180}{(490,320) \cdot (117.86)}} = 0.033065$$

$$\frac{0.033065}{0.042304} \tan[(0.042304) \cdot (18.00)] \tan[(0.033065) \cdot (28.14)] = 1.000 \text{ ok}$$

Knowing the critical load for a stepped column/shaft allows solution for the equivalent column length of a prismatic column. Using the section properties for the top of the column, the general equation for the buckling load can be rearranged to solve for the equivalent length as follows:

$$l_u = \frac{\pi}{K} \sqrt{\frac{EI}{P_{cr}}} = \frac{\pi}{2} \sqrt{\frac{(490,320) \cdot (72.00)}{63,180}} = 37.13 \text{ ft}$$

The radius of gyration for a rectangular section may be taken as 0.30 times the overall dimension in the direction in which stability is being considered. For the transverse direction:  $r = 0.30(6.00) = 1.80 \text{ ft}$ .

For members not braced against sidesway, K is determined based on the relative stiffness of the pier cap and the compression member as follows:

$$I_g = (0.50)(126.97) = 63.49 \text{ ft}^4$$

$$G_{top} = \frac{\sum \frac{E_c I_c}{l_c}}{\sum \frac{E_g I_g}{l_g}} = \frac{(490,320) \cdot (72.00)}{(555,984) \cdot (63.49)} = 0.646$$

$G_{bottom} = 1.00$  for a fixed end of the equivalent length for a drilled shaft.

From the alignment chart the value of  $K = 1.26$  is obtained.

The slenderness ratio is calculated as follows:

$$\frac{kl_u}{r} = \frac{(1.26) \cdot (37.13)}{1.80} = 26.0$$

Since the slenderness ratio is greater than 22, slenderness effects must be considered in the transverse direction.

The reduced EI must be determined. See Figure 24 for the reinforcing pattern for this problem.

$$I_s = \sum A_s d^2 = 1.56[(7(32.67)^2 + 2(21.78)^2 + 2(10.89)^2](2) = 27,011 \text{ in}^4$$

The permanent moment on the column includes the total dead load and secondary moment from prestress.

Strength I Limit State

$$M_u = 1.25(-88) + 0.65(0) + 0.50(-6) + 0.50(-333) = -280 \text{ ft-k}$$

$$\beta_d = (280) / (936) = 0.299$$

$$EI = \frac{\frac{(490,320) \cdot (72.00)}{5} + (27,011) \cdot (29000) \div 144}{1 + 0.299} = 9,623,000 \text{ k-ft}^2$$

$$EI = \frac{(490,320) \cdot (72.00)}{2.5}{1 + 0.299} = 10,871,000 \text{ k-ft}^2 \leq \text{Critical}$$

For the braced condition, the rigid frame has a rotational restraint between conditions (a) and (b) shown in Table C4.6.2.5-1. To be conservative use the higher value of  $K = 0.80$ . The axial load is the same as for the longitudinal direction.

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (10,871,000)}{[(0.80) \cdot (37.13)]^2} = 121,602 \text{ k}$$

$$\delta_b = \frac{C_m}{1 - \frac{P_u}{\phi_K P_e}} = \frac{1.0}{1 - \frac{2296}{(0.75) \cdot (121,602)}} = 1.026$$

The moment on the compression member,  $M_{2b}$ , is equal to the factored gravity loads that result in no appreciable sidesway. For this problem the dead load, prestress secondary moment, temperature and shrinkage and differential settlement loads result in no sidesway.

$$M_{2b} = 1.25(-88) + 0.65(0) + 0.50(-6) + 0.50(-333) = -280 \text{ ft-k}$$

For the unbraced condition:

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (10,871,000)}{[(1.26) \cdot (37.13)]^2} = 49,021 \text{ k}$$

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{\phi_K \sum P_e}} = \frac{1}{1 - \frac{4130}{(0.75) \cdot (49,021) \cdot (2)}} = 1.060$$

The moment on the compression member,  $M_{2s}$  is equal to the factored lateral or gravity load that results in sidesway. For this problem in the transverse direction, only the live load and dynamic load allowance are included.

$$M_{2s} = 1.75(-375) = -656 \text{ ft-k}$$

The magnified moments are increased to reflect effects of deformation as follows:

$$M_c = 1.026(280) + 1.060(656) = 983 \text{ ft-k}$$

This magnified moment is used with the factored axial load to design the compression member.

## Strength III Limit State

$$M_u = 1.25(-88) + 0.65(0) + 0.50(-6) + 0.50(0) + 0.50(-333) = -280 \text{ ft-k}$$

$$\beta_d = (280) / (1237) = 0.236$$

$$EI = \frac{(490,320) \cdot (72.00)}{1 + 0.236} = 11,518,000 \text{ k-ft}^2$$

## Braced condition:

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (11,518,000)}{[(0.80) \cdot (37.13)]^2} = 128,839 \text{ k}$$

$$P_u = 1532.8 + 1.25(64.8) = 1614 \text{ k}$$

$$\delta_b = \frac{C_m}{1 - \frac{P_u}{\phi_K P_e}} = \frac{1.0}{1 - \frac{1614}{(0.75) \cdot (128,839)}} = 1.017$$

## Unbraced condition:

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (11,518,000)}{[(1.26) \cdot (37.13)]^2} = 51,938 \text{ k}$$

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{\phi_K \sum P_e}} = \frac{1}{1 - \frac{(3068)}{(0.75) \cdot (51,938) \cdot (2)}} = 1.041$$

$$M_c = 1.017(280) + 1.041(1237 - 280) = 1281 \text{ ft-k}$$



## Strength V Limit State

$$M_u = 1.25(-88) + 0.65(0) + 0.50(-6) + 0.50(-333) = -280 \text{ ft-k}$$

$$\beta_d = (280) / (1208) = 0.232$$

$$EI = \frac{(490,320) \cdot (72.00)}{1 + 0.232} = 11,462,000 \text{ k-ft}^2$$

## Braced condition:

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (11,462,000)}{[(0.80) \cdot (37.13)]^2} = 128,212 \text{ k}$$

$$P_u = 2076.0 + 1.25(64.8) = 2157 \text{ k}$$

$$\delta_b = \frac{C_m}{1 - \frac{P_u}{\phi_K P_e}} = \frac{1.0}{1 - \frac{2157}{(0.75) \cdot (128,212)}} = 1.023$$

## Unbraced condition:

$$P_e = \frac{\pi^2 EI}{(Kl_u)^2} = \frac{\pi^2 \cdot (11,462,000)}{[(1.26) \cdot (37.13)]^2} = 51,686 \text{ k}$$

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{\phi_K \sum P_e}} = \frac{1}{1 - \frac{(3887)}{(0.75) \cdot (51,686) \cdot (2)}} = 1.053$$

$$M_c = 1.023(280) + 1.053(1208 - 280) = 1264 \text{ ft-k}$$

The results from the column analysis program are shown below.

