Hammerhead Pier On Spread Footing

This example illustrates the substructure design of a hammerhead pier cap with a single column supported on a spread footing on rock for a three span precast prestressed box beam bridge. The bridge has spans of 85'-3", 86'-6" and 85'-3" resulting in equal lengths of the modified AASHTO BII-48 box beam in all spans. The bridge has zero skew. Standard ADOT 32-inch f-shape barriers will be used resulting in a bridge configuration of 1'-5" barrier, 12'-0" outside shoulder, one 12'-0" lane, a 4'-0" inside shoulder and a 1'-5" barrier. The overall out-to-out width of the bridge is 30'-10". A plan view and typical section of the bridge are shown in Figures 1 and 2.

The following legend is used for the references shown in the left-hand column:

[2.2.2]	LRFD Specification Article Number
[2.2.2-1]	LRFD Specification Table or Equation Number
[C2.2.2]	LRFD Specification Commentary
[A2.2.2]	LRFD Specification Appendix
[BDG]	ADOT Bridge Design Guideline

Superstructure

Design Example 3 demonstrates the basic design features for the design of the superstructure for a three span precast prestressed box beam bridge using LRFD. Critical dimensions and loads are repeated here for ease of reference.

Bridge Geometry

Span lengths	85.25, 86.50, 85.25 ft
Bridge width	30.83 ft
Roadway width	28.00 ft
Superstructure depth	3.17 ft
Box beam moment of inertia	171,153 in^4

Loads

DC Superstructure	816.3 kips
DW Superstructure	66.2 kips

Substructure

This example demonstrates basic design features for design of a hammerhead pier supported on a spread footing on rock. The substructure has been analyzed in accordance with the AASHTO LRFD Bridge Design Specifications, 4th Edition, 2007 and the 2008 Interim Revisions.

Geotechnical

The spread footing is designed to be supported on non erodible rock found at a depth of 6 feet below existing groundline. Per the geotechnical engineer, the rock is sound (Class I) with a rock mass rating (RMR) of 85. The bearing resistance of the rock mass for service limit state is 10 ksf corresponding to a tolerable settlement of ½ inch while the factored bearing resistance for strength limit state is 60 ksf. For sliding resistance assume a factored interface coefficient of friction value of 0.45 and the sliding resistance factor of 0.80.

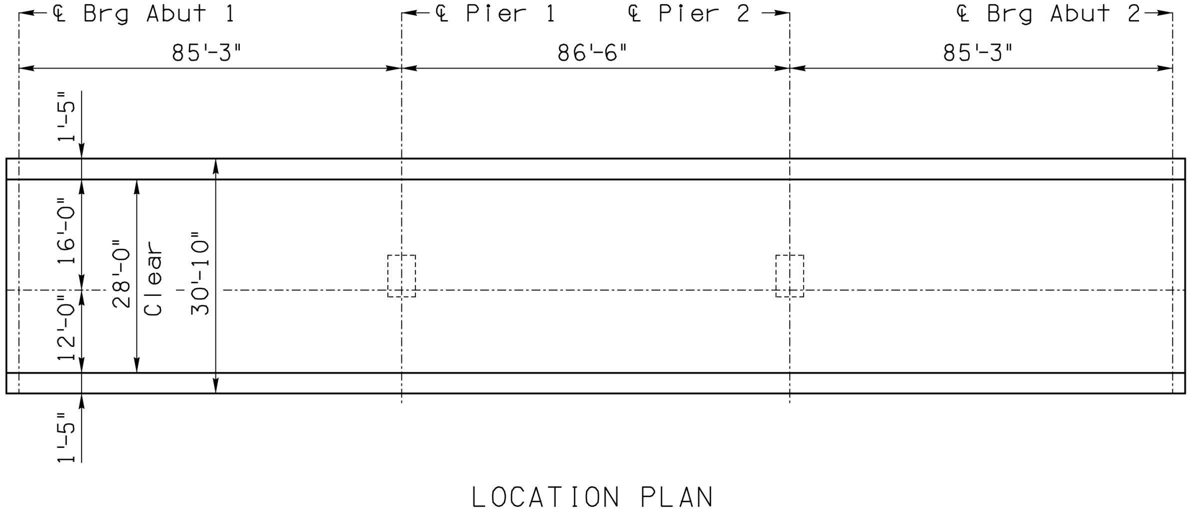


Figure 1

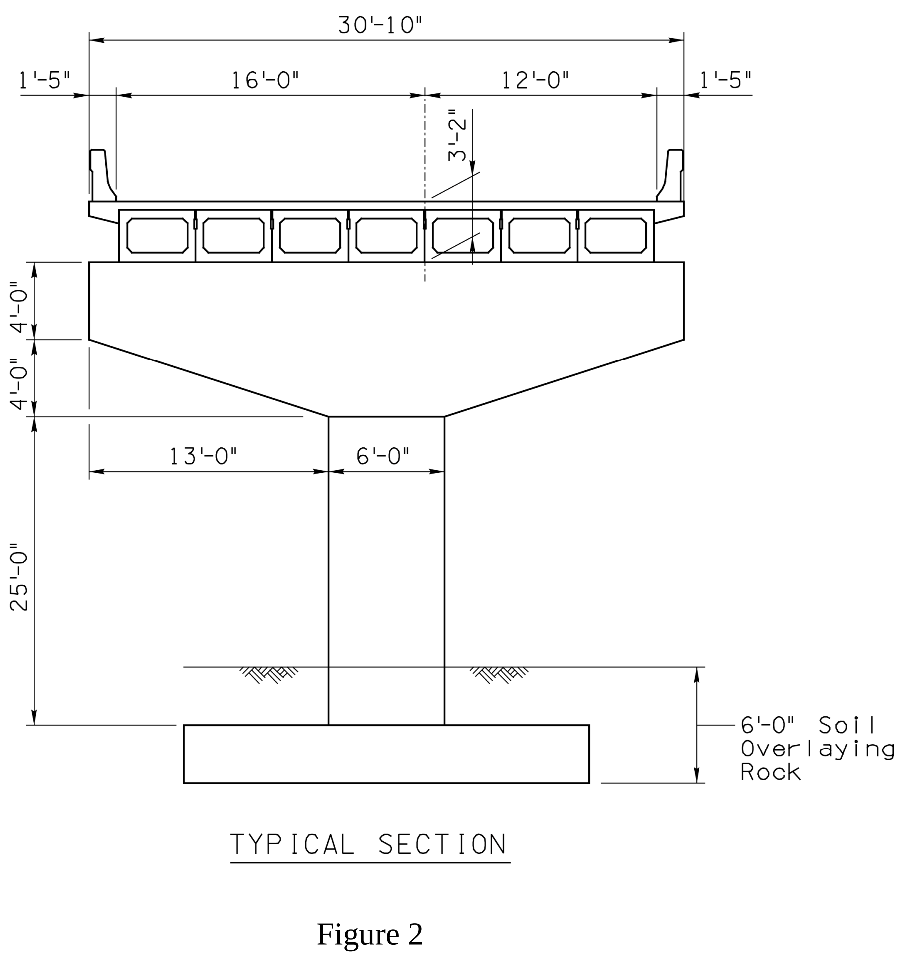


Figure 2

Material Properties

Reinforcing Steel

Yield Strength f_y = 60 ksi

[5.4.3.2] Modulus of Elasticity E_s = 29,000 ksi

Concrete

Precast Box Beam	Pier & Footing
f'_c = 5.0 ksi	f'_c = 3.5 ksi

[3.5.1-1] The unit weight for reinforced concrete is 0.005 kcf greater than plain concrete. The unit weight for normal weight concrete is listed below:

Unit weight for computing E_c = 0.145 kcf
Unit weight for DL calculation = 0.150 kcf

[C5.4.2.4] The modulus of elasticity for normal weight concrete where the unit weight is 0.145 kcf may be taken as shown below:

Precast Box Beam:

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{5.0} = 4070\ ksi$$

Pier and Footing:

$$E_c = 1820\sqrt{f'_c} = 1820\sqrt{3.5} = 3405\ ksi$$

[5.7.1] The modular ratio of reinforcing to concrete should be rounded to the nearest whole number.

Pier and Footing:

$$n = \frac{29{,}000}{3405} = 8.52$$ Use n = 9 for Pier and Footing

[5.7.2.2] β_1 = the ratio of the depth of the equivalent uniformly stressed compression zone assumed in the strength limit state to the depth of the actual compression zone stress block.

Pier and Footing

β_1 = 0.85

Modulus of Rupture [5.4.2.6]

The modulus of rupture for normal weight concrete has several values. When used to calculate service level cracking, as specified in Article 5.7.3.4 for side reinforcing or in Article 5.7.3.6.2 for determination of deflections, the following equation should be used for the substructure:

$$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{3.5} = 0.449\ ksi$$

When the modulus of rupture is used to calculate the cracking moment of a member for determination of the minimum reinforcing requirement as specified in Article 5.7.3.3.2, the following equation should be used for the substructure:

$$f_r = 0.37\sqrt{f'_c} = 0.37\sqrt{3.5} = 0.692\ ksi$$

Soil Properties

The soil above the rock has the following properties:

Depth	6.00 feet
γ_s	0.110 kcf

The rock layer has the following properties:

Bearing Resistance	10.0 ksf service with ½ inch settlement 60.0 ksf strength
Sliding Resistance	0.45 coefficient of friction 0.80 sliding resistance factor

Limit States
[1.3.2]
[1.3.2.1-1]

In the LRFD Specification, the general equation for design is shown below:

$$\sum \eta_i \gamma_i Q_i \leq \varphi R_n = R_r$$

For loads for which a maximum value of γ_i is appropriate:

[1.3.2.1-2]

$$\eta_i = \eta_D \eta_R \eta_I \geq 0.95$$

For loads for which a minimum value of γ_i is appropriate:

[1.3.2.1-3]

$$\eta_i = \frac{1}{\eta_D \eta_R \eta_I} \leq 1.0$$

[1.3.3]

Ductility
For strength limit state for conventional design and details complying with the LRFD Specifications and for all other limit states:

$$\eta_D = 1.0$$

[1.3.4]

Redundancy
For the strength limit state for conventional levels of redundancy and for all other limit states:

$$\eta_R = 1.0$$

[1.3.5]

Operational Importance
For the strength limit state for typical bridges and for all other limit states:

$$\eta_I = 1.0$$

For an ordinary structure with conventional design and details and conventional levels of ductility, redundancy, and operational importance, it can be seen that η_i = 1.0 for all cases. Since multiplying by 1.0 will not change any answers, the load modifier η_i has not been included in this example.

[BDG]

For actual designs, the importance factor may be a value other than one. The importance factor should be selected in accordance with the ADOT Bridge Design Guidelines.

SUBSTRUCTURE
Loads
Section 3

Loads
There are several major changes and some minor changes concerning the determination of loads. The permanent loads must be kept separate since different load factors apply to different load types for different conditions. The live load is new as seen in the superstructure design. The dynamic load allowance is a constant rather than a function of the span. The Longitudinal Force in the Standard Specifications has been modified and replaced by the Braking Force. The wind and wind on live load are similar but the wind has a modification factor for elevations above 30 feet. While the vertical wind pressure is the same, the specification now clarifies how to apply the force to the proper limit state.

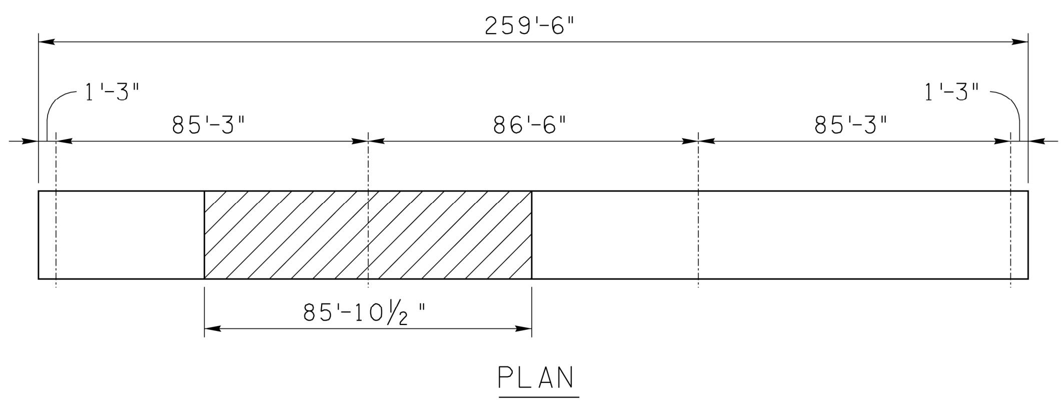


Figure 3

For this 3-span bridge, both abutments are expansion with the two piers resisting the longitudinal forces. The distribution of transverse forces is a complex problem involving the flexibility of the abutments, piers and the superstructure. For design of the pier, a conservative assumption is to design for the contributory area of 85.88 feet for transverse forces as shown in Figure 3.

[10.8.2]

Limit States
For substructure design, foundation design at the service limit state includes settlement and lateral displacement.

[10.8.3]

Foundation design at the strength limit state includes bearing resistance, sliding and overturning stability and structural resistance. For substructure design, three strength limit states require investigation. Strength I is the basic load combination without wind. Strength III is the load combination including wind exceeding 55 mph. Strength V is the load combination combining normal vehicular use with a wind of 55 mph. Strength II and IV are not used since Strength II is the limit state that includes permit overloads and Strength IV is for bridge with very large dead load to live load ratios consistent with long span bridges.