Load Combination	P <sub>u</sub> k	M <sub>trans</sub> ft-k	M <sub>long</sub> ft-k	Ratio
Strength I	2215	983	4037	1.370
Strength I	851	983	4037	1.137
Strength III	1533	1281	744	4.310
Strength III	699	1281	744	3.986
Strength V	2076	1264	3388	1.563
Strength V	834	1264	3388	1.325

Strength I with minimum axial load is the controlling load combination with a ratio of factored resistance to factored strength of 1.137. The column is reinforced with 24 #11 longitudinal bars as shown in Figure 23. The area of reinforcing is  $A_s = 1.56(24) = 37.44 \text{ in}^2$ . The gross area of the column is  $(48.0)(72.0) = 3456 \text{ in}^2$ . The percentage of reinforcement equals  $(37.44 / 3456)(100) = 1.08\%$  which is within the allowable limits.

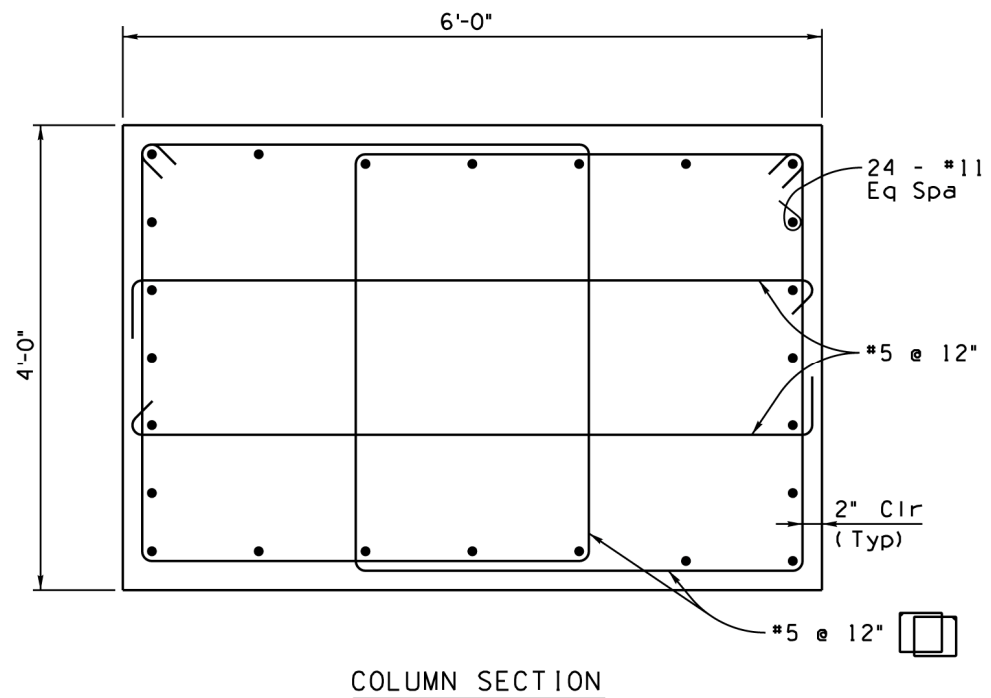


Figure 23

**Column/Shaft  
Splice****[5.11.5.2.1]**

The column reinforcing bars must be adequately spliced to the drilled shaft reinforcing. The LRFD Specifications discusses non-contact lap splices as splices where the reinforcing is within 6 inches of each other. However, in this case the 6 inch requirement is not satisfied. Where the distance between spliced rebar exceeds 6 inches, the development length must be increased to reflect the lack of a contact splice. This is done by assuming a 1:1 distribution between bars resulting in increasing the lap length by the distance of separation.

The development length of the reinforcement is determined as follows:

**[5.11.2.1.1]**

$$\text{For \#11 and smaller: } \frac{1.25 A_b f_y}{\sqrt{f'_c}} = \frac{1.25 \cdot (1.56) \cdot (60)}{\sqrt{3.5}} = 62.5 \text{ in}$$

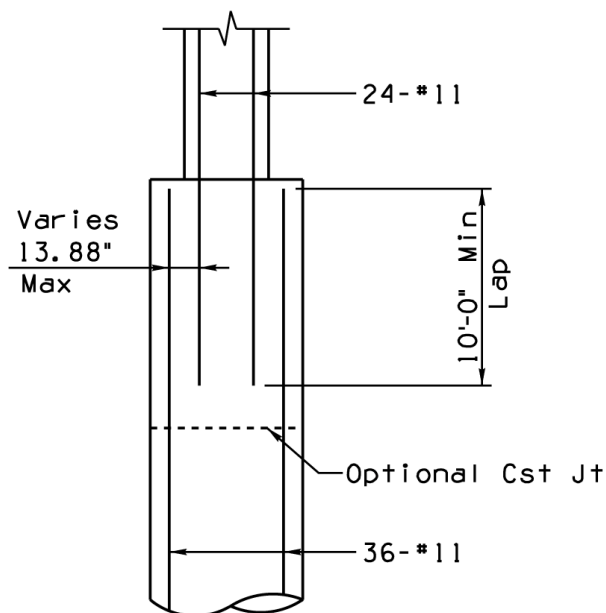
$$\text{But not less than } 0.4 d_b f_y = 0.4(1.41)(60) = 33.8 \text{ in}$$

**[5.11.5.3.1-1]**

For the column/shaft splice, all the reinforcing is spliced in the same location. Since there is less than twice the required reinforcing a Class C splice is required. The lap length should be  $1.7 l_d = 1.7(62.5) = 106.3$  inches.

**[5.11.5.2.1]**

To account for the lack of contact in some locations the splice length should be increased by the minimum separation of 13.9 inches. The lap length should be 106.3 plus 13.9 = 120.2 inches or 10.02 feet. Use a 10 foot length splice.



ELEVATION

Figure 24

**Column Shear****Step 1 – Determine Shear**

The transverse and longitudinal shears per column are taken from computer output as shown in the following table.

Load	$V_{trans}$	$V_{long}$
DC	-2.9	0
DW	0.3	0
PS	-0.2	0
LL 1	5.3	
LL 2	-6.6	
LL 3	-3.9	
LL 4	5.3	
LL 5	-12.2	
LL+ IM Pos	10.6	60.3
LL+ IM Neg	-12.2	-52.6
BR	0	27.4
WS super	32.0	24.3
WS sub	2.6	7.7
WS	34.6	32.0
WL	7.3	5.6
TU	$\pm 17.9$	0
CR + SH	8.5	0
SE	0	0

Shear design is based on strength limit states as follows:

**STRENGTH I**

$$\text{Max} = 1.25\text{DC} + 1.50\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.75(\text{LL} + \text{IM} + \text{BR}) + 0.50\text{TU} + 1.00\text{SE}$$

$$\text{Min} = 0.90\text{DC} + 0.65\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.75(\text{LL} + \text{IM} + \text{BR}) + 0.50\text{TU} + 1.00\text{SE}$$

$$P_{min} = 0.90(1076.7) + 0.65(0) + 0.50(-53.6) + 1.75(-45.2 - 0.3) \\ + 1.00(-11.7) = 850.9 \text{ k}$$

$$V_{trans} = 0.9(-2.9) + 1.50(0.3) + 0.50(-0.2) + 0.50(8.5) + 1.75(10.6 + 0) \\ + 0.5(17.9) = 29.5 \text{ k}$$

$$V_{trans} = 1.25(-2.9) + 0.65(0) + 0.50(-0.2) + 1.75(-12.2 + 0) + 0.50(-17.9) \\ = -34.0 \text{ k}$$

$$V_{long} = 1.75(60.3 + 27.4) = \underline{153.5 \text{ k}} \leq \text{Critical}$$

**STRENGTH III**

$$\text{Max} = 1.25\text{DC} + 1.50\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) + 1.40\text{WS} \\ + 0.50\text{TU} + 1.00\text{SE}$$

$$\text{Min} = 0.90\text{DC} + 0.65\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) + 1.40\text{WS} \\ + 0.50\text{TU} + 1.00\text{SE}$$

$$P_{\min} = 0.90(1076.7) + 0.65(0) + 0.50(-53.6) + 1.40(-57.0 - 108.7) \\ + 1.00(-11.7) = 698.6 \text{ k}$$

$$V_{\text{trans}} = 0.90(-2.9) + 1.50(0.3) + 0.50(-0.2) + 0.50(8.5) + 1.40(34.6) \\ + 0.50(17.9) = 59.4 \text{ k}$$

$$V_{\text{trans}} = 1.25(-2.9) + 0.65(0) + 0.50(-0.2) + 0.50(0) + 1.40(-34.6) \\ + 0.50(-17.9) = -61.2 \text{ k}$$

$$V_{\text{long}} = 1.40(32.0) = 44.8 \text{ k}$$

**STRENGTH V**

$$\text{Max} = 1.25\text{DC} + 1.50\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.35(\text{LL} + \text{IM} + \text{BR}) + 0.40\text{WS} + 1.00\text{WL} + 0.50\text{TU} + 1.00\text{SE}$$

$$\text{Min} = 0.90\text{DC} + 0.65\text{DW} + 0.50\text{PS} + 0.50(\text{CR} + \text{SH}) \\ + 1.35(\text{LL} + \text{IM} + \text{BR}) + 0.40\text{WS} + 1.00\text{WL} + 0.50\text{TU} + 1.00\text{SE}$$

$$P_{\min} = 0.90(1076.7) + 0.65(0) + 0.50(-53.6) + 1.35(-45.2 - 0.3) \\ + 0.40(-57.0) + 1.00(-12.4) + 1.00(-11.7) = 833.9 \text{ k}$$

$$V_{\text{trans}} = 0.90(-2.9) + 1.50(0.3) + 0.50(-0.2) + 0.50(8.5) + 1.35(10.6 + 0) \\ + 0.40(34.6) + 1.00(7.3) + 0.50(17.9) = 45.9 \text{ k}$$

$$V_{\text{trans}} = 1.25(-2.9) + 0.65(0) + 0.50(-0.2) + 0.50(0) + 1.35(-12.2 + 0) \\ + 0.40(-34.6) + 1.00(-7.3) + 0.50(-17.9) = -50.3 \text{ k}$$

$$V_{\text{long}} = 1.35(60.3 + 27.4) + 0.40(32.0) + 1.00(5.6) = 136.8 \text{ k}$$

Strength I Limit State has the maximum shear in the longitudinal direction. The determination of the shear resistance for this limit state will be shown below. Strength V Limit State has a smaller shear but also a smaller axial load and could be critical. One way to simplify the problem would be to analyze the column for the maximum shear (Strength I) with the minimum axial force (Strength III).

**Step 2 – Determine Analysis Method**

The sectional model of analysis is appropriate for the design of bridge columns since the assumptions of traditional beam theory are valid. Since concentrated loads are not applied directly to the column the sectional model may be used.

Three methods are available to determine shear resistance. Since none of the criteria required to use the simplified procedure is satisfied, this easy method may not be used. The first method described in the General Procedure requires minimum transverse reinforcement and will be used. The 2008 Interim Revisions now provide for a direct method of calculation of  $\beta$  and  $\theta$  using the following equations:

$$[5.8.3.4.2-1] \quad \beta = \frac{4.8}{(1 + 750\epsilon_s)}$$

$$[5.8.3.4.2-3] \quad \theta = 29 + 3500\epsilon_s$$

**Step 3 – Shear Depth,  $d_v$** 

The shear depth is the maximum of the following three criteria:

$$1) \quad d_v = 0.9 d_e \text{ where } d_e = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = d_s \text{ when } A_{ps} = 0$$

$$d_s = 48.00 - 2.00 \text{ clear} - 0.625 - 1.41 / 2 = 44.67 \text{ inches}$$

$$d_v = 0.9 d_s = 0.9(44.67) = 40.20 \text{ in} \leq \text{Critical}$$

$$2) \quad 0.72h = 0.72(48.00) = 34.56 \text{ in}$$

$$3) \quad d_v = \frac{M_n}{A_s f_y + A_{ps} f_{pu}}$$

For a column,  $M_n$ , is a function of the axial load making the determination of this value difficult. Therefore, ignore the third option and choose the greater of the first two values.

Based on the above, the shear depth,  $d_v$ , equals 40.20 inches.

**Step 4 – Calculate Strain,  $\epsilon_s$** 

The formula for the calculation of positive values of strain for sections containing at least the minimum amount of transverse reinforcing follows.