A diagram showing the general dimensions for the pier follows:

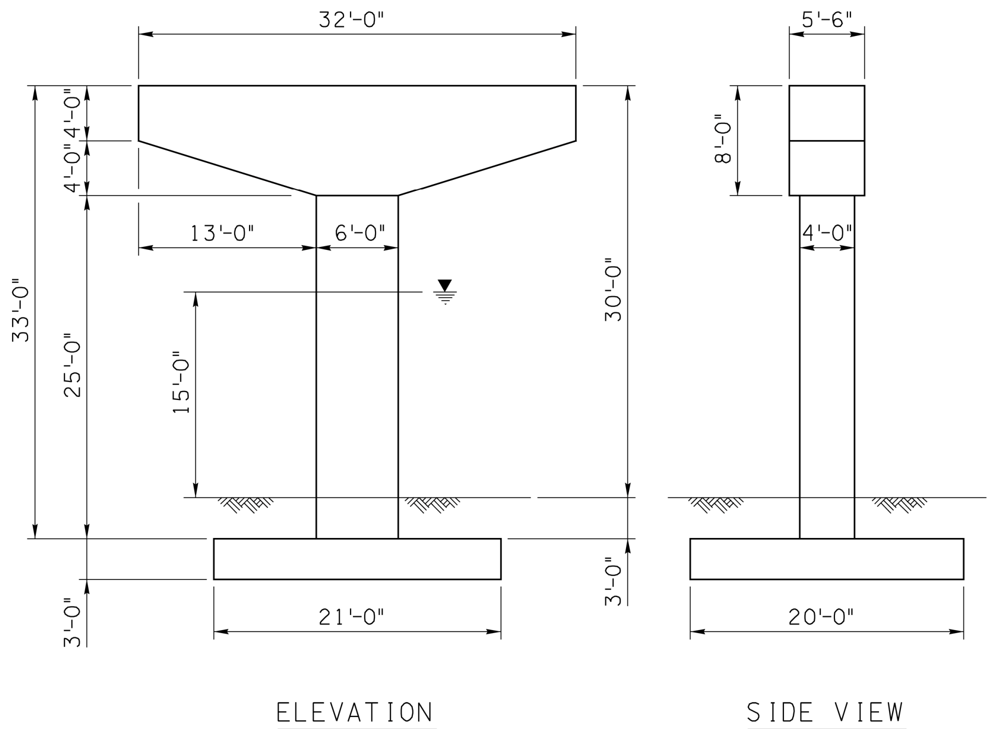


Figure 4

[3.5] **PERMANENT LOADS**

[3.5.1] **DC – Dead Load Structural Components**

The self-weight of the precast box beams and the weight from all other non-composite dead loads are based on simple span reaactions. The dead load pier reactions per box beam are:

Box Beam	0.798(85.50)	=	68.23 k
Int Diaphragms	0.821(3)	=	2.46 k
End Diaphragms	0.821(18.00 / 12.00)(2)	=	2.46 k
Non-Comp DL	0.373(84.00 + 2.50)	=	32.26 k
CIP Pier Diaph	0.150(2.75)(4.00)(1.0)	=	1.65 k
	Total	=	107.06 k

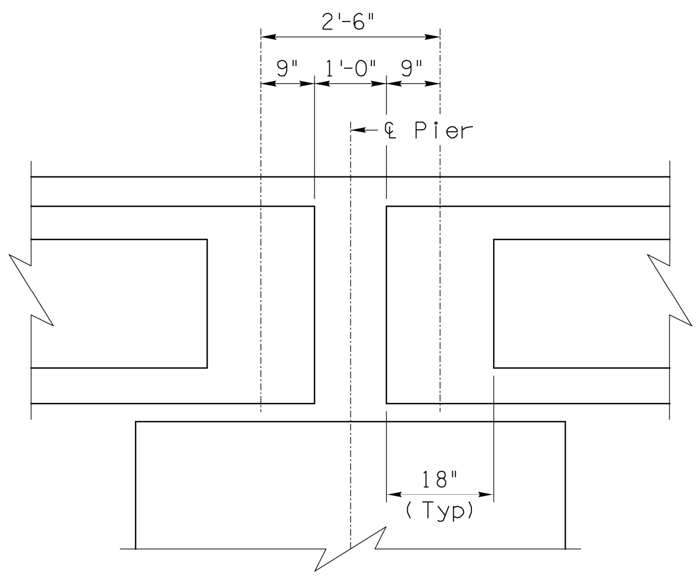


Figure 5

The dead load reaction from the barriers based on a continuous beam analysis is 9.55 kips per box beam assuming the load is equally distributed to all the beams.

DC superstructure = 107.06 + 9.55 = 116.61 kips per box beam.
Total DC superstructure = (7 beams)(116.61) = 816.3 k

DC Pier

DC pier cap = 0.150[(5.50)(32.00)(8.00) – 0.50(2)(5.50)(13.00)(4.00)]
= 168.3 k
DC column = 0.150[(4.00)(6.00)(25.00)] = 90.0 k

DC = 816.3 + 168.3 + 90.0 = 1074.6 k

DC Footing

DC footing = 0.150[(21.00)(20.00)(3.00)] = 189.0 k

DC = 1074.6 + 189.0 = 1263.6 k

DW – Dead Load Wearing Surfaces and Utilities

The DW load includes the future wearing surface and utility loads. This bridge has no utilities. The reaction from the future wearing surface based on a continuous beam analysis is 9.46 kips per box beam assuming the load is equally distributed to all beams.

DW = (7 beams)(9.46) = 66.22 k

EV – Vertical Earth Pressure

The LRFD Specification does not provide data on unit weights of well compacted soils. For this example use a vertical earth pressure based on a unit weight of 0.110 kcf for the soil placed over the footing. In actual design use the values specified in the Geotechnical Report.

EV = (0.110)[(21.00)(20.00) – (6.00)(4.00)](3.00) = 130.68 k

TRANSIENT LOADS

[3.6.1.1]

LL – Vehicular Live Load

[3.6.1.1.1]

The number of design lanes is the integer part of the traffic opening divided by 12. For this bridge the number of design lanes equals 28 ÷ 12 = 2. For the reaction apply the multiple presence factor, m.

The maximum pier reaction from live load for the HL93 equals 89.36 k (design truck) plus 59.32 k (design lane) = 148.68 kips per vehicle. For each wheel line the reaction is half that amount or 74.34 kips. The maximum uplift from live load equals -10.0 k (design truck) plus -5.2 k (design lane) = –15.2 kips per vehicle. While several combinations of possible live load locations exist, engineering judgment and experience can reduce the number to a few critical locations as follows:

Live Load 1
This live load is located 2 feet from the left side barrier producing maximum moments and shears on the pier cap overhang and maximum moments on the column and footing in one direction.

Live Load 2
This live load is located 6 feet from Live load 1 adding to the axial load but reducing the moment in the column and footing.

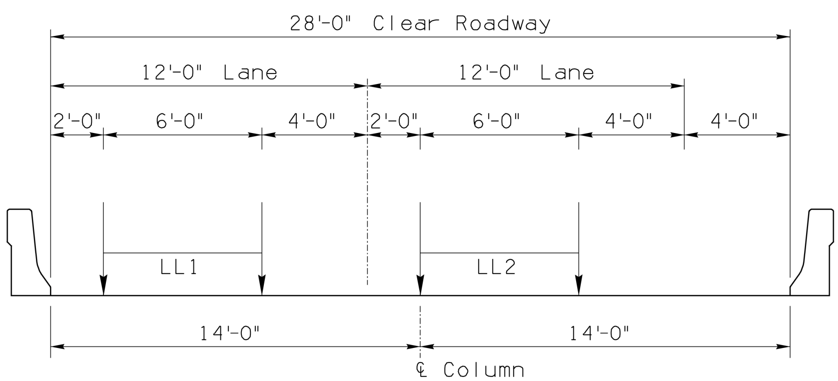


Figure 6

LL 1 M_{trans} = 74.34(12.00 + 6.00) = 1338 ft-k
LL 2 M_{trans} = 74.34(0.00 – 6.00) = -446 ft-k

[3.6.2] **IM – Dynamic Load Allowance**

[3.6.2.1-1] Dynamic load allowance equal to 33% is applied to the design truck but not the design lane load for the design of the pier cap and column. The dynamic load allowance is 0.33(89.36) = 29.49 kips per vehicle. The dynamic load allowance for uplift is 0.33(-10.0) = –3.3 kips per vehicle. The dynamic load allowance is not included for design of the footing.

Live load and Dynamic Load Allowance

For the superstructure design, the combination of the live load plus dynamic load allowance yields a load of (148.68 + 29.49) = 178.17 kips per vehicle or 89.09 kips per wheel. The uplift reaction is (-15.2 – 3.3) = -18.5 kips.

LL + IM 1 M_{trans} = 89.09(12.00 + 6.00) = 1604 ft-k
LL + IM 2 M_{trans} = 89.09(0.00 – 6.00) = -535 ft-k

[3.6.1.1.2] **Multiple Presence Factor**

[3.6.1.1.2-1] Since the critical moment and shear in the pier overhang is the result of a single vehicle the multiple presence factor of 1.20 must be used. When both live loads are used the multiple presence factor equals 1.00.

The critical axial force and moment from live load including the multiple presence factor is:

P_{max} = (148.68)(2 vehicles)(1.0) = 297.4 k
P_{min} = (-15.2)(2 vehicles)(1.0) = -30.4 k
M_{trans} = (1338)(1 vehicle)(1.20) = 1606 ft-k

The critical axial force and moment from live load plus dynamic load allowance including the multiple presence factor is:

P_{max} = (148.68 + 29.49)(2 vehicles)(1.0) = 356.4 k
P_{min} = (-15.2 – 3.3)(2 vehicles)(1.0) = -37.0 k
M_{trans} = (1604)(1 vehicles)(1.20) = 1925 ft-k

[3.6.4]

BR – Vehicular Braking Force

The braking force shall be taken as the greater of:

25 percent of the axle weights of the design truck or design tandem

V = (0.25)(32 + 32 + 8) = 18.00 k <= Critical
V = (0.25)(25 + 25) = 12.50 k

5 percent of the design truck plus lane load or 5 percent of the design tandem plus lane load

V = (0.05)[32 + 32 + 8 + (256.00)(0.640)] = 11.79 k
V = (0.05)[25 + 25 + (256.00)(0.640)] = 10.69 k

The braking force shall be placed in all design lanes that are considered to be loaded which carry traffic in the same direction. The bridge is a one directional structure with both design lanes headed in the same direction. The multiple presence factors shall apply.

BR = (18.00)(2 lanes)(1.00) = 36.00 k

This load is to be applied 6 feet above the deck surface. However, since the structure is pinned in the longitudinal direction and no moment can be transmitted through the connection, the load will be transmitted at the bearing elevation. Shears and moments at the base of the column and footing follow:

V_{long} = (36.00)÷(2 piers) = 18.00 k
V_{trans} = 0 kips

Bottom Column	Bottom Footing
M_{long} = (18.00)(33.00) = 594 ft-k	M_{long} = (18.00)(36.00) = 648 ft-k
M_{trans} = 0 ft-k	M_{trans} = 0 ft-k

[3.8]

WS – Wind Load on Structure

Wind pressures are based on a base design wind velocity of 100 mph. For structures with heights over 30 feet above the groundline, a formula is available to adjust the wind velocity. The wind is assumed to act uniformly on the area exposed to the wind. The exposed area is the sum of the areas of all components as seen in elevation taken perpendicular to the assumed wind direction.

Long Area = [3.17 + 2.67 + 0.02(30.83)][85.25(2) + 86.50] / 2 = 830 ft^2
Trans Area = [3.17 + 2.67 + 0.02(30.83)][85.25 + 86.50] / 2 = 554 ft^2

For this problem the wind sees a superstructure depth of 6.46 feet. Since the soffit is 30 feet above the ground, the wind velocity and corresponding superstructure pressure should be adjusted based on the average superstructure height above the ground equal to 30.00 + 6.46 / 2 = 33.23 feet.

The design wind velocity is:

$$V_{DZ} = 2.5V_o\left(\frac{V_{30}}{V_B}\right)\ln\left(\frac{Z}{Z_0}\right)$$

For a structure in open country the following variables are used.