**[5.8.3.4.2-4]**

$$\epsilon_s = \left[ \frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps}f_{po}}{E_s A_s + E_p A_{ps}} \right]$$

$A_{ps}$  = area of prestressing steel on the flexural tension side of the member.  
 $A_{ps} = 0 \text{ in}^2$

$A_s$  = area of nonprestressed steel on the flexural tension side of the member.  
 $A_s = 12(1.56) = 18.72 \text{ in}^2$

$f_{po} = 0 \text{ ksi}$  for a nonprestressed member.

$N_u$  = factored axial force taken as positive if tensile and negative if compressive.  
 $N_u = -850.9 \text{ kips}$

$V_u$  = factored shear force.  
 $V_u = 153.5 \text{ kips}$

$M_u$  = factored moment, not to be taken less than  $(V_u - V_p)d_v$ .  
 $M_u = 4037 \text{ ft-k}$  but not less than  $V_u d_v = (153.5)(40.20) / 12 = 514 \text{ ft-k}$ .

$$\epsilon_s = \left[ \frac{\frac{(4037) \cdot (12)}{40.20} + 0.5 \cdot (-850.9) + |153.5 - 0| - (0) \cdot (0)}{(29000) \cdot (18.72) + (28500) \cdot (0)} \right]$$

$$\epsilon_s = 0.00172$$

Since the strain is positive, the equation is satisfactory. If the strain is negative the denominator of the equation must be modified.

**Step 5 – Determine  $\beta$  and  $\theta$** 

$$\beta = \frac{4.8}{(1 + (750) \cdot (0.00172))} = 2.10$$

$$\theta = 29 + (3500) \cdot (0.00172) = 35.0 \text{ degrees}$$

**Step 6 - Calculate Concrete Shear Strength,  $V_c$** 

The nominal shear resistance from concrete,  $V_c$ , is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316\beta\sqrt{f'_c}b_vd_v$$

$$V_c = 0.0316 \cdot (2.10) \cdot \sqrt{3.5} \cdot (72.00) \cdot (40.20) = 359.3 \text{ kips}$$

**Step 7 - Determine Required Vertical Reinforcement,  $V_s$** 

[5.8.2.4-1]

Since  $V_u = 153.5 < 0.5\phi V_c = (0.5)(0.90)(359.3) = 161.7 \text{ k}$ , minimum reinforcing is not required. However, the column requires ties. Therefore calculate the strength of the ties.

[5.8.3.3-4]

$$V_s = \frac{A_v f_y d_v (\cot \theta + \cot \alpha) \sin \alpha}{s} = \frac{A_v f_y d_v \cot \theta}{s} \text{ where } \alpha = 90^\circ$$

Use #5 stirrups with 4 legs at 12 inch spacing

$$V_s = \frac{(1.24) \cdot (60) \cdot (40.20) \cot(35.0)}{12} = 356.0 \text{ kips}$$

The shear strength is the lesser of:

[5.8.3.3-1]

$$V_n = V_c + V_s + V_p = [359.3 + 356.0 + 0] = 715.3 \text{ kips}$$

[5.8.3.3-2]

$$V_n = 0.25f'_c b_v d_v + V_p = [0.25(3.5)(72.00)(40.20) + 0] = 2532.6 \text{ kips}$$

$$\phi V_n = (0.90)(715.3) = 643.8 \text{ k} > 153.5 \text{ k}$$

$\therefore$  The column is adequately reinforced for shear in the longitudinal direction.



[5.8.2.5]

**Step 8 – Minimum Reinforcement**

Since transverse reinforcement is not required by analysis the minimum reinforcement criteria need not be satisfied. However, the formula for strain that was used required that the minimum reinforcement requirement be satisfied.

[5.8.2.5-1]

$$A_v \geq 0.0316 \sqrt{f'_c} \frac{b_v s}{f_y}$$

$$A_v \geq 0.0316 \sqrt{3.5} \frac{(72.0) \cdot (12.0)}{60} = 0.85 \text{ in}^2$$

$$A_v \text{ provided} = 4(0.31) = 1.24 \text{ in}^2$$

Therefore, the minimum transverse reinforcement requirement is satisfied.

**Step 9 – Maximum Spacing Transverse Reinforcement**

The maximum spacing for transverse reinforcement shall not exceed the following.

[5.8.2.9-1]

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v} = \frac{|153.5 - 0|}{(0.90) \cdot (72.00) \cdot (40.20)} = 0.059 \text{ ksi}$$

$$v_u = 0.059 < 0.125 f'_c = (0.125)(3.5) = 0.438 \text{ ksi}$$

$$s_{\max} = 0.8d_v = (0.8)(40.20) = 32.2 \text{ inches but not to exceed 24 in}$$

**Step 10 - Longitudinal Reinforcement**

The area of longitudinal reinforcement need not be greater than the area required to resist the maximum moment alone. Since the column reinforcing is continuous this criteria is met.

**Drilled Shafts****Drilled Shafts**

The design of a drilled shaft foundation requires consideration of geotechnical and structural resistance and deformation limits. The design process requires establishment of criteria for acceptable stress and deformation levels and comparison of these criteria with stress and deformation levels calculated. The design of foundations supporting bridge piers should consider all limit states loading conditions applicable to the structure being designed. The following Strength Limit States may control the design and should be investigated:

Strength I Limit State will control for high live to dead load ratios.

Strength III or V will control for structures subjected to high wind loads

A drilled shaft foundation will be evaluated for the following failure conditions:

1. Bearing Resistance – Strength Limit States
2. Lateral Stability – Strength Limit States
3. Shaft Deformations – Service I Limit State
4. Structural Resistance – Strength Limit States

### 1. Bearing Resistance

Bearing resistance is one of the criteria used to determine the depth of the shaft. Bearing resistance mobilizes a combination of side friction and end bearing. Because the side friction mobilizes with smaller settlements than end bearing, the full end bearing is not normally used. The weight of the drilled shaft below ground is usually not included in the reaction. The Geotechnical Foundation Report will provide a chart of factored axial resistance versus depth for various diameter drilled shafts and will state whether to add the weight of the shaft. This check is a strength limit state. For foundations dynamic load allowance is not included. The factored axial loads for the critical reaction follow.

#### Strength I (max) Limit State

$$P_{\max} = 1.25DC + 1.50DW + 0.50PS + 0.50(CR + SH) + 1.75(LL + BR) + 0.50TU + 1.00SE$$

$$P_{\max} = 1.25(1141.5) + 1.50(81.5) + 0.50(-53.6) + 1.75(367.7 + 0.3) + 0.50(0) + 1.00(11.7) = \underline{2178} \text{ k} \leq \text{Critical Limit State}$$

#### Strength III (max) Limit State

$$P_{\max} = 1.25DC + 1.50DW + 0.50PS + 0.50(CR + SH) + 1.40WS + 0.50TU + 1.00SE$$

$$P_{\max} = 1.25(1141.5) + 1.50(81.5) + 0.50(-53.6) + 0.50(0) + 1.40(57.0) + 0.50(0) + 1.00(11.7) = 1614 \text{ k}$$

#### Strength V (max) Limit State

$$P_{\max} = 1.25DC + 1.50DW + 0.50PS + 0.50(CR + SH) + 1.35(LL + BR) + 0.40WS + 1.0WL + 0.50TU + 1.00SE$$

$$P_{\max} = 1.25(1141.5) + 1.50(81.5) + 0.50(-53.6) + 1.35(367.7 + 0.3) + 0.40(57.0) + 1.00(12.4) + 0.50(0) + 1.00(11.7) = 2066 \text{ k}$$

The maximum factored load is a Strength I Limit State with a value of 2178 kips. Figure 25 shows the factored axial resistance chart from Geotechnical Policy Memo 3.

#### [10.8.3.6.3]

The axial resistance chart in Figure 25 provides the resistance for a redundant shaft with spacing greater than 4.0 diameters. For a closer spacing an axial group reduction factor  $\eta$  is applied. The value of  $\eta$  is 0.65 for a spacing of 2.5

diameters and 1.00 for a spacing of 4.0 diameters with the value of  $\eta$  determined by linear interpolation for an intermediate spacing. For a 7.0 foot diameter shaft with a spacing of 24.0 feet the spacing equals  $24.0 / 7.0 = 3.43$  diameters. The resulting value of  $\eta$  is  $0.65 + 0.35(3.43 - 2.5) / (4.0 - 2.5) = 0.867$ . Since the pier consists of two drilled shafts, the pier is classified as redundant and no additional adjustment in the strength load is required. The easiest way to apply the group reduction factor is to increase the factored axial load by dividing by the group reduction factor and using this value to read the required depth from the chart. The modified factored axial load equals  $2178 / 0.867 = 2512$  kips. From Figure 25 the required embedment depth for a 7 foot diameter shaft equals 48 feet.

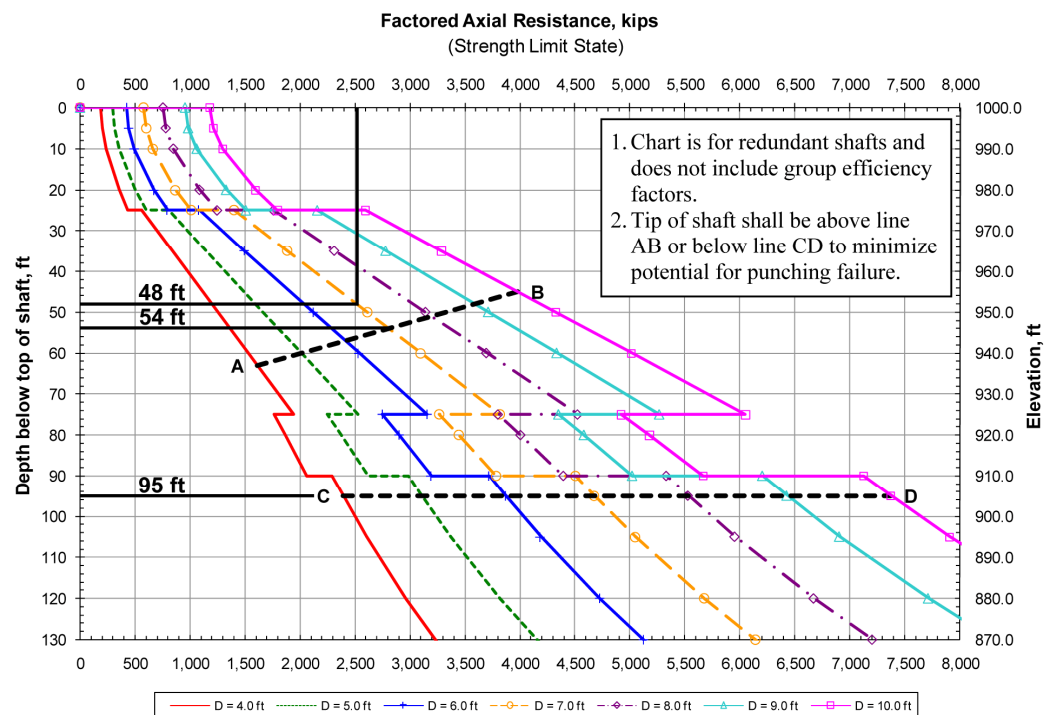


Figure 25

From the above chart a zone from 54 feet deep to 95 feet exists that the drilled shaft tip elevations must avoid. For the factored axial resistance the required depth is outside this zone and is thus satisfactory.

## 2. Lateral Stability

The lateral stability of the shaft must also be investigated as a criteria used to determine the depth of the shaft. Shafts that are embedded too short will have excessive horizontal deflections. While there are approximate methods to estimate this depth, most soil conditions will be complex enough to warrant a lateral analysis by a soil-structure interaction program such as L-Pile. The column shaft combination should be modeled and loaded with a load of the same magnitude that the structure will experience. The deflection should be plotted for various depths of embedment. The shaft depth should be varied until further increases in depth no longer result in reduced deflection. The lateral stability requirement will be satisfied for a shaft embedded this deep. However, careful examination of the deflection versus depth chart may reveal that a reduced embedment may have an increased but acceptable deflection. If the resulting lateral deflection satisfies the engineer's criteria for acceptable deflection of the structure, the reduced embedment depth may be used.

From Figure 26 the shaft is stabilized for an embedment depth of around 700 inches or 58 feet. However, this depth locates the shaft tip within the unacceptable soft region. For an embedment of 54 feet the deflection increases only 5 % which is acceptable. If the deflection were too much the embedment would have to be increased to 95 feet or a larger diameter shaft used.