V_0 = 8.20
Z_0 = 0.23

In the absence of better criteria assume that V_{30} = V_B = 100 mph.

$$V_{DZ} = (2.5)\cdot(8.20)\cdot\left(\frac{100}{100}\right)\ln\left(\frac{33.23}{0.23}\right) = 102 \text{ mph}$$

The design wind pressure may be determined as:

$$P_D = P_B\frac{V_{DZ}^{\ 2}}{10{,}000} = P_B\frac{(102)^2}{10{,}000} = 1.04P_B$$

Wind on Superstructure [3.8.1.2.2]

The base pressure for girder bridges corresponding to the 100 mph wind is 0.050 psf. The minimum wind loading shall not be less than 0.30 klf. Since the wind exposure depth of 6.46 feet is greater than 6 feet this criteria is satisfied. The AASHTO LRFD 2008 Interim Revision provided an approximate method for girder bridges with spans 125 feet or less and a maximum height of 30 feet above ground or water level. The spans are less than the 125 specified but the height of the bridge exceeds the 30 feet limit. Therefore, the general method will be used

0 Degree Skew Angle
V_{long} = (830)(0.000)(1.04) = 0.00 k
V_{trans} = (554)(0.050)(1.04) = <u>28.81</u> k

15 Degree Skew Angle
V_{long} = (830)(0.006)(1.04) = 5.18 k
V_{trans} = (554)(0.044)(1.04) = 25.35 k

30 Degree Skew Angle
V_{long} = (830)(0.012)(1.04) = 10.36 k
V_{trans} = (554)(0.041)(1.04) = 23.62 k

45 Degree Skew Angle

V_{long} = (830)(0.016)(1.04) = 13.81 k

V_{trans} = (554)(0.033)(1.04) = 19.01 k

60 Degree Skew Angle

V_{long} = (830)(0.019)(1.04) = 16.40 k

V_{trans} = (554)(0.017)(1.04) = 9.79 k

While the criteria for use of the simplified method for wind design for ordinary bridges is not satisfied, a conservative answer can be achieved by using the maximum values from the above. If wind controls the design, the complexities of combining 5 wind combinations should be performed. A summary of wind forces used in the design follows:

V_{long} = 16.40 k

V_{trans} = 28.81 k

Bottom Column

M_{long} = (16.40)(33.00) = 541 ft-k

M_{trans} = (28.81)(33.00 + 6.46 / 2) = 1044 ft-k

Bottom Footing

M_{long} = (16.40)(36.00) = 590 ft-k

M_{trans} = (28.81)(36.00 + 6.46 / 2) = 1130 ft-k

Wind on Substructure [3.8.1.2.3]

The transverse and longitudinal forces to be applied directly to the substructure are calculated from an assumed base wind pressure of 0.040 ksf.

V_{long} = 0.040[(32.00)(4.00)+(0.5)(2)(13.00)(4.00)+(6.00)(26.00)]
= 13.44 k

V_{trans} = 0.040[(5.50)(8.00) + (4.00)(22.00)] = 5.28 k

Bottom Column

M_{long} = 0.040[(32.00)(4.00)(31.00) + (0.5)(2)(13.00)(4.00)(27.67)
+ (6.00)(26.00)(16.00)] = 316 ft-k

M_{trans} = 0.040[(5.50)(8.00)(29.00) + (4.00)(22.00)(14.00)] = 100 ft-k

Bottom Footing

M_{long} = 0.040[(32.00)(4.00)(34.00) + (0.5)(2)(13.00)(4.00)(30.67)
+ (6.00)(26.00)(19.00)] = 356 ft-k

M_{trans} = 0.040[(5.50)(8.00)(32.00) + (4.00)(22.00)(17.00)] = 116 ft-k

[3.8.1.3]

WL – Wind Pressure on Vehicles

Wind pressure on vehicles is represented by a moving force of 0.10 klf acting normal to and 6.0 feet above the roadway. The wind pressure on live loads is not increased for height above the ground. Loads normal to the span should be applied at a height of 3.17 + 6.00 = 9.17 feet above the soffit.

[3.8.1.3-1]

0 Degree Skew Angle

$V_{long} = 128.50(0.000) = 0.00$ k

$V_{trans} = 85.88(0.100) = \underline{8.59}$ k

15 Degree Skew Angle

$V_{long} = 128.50(0.012) = 1.54$ k

$V_{trans} = 85.88(0.088) = 7.56$ k

30 Degree Skew Angle

$V_{long} = 128.50(0.024) = 3.08$ k

$V_{trans} = 85.88(0.082) = 7.04$ k

45 Degree Skew Angle

$V_{long} = 128.50(0.032) = 4.11$ k

$V_{trans} = 85.88(0.066) = 5.67$ k

60 Degree Skew Angle

$V_{long} = 128.50(0.038) = \underline{4.88}$ k

$V_{trans} = 85.88(0.034) = 2.92$ k

As with the wind load, a conservative answer for wind on live load can be achieved by using the maximum values from the above. If wind controls the design, the complexities of combining 5 wind directions should be performed.

$V_{long} = 4.88$ k

$V_{trans} = 8.59$ k

<u>Bottom Column</u>

$M_{long} = (4.88)(33.00) = 161$ ft-k

$M_{trans} = (8.59)(33.00 + 9.17) = 362$ ft-k

<u>Bottom Footing</u>

$M_{long} = (4.88)(36.00) = 176$ ft-k

$M_{trans} = (8.59)(36.00 + 9.17) = 388$ ft-k

Vertical Wind Pressure

A vertical upward wind force of 0.020 ksf times the width of the deck shall be applied at the windward quarterpoint of the deck. This load is only applied for limit states which do not include wind on live load (Strength III Limit State) and only when the direction of wind is taken to be perpendicular to the longitudinal axis of the bridge. When applicable the wind loads are as shown:

P = (0.020)(30.83)(85.88) = -52.95 upward
M_{trans} = (52.95)(30.83 / 4) = 408 ft-k

TU, SH, CR – Superimposed Deformations

Uniform Temperature

[BDG]

Internal force effects that cause the pier column to expand and contract in the longitudinal direction shall be considered in the design. For temperature deformations the member movement is taken to include the entire temperature range rather than the rise or fall from the mean temperature as was the case in the past. The Bridge Design Guidelines provide the appropriate temperature range to use. For this low elevation site the design temperature range is 70 degrees.

TU = 0.000006(70)(43.25)(12) = 0.2180 in

$I_g = (6.00)(4.00)^3 \div 12 = 32.00\ \text{ft}^4$

For a pier column fixed at the base but free to rotate at the top:

$$V_{long} = \frac{3EI\Delta}{l^3} = \frac{3\,(3405)\cdot(144)\cdot(32.00)\cdot(0.2180 \div 12)}{(33.00)^3} = 23.79\,\text{k}$$

M_{long} = (23.79)(33.00) = 785 ft-k (Bottom Column)
M_{long} = (23.79)(36.00) = 856 ft-k (Bottom Footing)

Creep & Shrinkage [BDG]

The creep and shrinkage of the prestressed superstructure member can be estimated as 0.60 inch per 100 feet. The shear and moment can be obtained by prorating the uniform temperature moments and shears as follows:

PS Shortening = (43.25)(0.60 / 100) = 0.2595 in

V_{long} = (23.79)(0.2595) / (0.2180) = 28.32 k

M_{long} = (28.32)(33.00) = 935 ft-k (Bottom Column)
M_{long} = (28.32)(36.00) = 1020 ft-k (Bottom Footing)

SE - Differential Settlement

Differential Settlement will be a minor load for this bridge. The foundation is supported on rock and the differential settlement will only apply for loads applied after the deck is made continuous (composite dead loads consisting of the barriers and the future wearing surface). Geotechnical information is usually not available early in the design, therefore the bridge engineer should determine a conservative amount of settlement that the bridge can sustain to eliminate the need for redesign later. For this structure most of the settlement will take place prior to the spans being made continuous. A very conservative assumption would be to assume that the structure may see ¼ inch settlement. The following fixed end moments result.

Spans 1 and 3:

$$FEM = \frac{6EI\Delta}{L^2} = \frac{6 \cdot (4070) \cdot (171{,}153) \cdot (7\,beams) \cdot (0.25)}{[(12) \cdot (85.25)]^2} \div 12 = 582 \text{ ft-k}$$

Span 2:

$$FEM = \frac{6EI\Delta}{L^2} = \frac{6 \cdot (4070) \cdot (171{,}153) \cdot (7\,beams) \cdot (0.25)}{[(12) \cdot (86.50)]^2} \div 12 = 566 \text{ ft-k}$$

The above fixed end moments were input into a continuous beam program to yield the corresponding output. Since the top of the pier and abutments are pinned the horizontal shear and moments equal zero for the substructures. The maximum reaction at Pier 1 is caused by settlement of Abutment 1 and Pier 2 only with the maximum reaction equal to 13.3 kips. The maximum uplift at Pier 1 is caused by settlement of Pier 1 and Abutment 2 only with the minimum reaction equal to -13.3 kips.

Bridge Group Design Guidelines for foundations have not been finalized at this time. Refer to these guidelines for clarification on this issue.

[3.7]

WA - Stream Force

For the design event, the Bridge Hydraulics Report has provided the following data:

Depth of flow	15.0 feet
Velocity	9.0 ft/sec
Scour depth	15.0 feet (theoretical calculated scour)

The Geotechnical Foundation Report has described the soil layer 6 feet below the groundline as a non-erodible rock. Therefore the design scour depth is 6 feet instead of the 15 feet calculated scour reported in the Drainage Report.

The stream is assumed to be directed at an angle of attack of 15 degrees from the theoretical normal direction. Debris is assumed to accumulate on the pier for the upper 12 feet. The LRFD Specification uses the term longitudinal to mean along the direction of the stream flow which is in the transverse direction of the bridge creating possible confusion. The longitudinal (transverse bridge direction) uniformly distributed pressure from stream flow may be taken as:

[3.7.3.1-1]

$$p = \frac{C_D V^2}{1000} = \frac{(1.4)\cdot(9.0)^2}{1000} = 0.113\,\text{ksf}$$

where

C_D = drag coefficient taken from Table 3.7.3.1-1.
= 1.4 for square-ended piers or for lodged debris

V = design velocity of water for the design event.

For the longitudinal (transverse bridge direction) direction with debris the effective width of the column is 4 feet plus 2 feet debris on each side for a width of 8 feet.

w = (0.113)(8.00) = 0.904 k/ft (Column with Debris)

w = (0.113)(4.00) = 0.452 k/ft (Column)

w = (0.113)(20.00) = 2.260 k/ft (Footing)

Longitudinal (transverse bridge direction) Shear and Moment:

V_{col} = (0.904)(12.00) + (0.452)(6.00) = 13.56 k

M_{col} = (0.904)(12.00)(12.00) + (0.452)(6.00)(3.00) = 138 ft-k

$V_{footing}$ = (0.904)(12.00) + (0.452)(6.00) + (2.260)(3.00) = 20.34 k

$M_{footing}$ = (0.904)(12.00)(15.00) + (0.452)(6.00)(6.00)
+ (2.260)(3.00)(1.50) = 189 ft-k

For a 15 degree angle of attack, the lateral (longitudinal bridge direction) uniformly distributed pressure may be taken as:

[3.7.3.2-1]

$$p = \frac{C_L V^2}{1000} = \frac{(0.8)\cdot(9.0)^2}{1000} = 0.065\,\text{ksf}$$

where

p = lateral pressure (ksf)

C_L = lateral drag coefficient specified in Table 3.7.3.2-1
= 0.8 by interpolation from Table

This lateral (longitudinal bridge direction) pressure is applied to the appropriate area as follows:

w = (0.065)(2.00 + 6.00 + 2.00) = 0.650 k/ft (Column with Debris)

w = (0.065)(6.00) = 0.390 k/ft (Column)

w = (0.065)(21.00) = 1.365 k/ft (Footing)

Shear and Moment:

V_{col} = (0.650)(12.00) + (0.390)(6.00) = 10.14 k

M_{col} = (0.650)(12.00)(12.00) + (0.390)(6.00)(3.00) = 101 ft-k

$V_{footing}$ = (0.650)(12.00) + (0.390)(6.00) + (1.365)(3.00) = 14.24 k

$M_{footing}$ = (0.650)(12.00)(15.00) + (0.390)(6.00)(6.00)
+ (1.365)(3.00)(1.50) = 137 ft-k

For this example problem, only the design event forces have been calculated. In an actual design the superflood must also be considered. Refer to the Bridge Hydraulic Guidelines for additional information. Since the footing is founded on non-erodible rock the scour depth will not change but the velocity and corresponding stream forces will increase for the super flood event.

A summary of unfactored axial forces, shears and moments is shown below.

Bottom of Column

Load	P_{max}	P_{min}	V_{long}	V_{trans}	M_{long}	M_{trans}
	k	k	k	k	ft-k	ft-k
DC	1074.6	1074.6	0	0	0	0
DW	66.2	0	0	0	0	0
EV	0	0	0	0	0	0
LL+IM	356.4	-37.0	0	0	0	1925
BR	0	0	18.00	0	594	0
WA	0	0	10.14	13.56	101	138
WS_{super}	0	0	16.40	28.81	541	1044
WS_{sub}	0	0	13.44	5.28	316	100
WS	0	0	29.84	34.09	857	1144
WS_{vert}	0	-53.0	0	0	0	408
WL	0	0	4.88	8.59	161	362
TU	0	0	23.79	0	785	0
CR+SH	0	0	28.32	0	935	0
SE	13.3	-13.3	0	0	0	0

Bottom of Footing

Load	P_{max}	P_{min}	V_{long}	V_{trans}	M_{long}	M_{trans}
	k	k	k	k	ft-k	ft-k
DC	1263.6	1263.6	0	0	0	0
DW	66.2	0	0	0	0	0
EV	130.7	0	0	0	0	0
LL	297.4	-30.4	0	0	0	1606
BR	0	0	18.00	0	648	0
WA	0	0	14.24	20.34	137	189
WS_{super}	0	0	16.40	28.81	590	1130
WS_{sub}	0	0	13.44	5.28	356	116
WS	0	0	29.84	34.09	946	1246
WS_{vert}	0	-53.0	0	0	0	408
WL	0	0	4.88	8.59	176	388
TU	0	0	23.79	0	856	0
CR+SH	0	0	28.32	0	1020	0
SE	13.3	-13.3	0	0	0	0

LOAD COMBINATIONS

[3.4.1-1] **STRENGTH I**

Max = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH)
+ 1.75(LL + IM + BR) + 1.00WA + 0.50TU + 1.00SE

Min = 0.90DC + 0.65DW + 1.00EV + 0.50(CR + SH)
+ 1.75(LL + IM + BR) + 1.00WA + 0.50TU + 1.00SE

STRENGTH III

Max = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH) + 1.00WA
+ 1.40(WS + WS_{vert}) + 0.50TU + 1.00SE

Min = 0.90DC + 0.65DW + 1.00EV + 0.50(CR + SH) + 1.00WA
+1.40(WS + WS_{vert}) + 0.50TU + 1.00SE

STRENGTH V

Max = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH)
+ 1.35(LL + IM + BR) + 1.00WA + 0.40WS + 1.00WL
+ 0.50TU + 1.00SE

Min = 0.90DC + 0.65DW + 1.00EV + 0.50(CR + SH)
+ 1.35(LL + IM + BR) + 1.00WA + 0.40WS + 1.00WL
+ 0.50TU + 1.00SE

SERVICE I

Max = 1.00(DC + DW + EV + CR + SH) + 1.00(LL + IM + BR)
+ 1.00WA + 0.30WS + 1.00WL + 1.00TU + 1.00SE

Since the creep and shrinkage forces were determined based on use of the gross moment of inertia of the columns a 0.50 load factor is appropriate for all strength limit states.