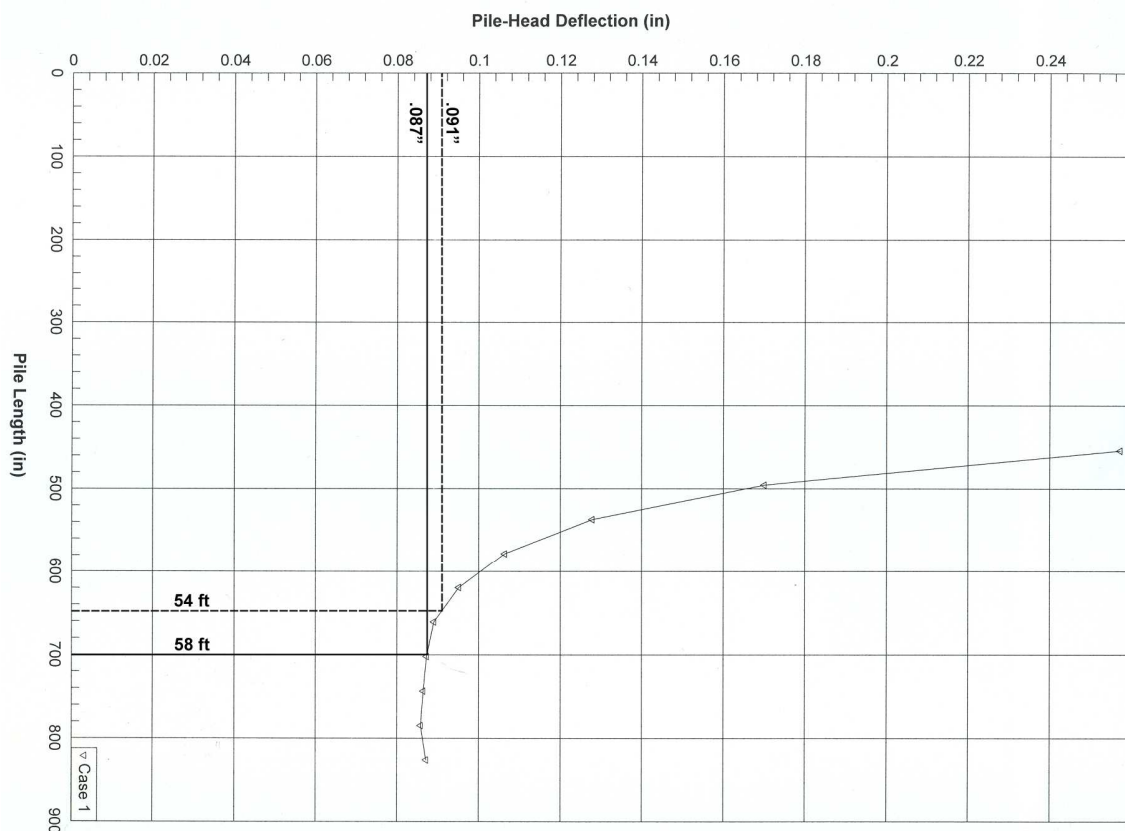


Figure 26

The greater of the embedment depth requirement for bearing resistance (48 feet) and lateral stability (54 feet) should be used to determine the drilled shaft tip elevation. This depth must also be checked for shaft deformations at service limit state.

### 3. Shaft Deformations

Deformation consists of both settlement and lateral displacement and is a service limit state. For a multi-span bridge uniform settlement will not cause structural distress to the bridge but differential settlement can. There are also limits to settlement to ensure a smooth ride. The Geotechnical Policy Memo 3 presents 6 charts for various settlements. Figure 27 presents a vertical load-vertical displacement curve based on a 7 foot diameter, 48 foot long shaft that was determined to satisfy the Strength Limit State (bearing resistance). Figure 27 was developed using the charts and procedures described in Geotechnical Policy Memo 3. Using this chart the bridge engineer will have to determine the amount of vertical settlement the structure can experience under service axial load and whether the bridge will be able to handle the estimated vertical displacement. The service limit state axial load is:

Service I Limit State

$$P_{\max} = 1.00(\text{DC} + \text{DW} + \text{PS} + \text{CR} + \text{SH}) + 1.00(\text{LL} + \text{BR}) \\ + 0.30\text{WS} + 1.00\text{WL} + 1.00\text{TU} + 1.00\text{SE}$$

$$P_{\max} = 1.00(1141.5 + 81.5 - 53.6 + 0) + 1.00(367.7 + 0.3) \\ + 0.30(57.0) + 1.00(12.4) + 1.00(0) + 1.00(11.7) \\ = 1579 \text{ k}$$

The axial resistance chart is for a single shaft or a group of shafts spaced greater than 4.0 diameters. For a closer spacing an axial group reduction factor  $\eta$  is applied. As previously determined  $\eta$  equals 0.867. The modified factored axial load equals  $1579 / 0.867 = 1821$  kips.

Using an axial load of 1821 kips, the total vertical displacement at the shaft top is approximately 0.32 inches as shown in Figure 27. This is considered to be acceptable for the example problem.

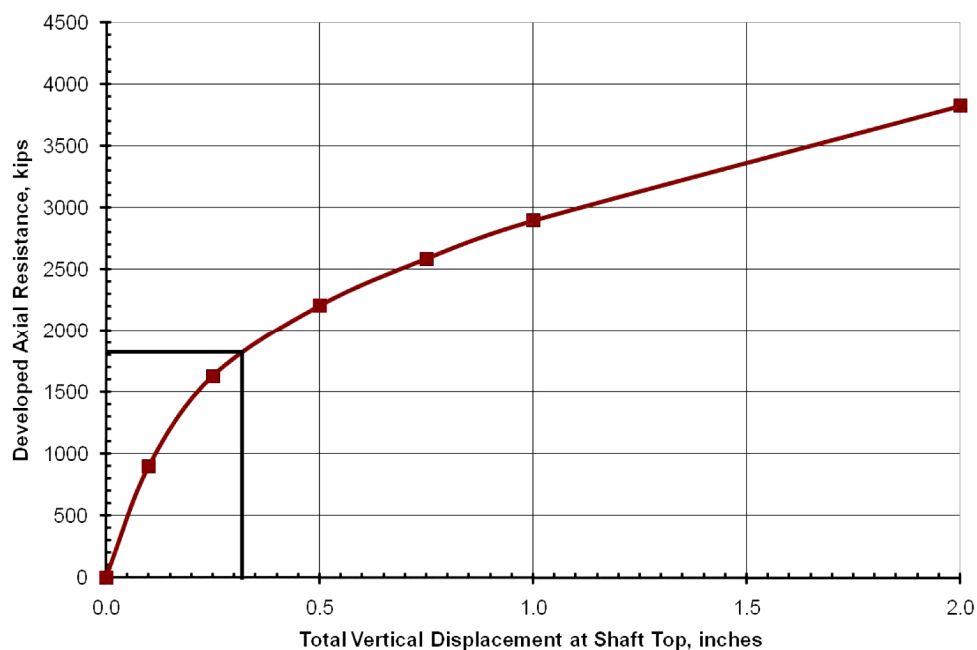


Figure 27

### Horizontal Movement

The bridge engineer must also determine the amount of horizontal movement the structure can accommodate and compare this to the service limit state movement to determine whether changes to the foundation system are required to reduce the movement to an acceptable level.