Pier Cap Design

A strut-and-tie analysis is demonstrated in this example but is not required by Bridge Group. Each designer should determine the appropriateness of the analysis method for each situation when performing an actual design.

[3.6.3.1]

Step 1 – Determine Structural Model

The sectional model of analysis is appropriate for the design of a pier cap where the assumptions of traditional beam theory are valid. However, the strut-and-tie model should be considered for situations in which the distance between the centers of the applied load and the supporting reactions is less than twice the member thickness.

[5.8.1.1]

Components in which the distance from the point of zero shear to the face of the support is less than twice the depth or in which a load causing more than one-half of the shear at a support is closer than twice the depth from the face of the support may be considered to be deep components and require a strut-and-tie analysis. For this problem the webs are closer than twice the section depth so the strut-and-tie model is used.

A strut-and-tie model is used to determine the internal force effects near supports and the points of application of concentrated loads. A strut-and-tie analysis is a strength limit state analysis. As such no service limit state analysis is performed.

The exterior node is under the bearing pad that is assumed to be 12 inches wide. Each interior node is under two bearing pads; one from each adjacent box beam.A section showing the relationship between the transverse frame model and the typical section is shown in Figure 7 below.

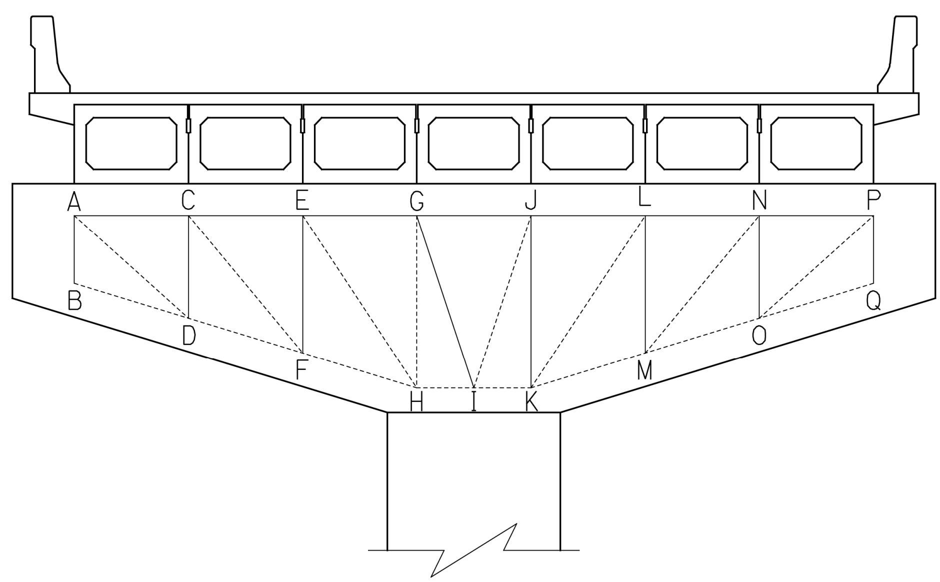


Figure 7

Step 2 – Create Strut-and-Tie Model

A half section of the strut-and-tie model showing the nodes, angles and dimensions in inches is shown in Figure 8. In determining the model, consideration must be given to the required depth of struts, ties and the nodal zone. Establishing geometry requires trial and error in which member sizes are assumed, the truss geometry is established, member forces are determined, and the assumed member sizes verified. The top tie is located 6.5 inches from the top and the bottom strut is located 5.5 inches normal to the bottom.

The support reactions and moments can be found from equilibrium since the structural element is statically determinate.

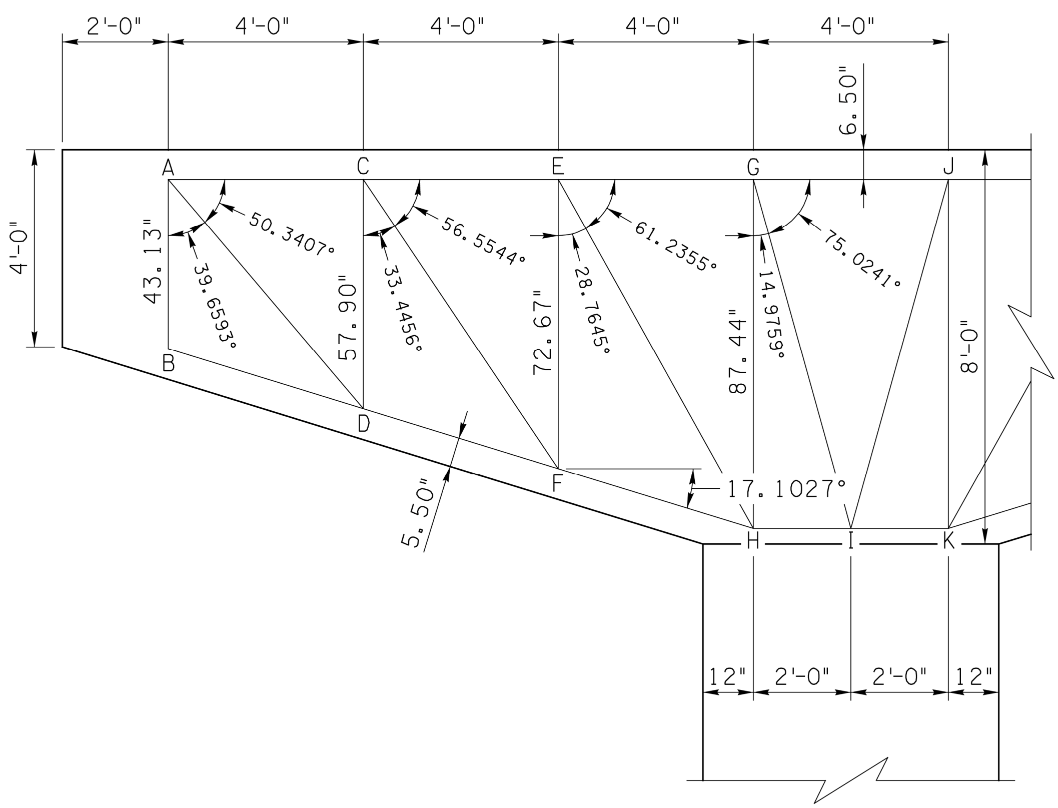


Figure 8

Step 3 – Determine Loads

In a strut-and-tie analysis the location of the applied load affects the answer. For a pier cap supporting precast members, the loads are applied at the top nodes.

For this problem, Strength I Limit State with one live load case will be investigated. The live load case will consist of the live load placed two feet from the barrier face to produce the maximum moment and shear.

The pier cap weight is distributed to the nodes based on the contributing span of the nodes based on simple span reactions. Figure 9 shows the nodal loads due to the cap self-weight. The factored node concentrated loads are shown below:

Node A/P: R = 1.25(0.150)(5.50)(4.00)(4.00+5.23) / 2 = 19.04 k
Node C/N: R = 1.25(0.150)(5.50)(4.00)(5.23+6.46) / 2 = 24.11 k
Node E/L: R = 1.25(0.150)(5.50)(4.00)(6.46+7.69) / 2 = 29.18 k
Node G/J: R = 1.25(0.150)(5.50)[1.00)(7.69+8.00) / 2 + (3.00)(8.00)] = 32.84 k

Total R = 2[19.04 + 24.11 + 29.18 + 32.84] = 210.34 k

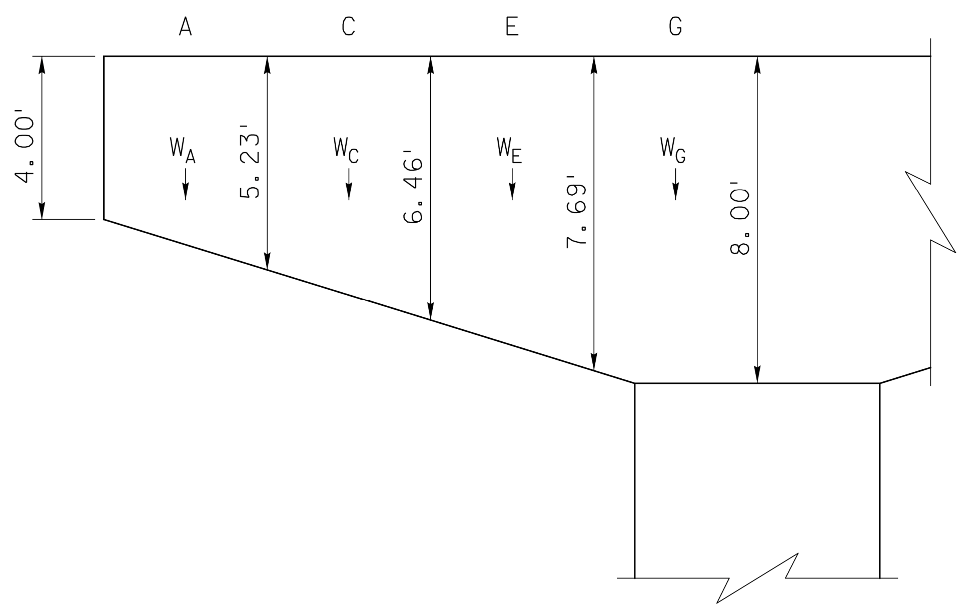


Figure 9

The dead load from the superstructure is concentrated in the box beam webs. Each beam end is supported by two elastomeric bearing pads. The support mechanism of bearing pads and a cast-in-place diaphragm complicates the assumption of how the load is transmitted to the pier cap. The non-composite dead loads clearly are transmitted through the bearing pads. The other loads are mainly carried through the cast-in-place concrete diaphragm. For this problem the loads will be assumed to act as concentrated loads through the bearing pads.

Node A/P:	R = 1.25(116.61) / 2 + 1.50(9.46) / 2 = 79.98 k
Node C/N:	R = 1.25(116.61) + 1.50(9.46) = 159.95 k
Node E/L:	R = 1.25(116.61) + 1.50(9.46) = 159.95 k
Node G/J:	R = 1.25(116.61) + 1.50(9.46) = 159.95 k
Total	R = 2[79.98 + (3)(159.95)] = 1119.66 k

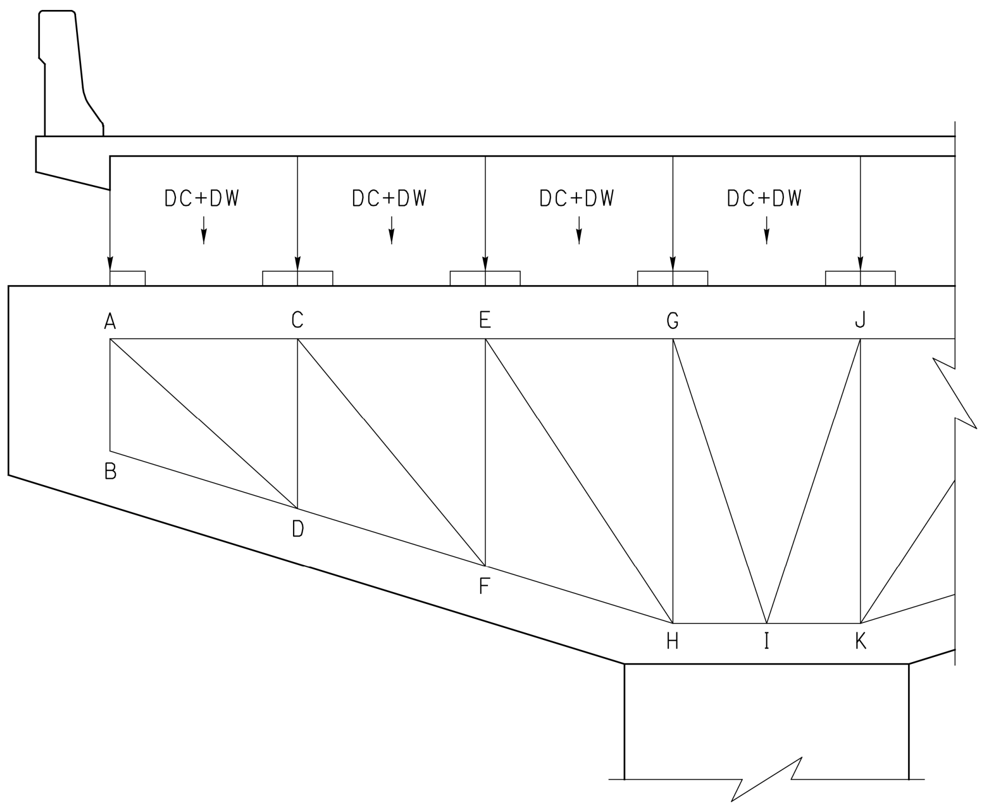


Figure 10

The live load is divided between the nodes assuming simple spans as follows.