### Transverse

The externally applied transverse force for Service I Limit State is:

$$V_s = 1.00(LL + BR) + 0.30WS + 1.00WL$$

$$V_s = 1.00(0) + 0.30(34.6) + 1.00(7.3) = 17.7 \text{ kips}$$

The frame will deflect 0.025 inches for this force while the tolerable horizontal deflection is ½ inch.

### [10.7.2.4-1]

The lateral spacing of the shafts in the transverse direction is 3.43 diameters. For shafts spaced less than 5.0 diameters the P-multiplier should be applied to determine the reduced strength of the frame. For row 1, the reduction factor is  $0.70 + 0.30(3.43 - 3.00) / (5.00 - 3.00) = 0.765$ . For row 2, the reduction factor is  $0.50 + 0.35(3.43 - 3.00) / (5.00 - 3.00) = 0.575$ . The transverse frame will deflect ½ inch for a lateral load of 349.10 kips. The frame can only resist  $(349.10 / 2)(0.765 + 0.575) = 233.9$  kips. Since the factored applied load is only 17.7 kips, the pier frame is adequate for horizontal movement in the transverse direction.

**Longitudinal**

For the longitudinal direction the externally applied force on the bridge for Service I Limit State is:

$$V_s = 1.00(LL + BR) + 0.30WS + 1.00WL$$

$$V_s = 1.00(60.3 + 27.4) + 0.30(32.0) + 1.00(5.6) = 102.9 \text{ kips}$$

The frame will deflect 0.166 inches for this force while the tolerable horizontal deflection is ½ inch.

The lateral spacing of the shafts is less than 5 times the diameter requiring consideration of group effects in the longitudinal direction. Since the load is applied normal to the single row of shafts, all the shafts are considered as row 1 shafts. The reduction factor for row 1 is 0.765 as calculated above. The longitudinal frame will deflect ½ inch for a lateral load of 312.7 kips. The frame can only resist  $(312.7)(0.765) = 239.2$  kips to limit the deflection to ½ inch. Since the factored applied load is only 102.9 kips, the longitudinal frame is adequate for horizontal movement in the longitudinal direction.

The horizontal and vertical displacements are acceptable for an embedment of 48 feet. The analysis could be performed again for an embedment depth of 54 feet if the displacements were excessive. However, the additional analysis is not required for this example problem.

**Summary**

The minimum embedment required by bearing resistance is 48 feet. The minimum embedment required by lateral stability is 54 feet. The vertical and horizontal shaft deformations are within acceptable limits for a shaft embedded 48 feet. The required embedment is the maximum of the above three requirements or 54 feet.



#### 4. Structural Resistance

The drilled shaft must satisfy the appropriate strength requirements. The shaft should be analyzed as a member subjected to axial loads and moments similar to a column. Ideally, L-Pile type analysis should be performed to determine the moments and shears along the length of the shaft. As an approximation, the frame analysis based on an equivalent length shaft may be used to estimate the moments at the top of the column/shaft but such an approach will overestimate the moments at the base of the shaft. These values may still be used for design if the design results in no more than one percent longitudinal reinforcing. If the required reinforcement exceeds this amount a more refined analysis using a soil-structure interaction program should be used to determine a reduced maximum moment to be used in design.

The moments and axial loads at the top of the shaft are the same as the moments and axial loads at the bottom of the column and do not have to be recalculated. For this problem the moments at the base of the shaft from the frame analysis are used and are summarized below:

##### Bottom of Shaft – Unfactored Axial Forces and Moments

Load	P <sub>max</sub>	P <sub>min</sub>	M <sub>trans</sub>	M <sub>long</sub>
DC	1304.0	1141.5	53	-283
DW	81.5	0	-6	-23
PS	-53.6	-53.6	4	399
LL 1	275.7		-74	
LL 2	159.4		113	
LL 3	38.9		81	
LL 4		-37.7	-121	
LL 5	238.0		224	
LL Pos *	367.7	-38.2	189	1257
LL Neg *	367.7	-38.2	-165	-1092
BR	0.3	-0.3	0	±826
WS super	54.5	-54.5	±915	±734
WS sub	2.5	-2.5	±67	±103
WS	57.0	-57.0	±982	±837
W vert	0	-108.7	0	0
WL	12.4	-12.4	±208	±170
TU	0	0	±546	0
CR + SH	0	0	-260	0
SE	11.7	-11.7	0	±17

The positive and negative live loads designated with an \* do not include the dynamic load allowance, IM, since the shaft is buried below the ground.

While the weight of the shaft is not used in determine the bearing resistance, the weight should be included for structural design of the shaft. Side friction of the shaft will reduce the magnitude of the load but a detailed analysis of the axial load versus depth is not warranted. The maximum reaction at the point of fixity for the shaft =  $0.15(38.49)(28.14) + 1141.5 = 1304.0$  kips.

The drilled shaft must be designed for strength limit states.

### **STRENGTH I (max)**

#### Bottom of Shaft

$$P_{\max} = 1.25(1304.0) + 1.50(81.5) + 0.50(-53.6) + 1.75(367.7 + 0.3) + 1.0(11.7) = \underline{2381.2} \text{ k} \leq \text{Critical}$$

$$P_{\min} = 0.90(1141.5) + 0.65(0) + 0.50(-53.6) + 1.75(-38.2 - 0.3) + 1.0(-11.7) = \underline{921.5} \text{ k}$$

$$M_{\text{trans}} = 1.25(53) + 0.65(0) + 0.50(4) + 0.50(0) + 1.75(189 + 0) + 0.50(546) = \underline{672} \text{ ft-k}$$

$$M_{\text{trans}} = 0.90(53) + 1.50(-6) + 0.50(4) + 0.50(-260) + 1.75(-165 + 0) + 0.50(-546) = -651 \text{ ft-k}$$

$$M_{\text{long}} = 0.90(-283) + 0.65(0) + 0.50(399) + 1.75(1257 + 826) + 1.00(17) = \underline{3608} \text{ ft-k} \leq \text{Critical}$$

$$M_{\text{long}} = 1.25(-283) + 1.50(-23) + 0.50(399) + 1.75(-1092 - 826) + 1.00(-17) = -3562 \text{ ft-k}$$