Node A: R = 1.75(89.09)(1.20)(2.00 / 4.00) = 93.54 k
Node C: R = 1.75(89.09)(1.20)(2.00 / 4.00) = 93.54 k
Node E: R = 1.75(89.09)(1.20) = 187.09 k

Total R = 93.54 + 93.54 + 187.09 = 374.17 k

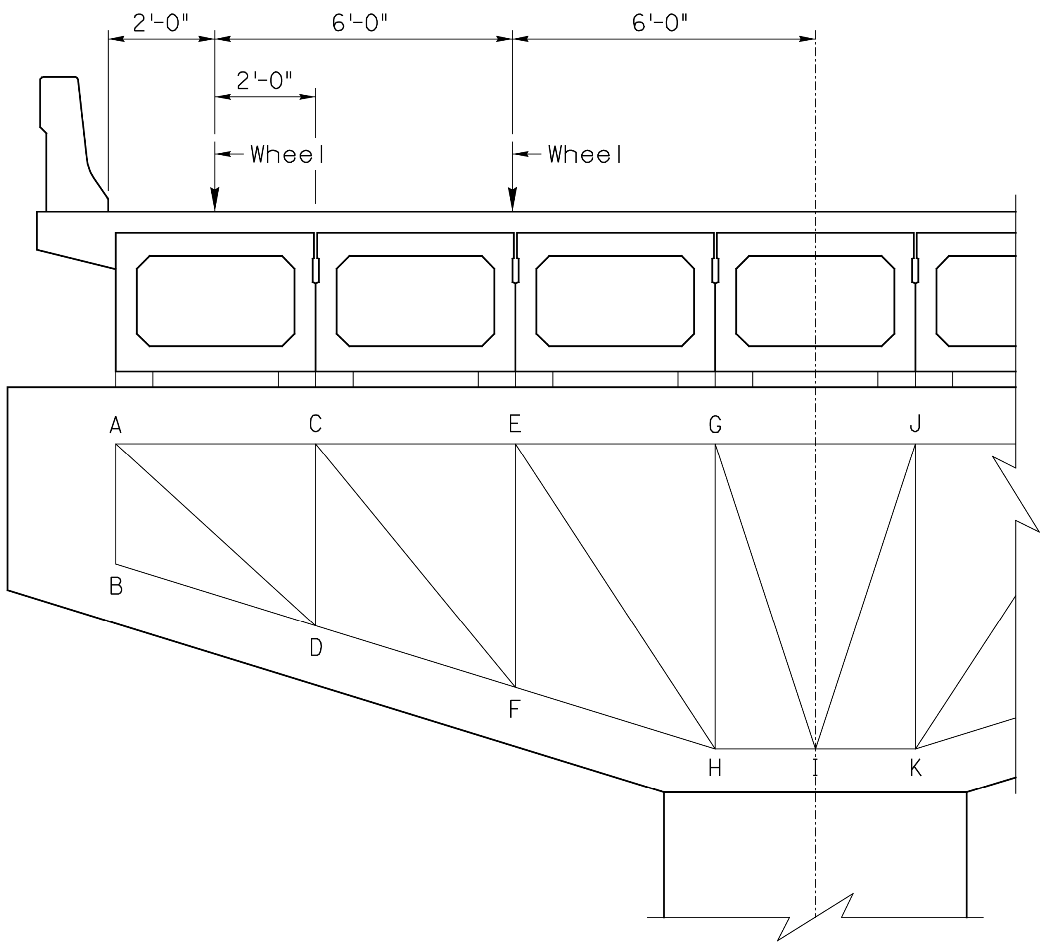


Figure 11

Step 4 – Determine Reactions

The pier cap in this problem is integral with the column. This restraint is modeled to preserve the equations of equilibrium and compatibility. The column is divided into three equally spaced nodes. For the 72 inch wide column Node H is 12 inches from one face, Node I is at the center of the column and Node K is 12 inches from the opposite face. For an applied uniform vertical load each node will have the same contributory area and same resulting reaction. The bending moment is handled with a couple between Node H and Node K resulting in equal but opposite axial loads at the nodes.

The pier cap has symmetrical dead loads so only the live load produces a moment. See Figure 12.

R = 210.34 + 1119.66 + 374.17 = 1704.17 k

M = 1.75(89.09)(1.20)(12.00) + 1.75(89.09)(1.20)(6.00) = 3367.60 ft-k

The reactions at the restraint nodes are as follows:

Node H:
R = 1704.17 / 3 + 3367.60 / 4.00 = 1409.96 ft-k

Node I:
R = 1704.17 / 3 = 568.05 k

Node K:
R = 1704.17 / 3 – 3367.60 / 4.00 = -273.84 ft-k

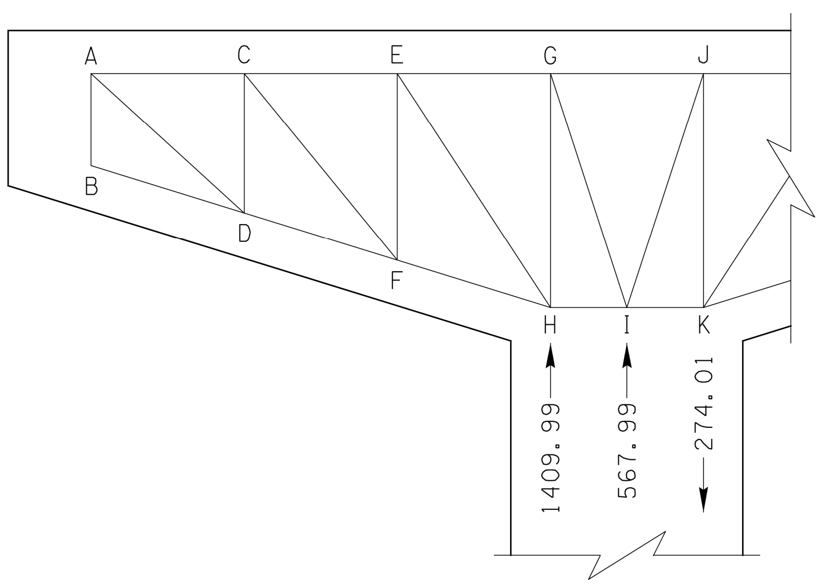


Figure 12

Step 5 – Determine Member Forces

A summation of external loads applied at the various nodes follows:

Node	Cap DL	Super	LL+IM	Reaction	Total
A	19.04	79.98	93.54		192.56
C	24.11	159.95	93.54		277.60
E	29.18	159.95	187.09		376.22
G	32.84	159.95			192.79
H				1`409.96	1409.96
I				568.05	568.05
J	32.84	159.95			192.79
K				-273.84	-273.84
L	29.18	159.95			189.13
N	24.11	159.95			184.06
P	19.04	79.98			99.02

With the geometry of the model established and the applied loads and reactions determined, the force in each member is calculated. Starting with Node B the unknown forces in Members B-D and B-A are calculated by summing the vertical and horizontal forces equal to zero. Next sum the vertical and horizontal forces equal to zero for Node A where Members A-C and A-D are calculated. This method is continued until all the member forces are known.

When the last node is evaluated there will be no unknown member forces. Summing the forces in both directions provides a summation of the error in the model. If the transverse frame model is not the same of that used for the strut-and-tie model, errors will be introduced in addition to the expected rounding errors due the number of separate calculations.

The strut-and-tie model is shown in Figure 13 with applied loads, reactions, and tension and compression member forces. The maximum tension of 839 kips occurs in top Member E-G near the face of the column. The maximum shear occurs in Member E-F with a tension force of 333 kips. The maximum compressive force of 812 kips occurs in Member E-H.

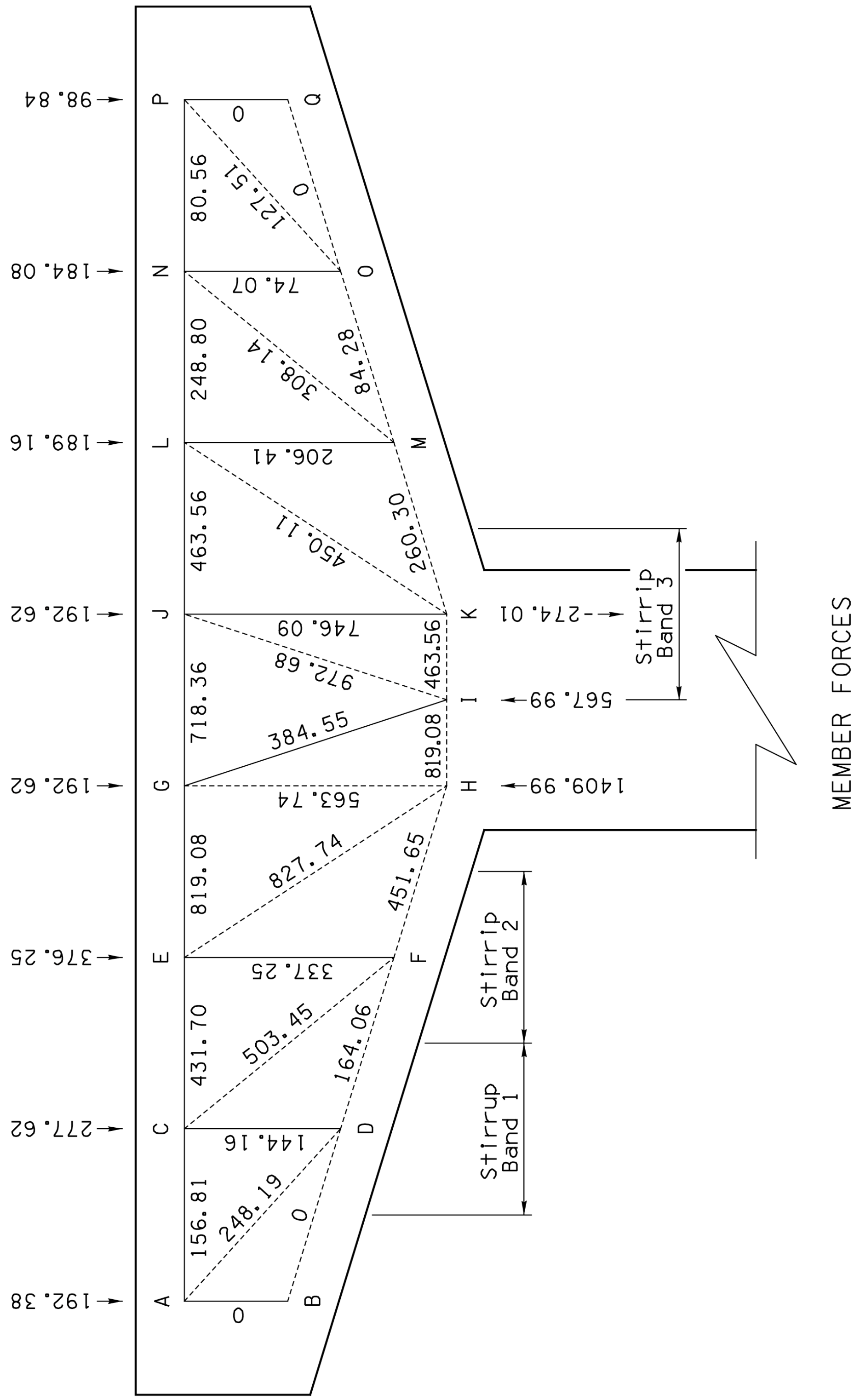

MEMBER FORCES
Stirrup
Band 1
Stirrip
Band 2
Stirrip
Band 3
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
192.38
277.62
376.25
192.62
192.62
189.16
184.08
98.84
1409.99
567.99
274.01
0
156.81
248.19
144.16
431.70
503.45
164.06
337.25
819.08
827.74
451.65
563.74
718.36
384.55
819.08
972.68
463.56
746.09
463.56
450.11
260.30
206.41
248.80
308.14
84.28
74.07
80.56
127.51

Figure 13

Step 6 – Provide Crack Control Reinforcing

Due to the presence of concentrated loads within a distance less than the member depth from the face of the column support, all the nodes are considered D-regions.

In D-regions crack control reinforcing in the form of orthogonal grids on both faces is required. Some reinforcing may be required to be located in the interior of the member. The minimum ratio of reinforcing to gross concrete area is 0.003. The required area of reinforcing per foot depth is:

$A_s = (0.003)(12.00)(66.00) = 2.38 \text{ in}^2$

Use 6 - #6 as minimum reinforcement in both horizontal and vertical directions with $A_s = (6)(0.44) = 2.64 \text{ in}^2$. The maximum spacing of this reinforcing is 12 inches. For the vertical reinforcing, use #6 stirrups with six legs. Use #6 at 12 inch spacing for the horizontal reinforcing.

Since the minimum crack control reinforcing often controls the design, it is useful to know the required minimum prior to determining actual required reinforcing.

Step 7 – Capacity of Tension Ties

(a) Top Reinforcement over column

The maximum tension at the face of the column occurs in Member E-G. The required area of tension tie reinforcement, A_{st}, is calculated as follows:

$$A_{st} = \frac{P_u}{\varphi f_y} = \frac{839}{(0.90)(60)} = 15.54 \text{ in}^2$$

Use 16 - #9 bars with an $A_s = 16.00 \text{ in}^2$

(b) Top Reinforcement for Member C-E

The required area of tension tie reinforcement, A_{st}, for Member C–E is calculated as follows:

$$A_{st} = \frac{P_u}{\varphi f_y} = \frac{444}{(0.90)(60)} = 8.22 \text{ in}^2$$

Use 10 - #9 bars with an $A_s = 10.00 \text{ in}^2$

(c) Vertical Stirrup Band 1

The vertical tension force in Stirrup Band 1 of 143 kips occurs in Member C-D. This tension force is resisted by stirrups within a certain length of the beam as indicated by the stirrup bands. Using #6 stirrups with 6 legs, the number of stirrups, n, required in the band is:

$$n = \frac{P_u}{\varphi A_{st} f_y} = \frac{143}{(0.90)(6)(0.44)(60)} = 1.0 \quad \text{Use 4 stirrups minimum}$$

Use minimum reinforcing of #6 stirrups with 6 legs spaced at 12 inches.

(d) Vertical Stirrup Band 2

The vertical tension force in Stirrup Band 2 of 333 kips occurs in Member E-F. Using #6 stirrups with 6 legs, the number of stirrups, n, required in the band is:

$$n = \frac{P_u}{\varphi A_{st} f_y} = \frac{333}{(0.90)(6)(0.44)(60)} = 2.3 \quad \text{Use 4 stirrups minimum}$$

Use minimum reinforcing of #6 stirrups with 6 legs spaced at 12 inches.

(e) Vertical Stirrup Band 3

The vertical tension force in Stirrup Band 3 of 746 kips occurs in Member G-H. This tension force is resisted by stirrups within the stirrup band width. Using #6 stirrups with 6 legs, the number of stirrups, n, required in the band is:

$$n = \frac{P_u}{\varphi A_{st} f_y} = \frac{746}{(0.90)(6)(0.44)(60)} = 5.23 \quad \text{Use 6 stirrups}$$

The required spacing, s, within the 48.0 inch band is:

$$s \le \frac{48.00}{5.23} = 9.2 \text{ in}$$

Use 6 - #6 stirrups with 6 legs spaced at 9 inches maximum.

Step 8 – Check Anchorage of Tension Tie

(a) Top Longitudinal Reinforcement

The top 16 - #9 rebar must be developed at the inner edge of Node E. By inspection this node will have adequate embedment. However, half the reinforcing can be terminated after the development length beyond Node E. The required development length is:

$$l_d = \frac{1.25 A_b f_y}{\sqrt{f'_c}} = \frac{(1.25)\cdot(1.00)\cdot(60)}{\sqrt{3.5}} = 40.1 \text{ in}$$

but not less than $0.4 d_b f_y = 0.4(1.128)(60) = 27.1$ in

Since more than 12 inches of fresh concrete is poured below the reinforcement, the bars are considered top bars for development length. Since the bars are spaced at 8 inches, they qualify for a reduction of 0.8 for widely spaced bars. The adjusted development length is:

$l_d = (40.1)(1.4)(0.8) = 44.9$ inches .

Cut 6 #9 top reinforcing bars a minimum of 44.9 inches beyond the inside face of Node E.

By independent analysis 4 - #9 reinforcing bars must be properly developed beyond the inside face of Node A to resist the force in Member A-C. The available embedment length equals 24.00 + 12.00 – 2.00 clear = 34.00 inches. Since the required development length exceeds this value the bars must be hooked.

The basic development length of a hooked #9 reinforcing bar is:

$$l_{dh} = \frac{38.0 d_b}{\sqrt{f'_c}} = \frac{(38.0)(1.128)}{\sqrt{3.5}} = 22.9 \text{ inches}$$

Where side cover normal to the plane of the hook is not less than 2.5 inches and for a 90 degree hook the cover on the bar extension beyond the hook is not less than 2.0 inches a modification factor of 0.7 may be applied. The modified development length is (0.7)(22.9) = 16.0 inches.

Therefore, the #9 reinforcement is adequate developed beyond Node A with a standard hook.

(b) Vertical Stirrups

The vertical stirrups must be developed beyond the inner edge of the nodes. The required development length is:

$$l_d = \frac{1.25 A_b f_y}{\sqrt{f'_c}} = \frac{(1.25)\cdot(0.44)\cdot(60)}{\sqrt{3.5}} = 17.6 \text{ in}$$

but not less than $0.4 d_b f_y$ = 0.4(0.750)(60) = 18.0 in

The available embedment length is 13 inches minus 2.5 inches clear or 10.5 inches. Therefore the stirrups must be hooked. The basic development length of a hooked #6 stirrup is:

$$l_{dh} = \frac{38.0 d_b}{\sqrt{f'_c}} = \frac{(38.0)(0.750)}{\sqrt{3.5}} = 15.2 \text{ inches}$$

Where side cover normal to the plane of the hook is not less than 2.5 inches and for a 90 degree hook the cover on the bar extension beyond the hook is not less than 2.0 inches a modification factor of 0.7 may be applied. The modified development length is (0.7)(15.2) = 10.6 inches.

Since the available embedment length is approximately equal to the required development length, the stirrups are adequately developed.

Step 9 – Capacity of Struts

Strut E-H carries the highest compression force of 812 kips. Since this strut is anchored at Joint E that also anchors tension Ties C-E, E-G and E-F (See Figure 30), this is the most critical strut.

The effective cross sectional area of a strut by considering the available concrete area and the anchorage conditions at the ends of the strut. When a strut is anchored by reinforcement, the effective concrete area may be considered to extend a distance of up to six bar diameters from the anchored bar. See Figures 14 and 15 for determination of the depth and width of the strut.

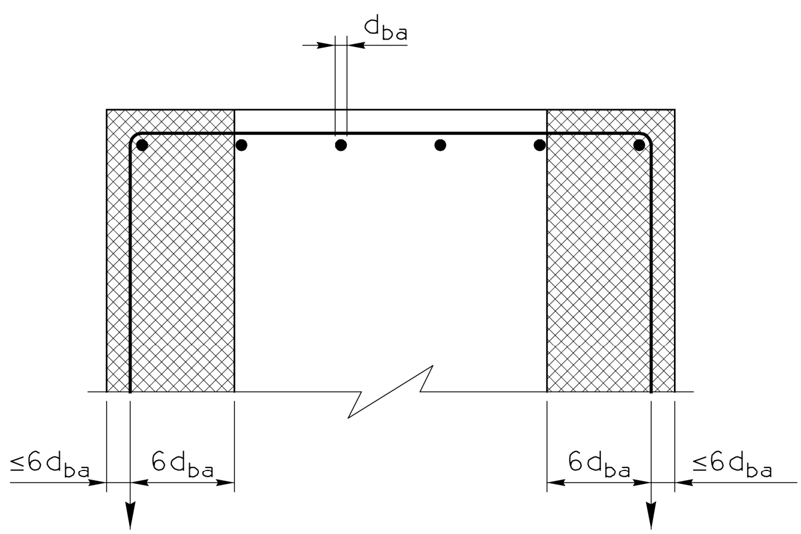


Figure 14

The effective cross sectional area is:

$l_a \sin\theta_s$ = [6(1.128)(2) + 3(12.00)] sin(60.94) = 43.30 inches

$w_{effective}$ = 2[2.38 + 6(1.128)] + 4[(2)(6)(1.128)] = 72.44 inches

Therefore, the entire width is effective

A_{cs} = (43.30)(66.00) = 2858 in^2

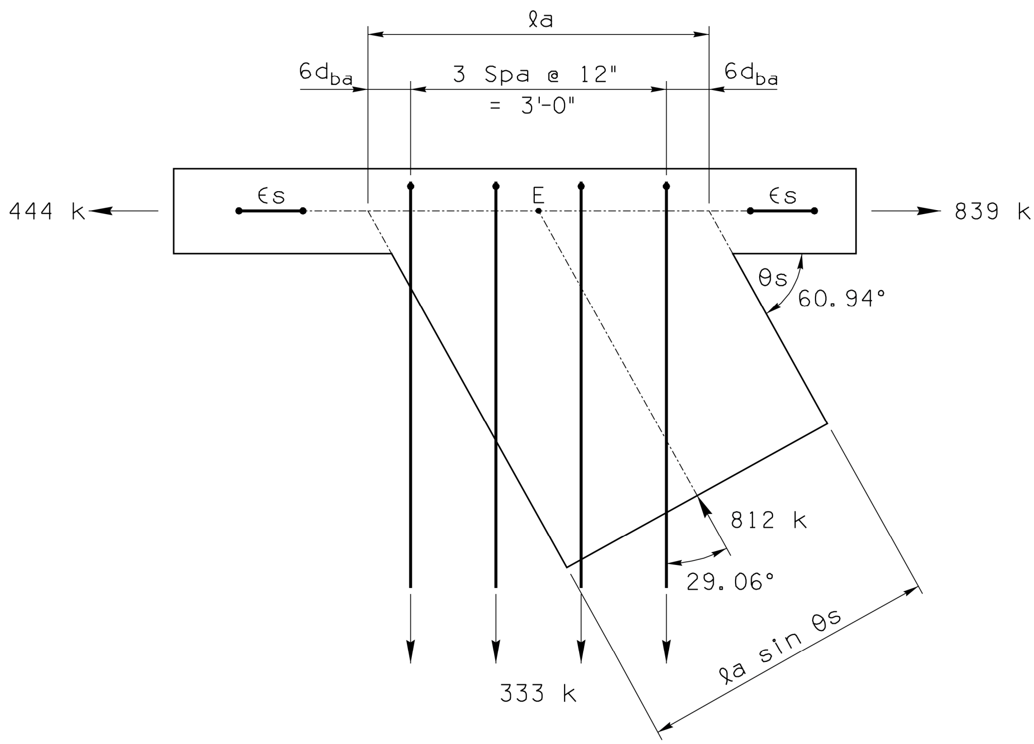


Figure 15

The limiting compressive stress in the strut, f_{cu}, is usually controlled by the tensile strain in the tie that is at the smallest angle to the strut. For this problem both tensile strains will be investigated. From the geometry of the truss, the angle between tension Tie E-G and Strut E-H is 60.94 degrees. The tensile strain in Tie E-G is:

$$\varepsilon_s = \frac{P_u}{A_{st}E_s} = \frac{839}{(16.00)(29000)} = -1.808x10^{-3}$$

The tensile strain the horizontal Tie C-E is:

$$\varepsilon_s = \frac{P_u}{A_{st} E_s} = \frac{444}{(16.00)\ (29000)} = -0.957x10^{-3}$$

At the center of the node the horizontal strain will be the average of the two horizontal members joining at the node. The average strain is:

ε_s = (1.808 x 10^{-3} + 0.957 x 10^{-3}) / 2 = 1.383 x 10^{-3}

The principal strain, ε_1, is determined as follows:

$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2\alpha_s$

ε_1 = 1.383 x 10^{-3} + (1.383 x 10^{-3} + 0.002) $\cot^2$(60.94) = 2.428 x 10^{-3}

The limiting compressive stress, f_{cu}, in the Strut E-H is:

$$f_{cu} = \frac{f'_c}{0.8 + 170\varepsilon_1} \leq 0.85 f'_c$$

$$f_{cu} = \frac{3.5}{0.8 + (170)\left(2.428x10^{-3}\right)} = 2.89 \leq 0.85\,(3.5) = 2.98 \text{ ksi}$$

If the vertical Tie E-F is considered, the tension strain is:

$$\varepsilon_s = \frac{P_u}{A_{st} E_s} = \frac{333}{(6)(0.44)(4)(29000)} = 1.087x10^{-3}$$

From this tie the principle strain is:

ε_1 = 1.087 x 10^{-3} + (1.087 x 10^{-3} + 0.002) $\cot^2$(29.06) = 11.09 x 10^{-3}

Since this is greater than the previously calculated value for principle strain, this tie will govern the compressive capacity of the strut.

$$f_{cu} = \frac{3.5}{0.8 + (170)\left(11.09x10^{-3}\right)} = 1.30 \leq 0.85\,(3.5) = 2.98$$

The nominal resistance of the strut is based on the limiting stress, f_{cu}, and the strut dimensions. The nominal resistance is:

$P_n = f_{cu}A_{cs} = (1.30)(2858) = 3715$ kips

The factored resistance of the strut is:

$P_r = \varphi P_n = (0.70)(3715) = 2601$ kips $> P_u = 812$ kips

Step 10 – Node Regions

The concrete compression stress in the node region of the strut shall not exceed 0.65 φ f'_c for node regions anchoring tension ties in more than one direction. For Node E:

$$f_c = 0.65\varphi f'_c = (0.65)(0.70)(3.5) = 1.59 \text{ ksi}$$

The nodal zone compressive stress is:

$$f_c = \frac{812}{2858} = 0.284 \text{ ksi}$$

The tension tie reinforcing shall be uniformly distributed over an effective area of concrete. Check to ensure that the tension ties are sufficiently spread out in the effective anchorage area. The effective anchorage area is equal to twice the depth to the top tie or 2(6.50) = 13.0 inches.