### **STRENGTH III (max)**

#### Bottom of Shaft

$$P_{\max} = 1.25(1304.0) + 1.50(81.5) + 0.50(-53.6) + 1.40(57.0) + 1.00(11.7) = \underline{1817.0} \text{ k}$$

$$P_{\min} = 0.90(1141.5) + 0.65(0) + 0.50(-53.6) + 1.40(-57.0 - 108.7) + 1.00(-11.7) = \underline{756.9} \text{ k} \leq \text{Critical}$$

$$M_{\text{trans}} = 1.25(53) + 0.65(0) + 0.50(4) + 0.50(0) + 1.40(982) + 0.50(546) = 1716 \text{ ft-k}$$

$$M_{\text{trans}} = 0.90(53) + 1.50(-6) + 0.50(4) + 0.50(-260) + 1.40(-982) + 0.50(-546) = \underline{-1736} \text{ ft-k}$$

$$M_{\text{long}} = 0.90(-283) + 0.65(0) + 0.50(399) + 1.40(837) + 1.00(17) = 1134 \text{ ft-k}$$

$$M_{\text{long}} = 1.25(-283) + 1.50(-23) + 0.50(399) + 1.40(-837) + 1.00(-17) = \underline{-1378} \text{ ft-k}$$

**STRENGTH V (max)**Bottom of Shaft

$$P_{\max} = 1.25(1304.0) + 1.50(81.5) + 0.50(-53.6) + 1.35(367.7 + 0.3) + 0.40(57.0) + 1.00(12.4) + 1.00(11.7) = \underline{2269.2 \text{ k}}$$

$$P_{\min} = 0.90(1141.5) + 0.65(0) + 0.50(-53.6) + 1.35(-38.2 - 0.3) + 0.40(-57.0) + 1.00(-12.4) + 1.00(-11.7) = \underline{901.7 \text{ k}}$$

$$M_{\text{trans}} = 1.25(53) + 0.65(0) + 0.50(4) + 0.50(0) + 1.35(189 + 0) + 0.40(982) + 1.00(208) + 0.50(546) = \underline{1197 \text{ ft-k}}$$

$$M_{\text{trans}} = 0.90(53) + 1.50(-6) + 0.50(4) + 0.50(-260) + 1.35(-165 - 0) + 0.40(-982) + 1.00(-208) + 0.50(-546) = -1186 \text{ ft-k}$$

$$M_{\text{long}} = 0.90(-283) + 0.65(0) + 0.50(399) + 1.35(1257 + 826) + 0.40(837) + 1.00(170) + 1.00(17) = 3279 \text{ ft-k}$$

$$M_{\text{long}} = 1.25(-283) + 1.50(-23) + 0.50(399) + 1.35(-1092 - 826) + 0.40(-837) + 1.00(-170) + 1.00(-17) = \underline{-3300 \text{ ft-k}}$$

The effects of slenderness must also be considered in the shaft design. Using the maximum moment magnifiers previously calculated for the column for each direction yields the following conservative design moments.

**Strength I Limit State**

$$\text{Trans} \quad M_u = (1.060)(672) = 712 \text{ ft-k}$$

$$\text{Long} \quad M_u = (1.053)(3608) = 3799 \text{ ft-k}$$

$$\text{Resultant: } M_u = \sqrt{(712)^2 + (3799)^2} = 3865 \text{ ft-k} \leq \text{Critical}$$

**Strength III Limit State**

$$\text{Trans} \quad M_u = (1.041)(1736) = 1807 \text{ ft-k}$$

$$\text{Long} \quad M_u = (1.047)(1378) = 1443 \text{ ft-k}$$

$$\text{Resultant: } M_u = \sqrt{(1807)^2 + (1443)^2} = 2312 \text{ ft-k}$$

**Strength V Limit State**

$$\text{Trans} \quad M_u = (1.053)(1197) = 1260 \text{ ft-k}$$

$$\text{Long} \quad M_u = (1.050)(3300) = 3465 \text{ ft-k}$$

$$\text{Resultant: } M_u = \sqrt{(1260)^2 + (3465)^2} = 3687 \text{ ft-k}$$

The drilled shaft has a minimum ratio of factored resistance to factored load of 2.58 meaning the drilled shaft is adequately reinforced with 1 percent reinforcing using 36 # 11 bars as shown in Figure 28 below.

The drilled shaft is larger than required by service and strength requirements. However, the large shaft is required due to the architectural rectangular shape of the column and the need to develop the column reinforcing into the shaft.

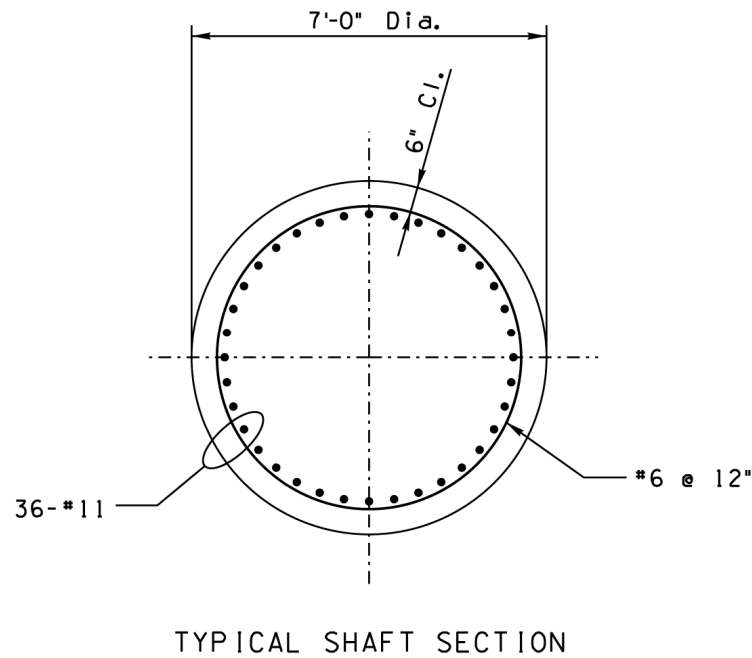


Figure 28

In an actual design the shear strength of the shaft would be evaluated. For this problem due to the small factored shear values compared to the large shear resistance of the column section, the shear in the shaft will not be investigated. The horizontal reinforcing will consist of #6 ties at 12 inches mainly to provide stiffness to the cage for constructability.