The nodal zone stress to anchor the tension tie is:

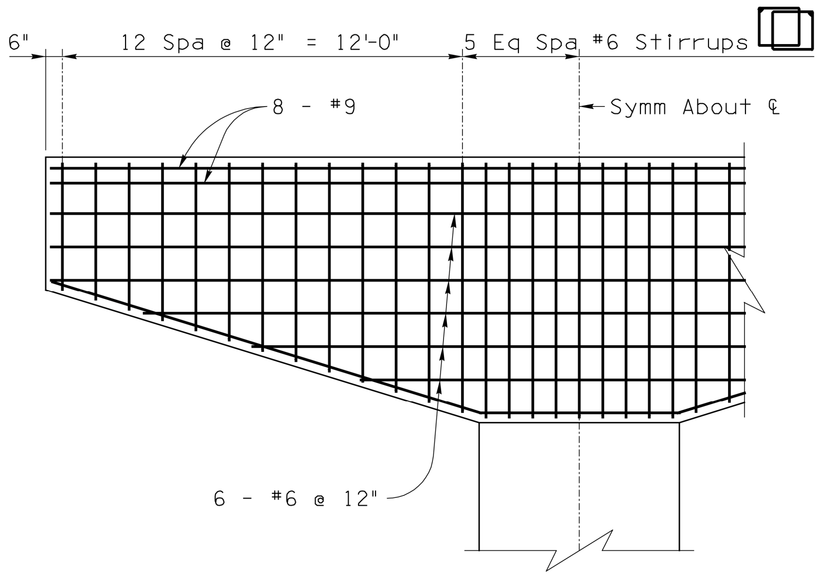
$$f_c = \frac{839}{(13.00)\cdot(66.00)} = 0.978 \text{ ksi}$$

For this nodal zone the limiting nodal zone stress is:

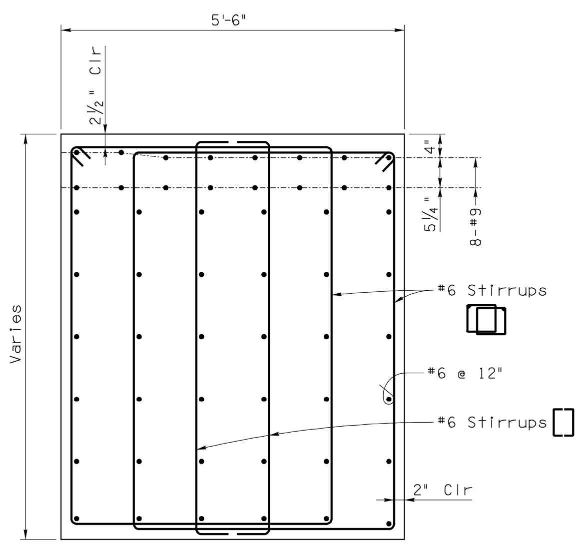
$f_c = 0.65\ \varphi\ f'_c = (0.65)(0.70)(3.5) = 1.593$ ksi

Since the stress is less than the limiting stress the criteria is satisfied.

Step 11 – Sketch Required Reinforcing



ELEVATION



TYPICAL SECTION

Figure 16

Column Design [5.7.4]

The column is designed as a compression member under axial load and moment. A summary of moments and axial loads at the bottom of the column in both the longitudinal and transverse directions is required. A summary of values is shown below:

Bottom of Column Axial Loads and Moments

Load	P_{max}	P_{min}	M_{trans}	M_{long}
DC	1074.6	1074.6	0	0
DW	66.2	0	0	0
LL + IM	356.4	-37.0	1925	0
WA	0	0	138	101
BR	0	0	0	594
WS_{super}	0	0	1044	541
WS_{sub}	0	0	100	316
WS	0	0	1144	857
WS_{vert}	0	-53.0	408	0
WL	0	0	362	161
TU	0	0	0	785
CR + SH	0	0	0	935
SE	13.3	-13.3	0	0

The strength limit states load factor for TU is 0.50 since the analysis is based on gross section properties.

The column must be designed for the strength limit states. Several strength limit states will be investigated to determine the critical one.

[3.4.1]

STRENGTH I

Max = 1.25DC + 1.50DW + 0.50(CR + SH) + 1.75(LL + IM + BR) + 1.00WA + 0.50TU + 1.00SE

Min = 0.90DC + 0.65DW + 0.50(CR + SH) + 1.75(LL + IM + BR) + 1.00WA + 0.50TU + 1.00SE

Bottom of Column

P_{max} = 1.25(1074.6) + 1.50(66.2) + 1.75(356.4) + 1.00(13.3) = 2080 k
P_{min} = 0.90(1074.6) + 0.65(0) + 1.75(-37.0) + 1.00(-13.3) = 889 k

M_{trans} = 1.25(0) + 1.50(0) + 1.75(1925) + 1.00(138) = 3507 ft-k

M_{long} = 1.25(0) + 1.50(0) + 0.50(935) + 1.75(594) + 1.00(101) + 0.50(785) = 2001 ft-k

STRENGTH III

$$Max = 1.25DC + 1.50DW + 0.50(CR + SH) + 1.00WA + 1.40(WS + WS_{vert}) + 0.50TU + 1.00SE$$

$$Min = 0.90DC + 0.65DW + 0.50(CR + SH) + 1.00WA + 1.40(WS + WS_{vert}) + 0.50TU + 1.00SE$$

Bottom of Column

$$P_{max} = 1.25(1074.6) + 1.50(66.2) + 1.00(13.3) = 1456 \text{ k}$$

$$P_{min} = 0.90(1074.6) + 0.65(0) + 1.40(-53.0) + 1.00(-13.3) = 880 \text{ k}$$

$$M_{trans} = 1.25(0) + 1.50(0) + 1.00(138) + 1.40(1144 + 408) = 2311 \text{ ft-k}$$

$$M_{long} = 1.25(0) + 1.50(0) + 0.50(935) + 1.00(101) + 1.40(857) + 0.50(785) = 2161 \text{ ft-k}$$

STRENGTH V

$$Max = 1.25DC + 1.50DW + 0.50(CR + SH) + 1.35(LL + IM + BR) + 1.00WA + 0.40WS + 1.00WL + 0.50TU + 1.00SE$$

$$Min = 0.90DC + 0.65DW + 0.50(CR + SH) + 1.35(LL + IM + BR) + 1.00WA + 0.40WS + 1.00WL + 0.50TU + 1.00SE$$

Bottom of Column

$$P_{max} = 1.25(1074.6) + 1.50(66.2) + 1.35(356.4) + 1.00(13.3) = 1937 \text{ k}$$

$$P_{min} = 0.90(1074.6) + 0.65(0) + 1.35(-37.0) + 1.00(-13.3) = 904 \text{ k}$$

$$M_{trans} = 1.25(0) + 1.50(0) + 1.35(1925) + 1.00(138) + 0.40(1144) + 1.00(362) = 3556 \text{ ft-k}$$

$$M_{long} = 1.25(0) + 1.50(0) + 0.50(935) + 1.35(594) + 1.00(101) + 0.40(857) + 1.00(161) + 0.50(785) = 2267 \text{ ft-k}$$

Longitudinal Column Design

A review of the group load combinations does not indicate an obvious critical limit state. Therefore the slenderness effects will be considered for all three strength limit states.

Slenderness Effects [5.7.4.3]

For members not braced against sidesway, the effects of slenderness shall be considered where the slenderness ratio Kl_u/r is greater than 22. The effects of slenderness must be considered separately in each direction as the moment of inertia of the column varies with each direction.

[C5.7.4.3]

The radius of gyration for a rectangular section may be taken as 0.30 times the overall dimension in the direction in which stability is being considered. For the longitudinal direction: r = 0.30(4.00) = 1.20 ft.

[C4.6.2.5-1]

For members not braced against sidesway, k is determined with due consideration for the effects of cracking and reinforcement on the relative stiffness and may not be taken less than 1.0. For a member free at the top but fixed at the base, the design value of k = 2.1 is obtained from the alignment chart.

The slenderness ratio is calculated as follows:

$$\frac{kl_u}{r} = \frac{(2.1)\cdot(33.00)}{1.20} = 57.8$$

Since the slenderness ratio is greater than 22, slenderness effects must be considered. Since the slenderness ratio is less than 100 the approximate moment magnifier method may be used.

In lieu of a more precise method, EI shall be taken as the greater of:

[5.7.4.3-1]

$$EI = \frac{\frac{E_c I_g}{5} + E_s I_s}{1+\beta_d}$$

[5.7.4.3-2]

$$EI = \frac{\frac{E_c I_g}{2.5}}{1+\beta_d}$$

The first equation will control for large percentages of reinforcement. The second equation will control when the reinforcing percentage is small. At this stage the reinforcing pattern is usually not known, requiring that I_s be estimated or ignored. See Figure 17 for the reinforcing pattern for this problem.

$I_s = \sum A_s d^2 = 1.56[(7(20.67)^2 + 2(13.78)^2 + 2(6.89)^2](2) = 10{,}812 \text{ in}^4$

$I_g = (6.00)(4.00)^3 \div 12 = 32.00 \text{ ft}^4$

$E_c = 3405(144) = 490{,}320 \text{ ksf}$

The permanent moment in the column includes the dead load moment. For a column assumed free at the top with a symmetrical load, the moment from dead load is zero resulting in the value β_d being zero.

$$EI = \frac{\frac{(490{,}320)\cdot(32.00)}{5} + (10{,}812)\cdot(29000)\div 144}{1+0} = 5{,}315{,}000 \text{ k-ft}^2$$

$$EI = \frac{\frac{(490{,}320)\cdot(32.00)}{2.5}}{1+0} = 6{,}276{,}000 \text{ k-ft}^2 \Leftarrow \text{Critical}$$

The critical load, required by the moment magnifier method, is determined as follows:

[4.5.3.2.2b-5]

$$P_e = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \cdot (6{,}276{,}000)}{[(2.1)\cdot(33.00)]^2} = 12{,}898 \text{ k}$$

For a single column pier, the equations for δ_b and δ_s will produce the same amplification value. The equation for δ_b will be used.

Strength I Limit State

The axial load should include the applied load at top plus half the column weight. Since the column load is small use the reaction at the column base.

[4.5.3.2.2b-3]
[4.5.3.2.2b-4]

$$\delta_b = \delta_s = \frac{C_m}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1 - \frac{2080}{(0.75)\cdot(12{,}898)}} = 1.274$$

where φ_K = a stiffness reduction factor = 0.75 for concrete members

The moment on the compression member, M_{2b}, is equal to the factored gravity loads that result in no appreciable sidesway. For this problem the dead load moment is included. This factored moment equals 0 ft-k.

The moment on the compression member, M_{2s} is equal to the factored lateral or gravity load that results in sidesway. For this problem in the longitudinal direction, the lateral loads result in sidesway.

The magnified longitudinal moments are increased to reflect effects of deformation as follows:

[4.5.3.2.2b-1]

$M_c = \delta_b M_{2b} + \delta_s M_{2s}$

$M_c = 1.274(0) + 1.274(2001) = 2549$ ft-k

Strength III Limit State

$$\delta_b = \delta_s = \frac{1}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1}{1 - \frac{1456}{(0.75)\cdot(12{,}898)}} = 1.177$$

$M_c = 1.177(0) + 1.177(2161) = 2543$ ft-k

Strength V Limit State

$$\delta_b = \delta_s = \frac{1}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1}{1 - \frac{1937}{(0.75)\cdot(12{,}898)}} = 1.250$$

$M_c = 1.250(0) + 1.250(2267) = 2834$ ft-k

These magnified moments are used with the factored axial load to design the compression member.

Transverse Column Design

[5.7.4.3]
[4.6.2.5-1]

The effects of deformation on the moments in the transverse direction must also be considered. The radius of gyration for a rectangular section may be taken as 0.30 times the overall dimension in the direction in which stability is being considered. For the transverse direction: r = 0.30(6.00) = 1.80 ft. For a member free at the top but fixed at the base, the design value of k = 2.1 is obtained from the alignment chart.

The slenderness ratio is calculated as follows:

$$\frac{kl_u}{r} = \frac{(2.1)\cdot(33.00)}{1.80} = 38.5$$

Since the slenderness ratio is greater than 22, slenderness effects must be considered in the transverse direction. Since the slenderness ratio is less than 100 the moment magnifier method can be used.

In lieu of a more precise method, EI shall be taken as the greater of:

[5.7.4.3-1]

$$EI = \frac{\frac{E_c I_g}{5} + E_s I_s}{1 + \beta_d}$$

[5.7.4.3-2]

$$EI = \frac{\frac{E_c I_g}{2.5}}{1 + \beta_d}$$

The first equation will control for large percentages of reinforcement. The second equation will control when the reinforcing percentage is small. At this stage the reinforcing pattern is usually not known requiring that I_s be estimated or ignored. See Figure 17 for the reinforcing pattern for this problem.

$$I_s = \sum A_s d^2 = 1.56[(7(32.67)^2 + 2(21.78)^2 + 2(10.89)^2](2) = 27{,}011 \text{ in}^4$$

$$I_g = (4.00)(6.00)^3 \div 12 = 72.00 \text{ ft}^4$$

The permanent moment on the column includes dead load moment. For a column assumed free at the top with a symmetrical load, the moment from dead load is zero resulting in the value β_d also equal to zero.

$$EI = \frac{\frac{(490{,}320)\cdot(72.00)}{5} + (27{,}011)\cdot(29000)\div 144}{1+0} = 12{,}500{,}000 \text{ k-ft}^2$$

$$EI = \frac{\frac{(490{,}320)\cdot(72.00)}{2.5}}{1+0} = 14{,}121{,}000 \text{ k-ft}^2 \text{ <= Critical}$$

The critical load, required by the moment magnifier method, is determined as follows:

[4.5.3.2.2b-5]

$$P_e = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2\cdot(14{,}121{,}000)}{[(2.1)\cdot(33.00)]^2} = 29{,}020 \text{ k}$$

Strength I Limit State

[4.5.3.2.2b-3]
[4.5.3.2.2b-4]

$$\delta_b = \delta_s = \frac{C_m}{1-\frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1-\frac{2080}{(0.75)\cdot(29{,}020)}} = 1.106$$

The magnified moments are increased to reflect effects of deformation as follows:

[4.5.3.2.2b-1]

$M_c = \delta_b M_{2b} + \delta_s M_{2s}$

M_c = 1.106(0) + 1.106(3507) = 3879 ft-k

Strength III Limit State

$$\delta_b = \delta_s = \frac{1}{1-\frac{P_u}{\varphi_K P_e}} = \frac{1}{1-\frac{1456}{(0.75)\cdot(29{,}020)}} = 1.072$$

M_c = 1.072(0) + 1.072(2311) = 2477 ft-k

Strength V Limit State

$$\delta_b = \delta_s = \frac{1}{1-\frac{P_u}{\varphi_K P_e}} = \frac{1}{1-\frac{1937}{(0.75)\cdot(29{,}020)}} = 1.098$$

M_c = 1.098(0) + 1.098(3556) = 3904 ft-k

These magnified moments are used with the factored axial load to design the compression member. A summary of the load combinations is shown in the table below.

	P	M_{trans}	M_{long}	Ratio
Strength I	2080	3879	2549	1.301
Strength I	889	3879	2549	1.274
Strength III	1456	2477	2543	1.587
Strength III	880	2477	2543	1.520
Strength V	1937	3904	2834	1.231
Strength V	904	3904	2834	1.204

From the results from the column analysis program the column has a minimum ratio of factored strength to factored load of 1.204 for the critical limit state (Strength V with the minimum axial load). The column is adequately reinforced with 24 # 11 longitudinal bars as shown in Figure 17.

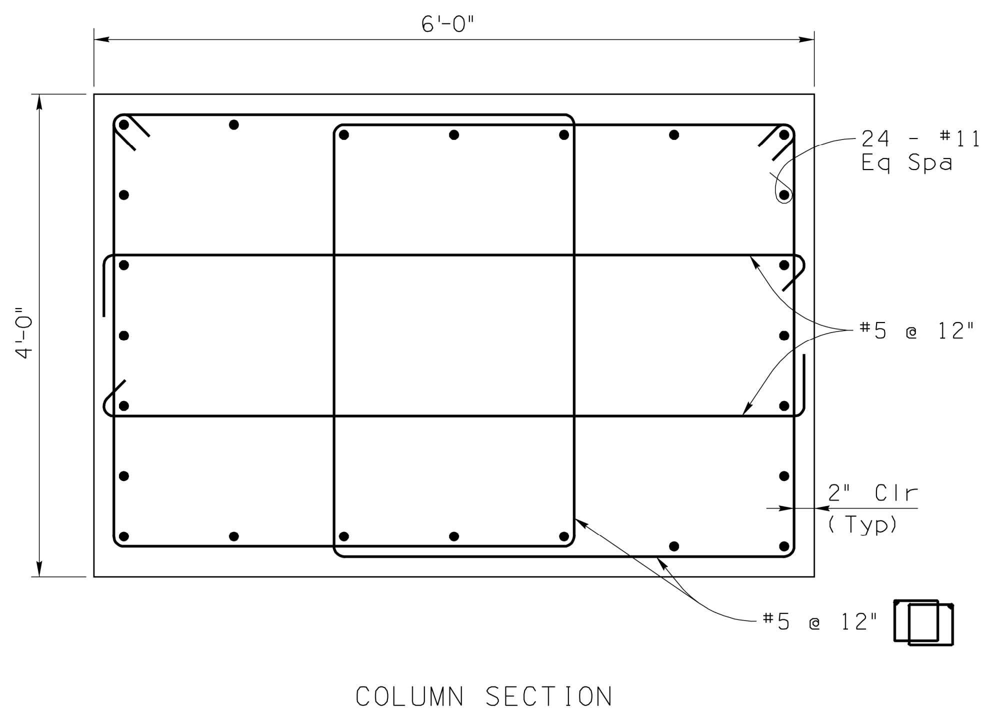


Figure 17

Column Shear
[5.8]

Step 1 – Determine Shear

Shear design is based on strength limit states. The transverse and longitudinal column shears are taken from the loads as shown in the following table.

Load	V_{trans}	V_{long}
	k	k
DC	0	0
DW	0	0
LL + IM	0	0
BR	0	18.0
WA	13.6	10.1
WS_{super}	28.8	16.4
WS_{sub}	5.3	13.4
WS	34.1	29.8
WL	8.6	4.9
TU	0	23.8
CR + SH	0	28.3
SE	0	0

Weight of the column 0.15(4.0)(6.0)(25.0) = 90.0 k

[3.4.1]

STRENGTH I

Max = 1.25DC + 1.50DW + 0.50(CR + SH) + 1.75(LL + IM + BR)
+ 1.00WA + 0.50TU + 1.00SE

P_{min} = 0.90(1074.6 – 90.0) + 1.75(-37.0) + 1.00(-13.3) = 808 k

V_{trans} = 1.25(0) + 1.50(0) + 0.50(0) + 1.75(0) + 1.00(13.6) + 0.5(0)
+ 1.00(0) = 13.6 k

V_{long} = 1.25(0) + 1.50(0) + 0.50(28.3) + 1.75(18.0) + 1.00(10.1)
+ 0.50(23.8) + 1.00(0) = 67.7 k

STRENGTH III

Max = 1.25DC + 1.50DW + 0.50(CR + SH) + 1.00WA
+ 1.40(WS + WS_{vert}) + 0.50TU + 1.00SE

P_{min} = 0.90(1074.6 – 90.0) + 1.40(-53.0) + 1.00(-13.3) = 799 k

V_{trans} = 1.25(0) + 1.50(0) + 0.50(0) + 1.00(13.6) + 1.40(34.1)
+ 0.50(0) + 1.00(0) = 61.3 k

V_{long} = 1.25(0) + 1.50(0) + 0.50(28.3) + 1.00(10.1) + 1.40(29.8)
+ 0.50(23.8) + 1.00(0) = <u>77.9</u> k <= Critical

STRENGTH V

Max = 1.25DC + 1.50DW + 0.50(CR + SH) + 1.35(LL + IM + BR)
+ 1.00WA + 0.40WS + 1.00WL + 0.50TU + 1.00SE

P_{min} = 0.90(1074.6 – 90.0) + 1.35(-37.0) + 1.00(-13.3) = 823 k

V_{trans} = 1.25(0) + 1.50(0) + 0.50(0) + 1.35(0) + 1.00(13.6) + 0.40(34.1)
+ 1.00(8.6) + 0.50(0) + 1.00(0) = 35.8 k

V_{long} = 1.25(0) + 1.50(0) + 0.50(28.3) + 1.35(18.0) + 1.00(10.1)
+ 0.40(29.8) + 1.00(4.9) + 0.50(23.8) + 1.00(0) = 77.3 k

Based on the above, Strength III Limit State in the longitudinal direction with P_{min} = 799 kips and V_U = 77.9 kips controls the column shear design.

[5.8.3]

Step 2 – Determine Analysis Model

The sectional model of analysis is appropriate for the design of bridge columns since the assumptions of traditional beam theory are valid. Since concentrated loads are not applied directly to the column the sectional model may be used.

Longitudinal Shear [5.8.2.9]

Step 3 – Shear Depth, d_v

The shear depth is the maximum of the following three criteria:

1) d_v = 0.9 d_e where $d_e = \dfrac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} = d_s$ when A_{ps} =0

d_s = 48.00 – 2.00 clear – 0.625 – 1.41 / 2 = 44.67 in
$d_v = 0.9 d_s = 0.9(44.67) = 40.20$ in <= Critical

2) 0.72h = 0.72(48.00) = 34.56 in

3) $d_v = \dfrac{M_n}{A_s f_y + A_{ps} f_{pu}}$

For a column, M_n, is a function of the axial load making determination of this value difficult. Ignore this option and choose the greater of the first two values.

Based on the above, the shear depth, d_v, equals 40.20 inches.

Step 4 – Calculate, V_p

Since the column is not prestressed, V_p = 0 kips

Step 5 – Calculate strain, ε_s

The formula for the calculation of strain for sections containing at least the minimum amount of transverse reinforcing with positive values of strain is shown below.

[5.8.3.4.2-4]

$$\varepsilon_s = \left[\frac{\frac{|M_u|}{d_v} + 0.5N_u + |V_u - V_p| - A_{ps} f_{po}}{E_s A_s + E_p A_{ps}} \right]$$

A_{ps} = area of prestressing steel on the flexural tension side of the member.
A_{ps} = 0 in^2

A_s = area of nonprestressed steel on the flexural tension side of the member.
A_s = 12(1.56) = 18.72 in^2

f_{po} = 0 ksi for a nonprestressed member.

N_u = factored axial force taken as positive if tensile.
N_u = -799 kips

V_u = factored shear force.
V_u = 77.9 kips

M_u = factored moment but not to be taken less than $V_u d_v$.
M_u = 2543 ft-k

The moment at the critical shear location is required. Use the factored magnified moment for Strength III Limit State at the base of the column equal to 2543 ft-k but not less than $V_u d_v$ = (77.9)(40.20) / 12 = 261 ft-k.

[5.8.3.4.2-4]

$$\varepsilon_s = \left[\frac{\frac{|(2543)\cdot(12)|}{40.20} + 0.50\cdot(-799) + |77.9 - 0| - (0)\cdot(0)}{(29000)\cdot(18.72) + (28500)\cdot(0)} \right]$$

$\varepsilon_s = 0.000806$

Since the value is positive the correct formula was used.

The 2008 Interim Revisions now provide for a method of direct calculation of the variables θ and β.

[5.8.3.4.2-1]

$$\beta = \frac{4.8}{(1 + 750\varepsilon_s)} = \frac{4.8}{(1 + 750\cdot(0.000806))} = 2.99$$

[5.8.3.4.2-3]

$$\theta = 29 + 3500\varepsilon_s = 29 + (3500)\cdot(0.000806) = 31.8 \text{ degrees}$$

Step 6 - Calculate Concrete Shear Strength, V_c

The nominal shear resistance from concrete, V_c, is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316\beta\sqrt{f'_c}\,b_v d_v$$

$$V_c = 0.0316\cdot(2.99)\cdot\sqrt{3.5}\cdot(72.00)\cdot(40.20) = 511.6 \text{ kips}$$

Step 7 - Determine Required Vertical Reinforcement, V_s

[5.8.2.4]

Since $V_u = 77.9 < 0.5\varphi V_c = (0.5)(0.9)(511.6) = 230.2$ k transverse reinforcing is not required. However, the formula for strain with the minimum transverse reinforcement was used. Verify that the minimum requirement is satisfied with use of transverse reinforcement of #5 at 12 inches.

Step 8 – Minimum Transverse Reinforcement

[5.8.2.5-1]

$$A_v \geq 0.0316\sqrt{f'_c}\,\frac{b_v s}{f_y}$$

$$A_v \geq 0.0316\sqrt{3.5}\cdot\frac{(72.00)\cdot(12.0)}{60} = 0.85\,\text{in}^2$$

A_V provided = 4(0.31) = 1.24 in^2

Therefore the minimum transverse reinforcement is supplied.

Step 9 – Maximum Spacing Transverse Reinforcement

[5.8.2.9-1]

$$v_u = \frac{\left|V_u - \varphi V_p\right|}{\varphi b_w d_v} = \frac{\left|77.9 - 0.90\cdot(0)\right|}{0.90\cdot(72.00)\cdot(40.20)} = 0.030\,\text{ksi}$$

[5.8.2.7-1]

Since v_u = 0.030 < 0.125f'$_c$ = (0.125)(3.5) = 0.438 ksi

s_{max} = 0.8d_v = 0.8(40.20) = 32.2 but not greater than 24.0 inches

Therefore, the maximum transverse spacing requirement is satisfied.

Step 10 - Longitudinal Reinforcement

In addition to vertical reinforcement, shear requires a minimum amount of longitudinal reinforcement. The requirement for longitudinal reinforcement follows:

[5.8.3.5-1]

$$A_s f_y + A_{ps} f_{ps} \geq \frac{M_u}{d_v \phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\frac{V_u}{\phi_v} - 0.5V_s - V_p\right)\cot\theta$$

Note that the Specification states that the area of longitudinal reinforcement need not be greater than the area required to resist the maximum moment alone. This requirement cannot be applied to members that are designed for axial load and moment. Since the column is adequately reinforced the above exemption is used.

Therefore the column is adequately reinforced for longitudinal shear using #5 at 12 inches. In an actual design the shear in the transverse direction would require investigation.

Footing Design
[5.13.3]

General

The methods used to estimate loads for the design of foundations using LRFD are fundamentally no different than procedures used in the past for ASD. What has changed is the use of factored loads for evaluation of foundation stability (bearing resistance, sliding resistance and limiting eccentricity for spread footing foundations). The design of foundations supporting bridge piers should consider all limit states loading conditions applicable to the structure being designed. The following Strength Limit States may control the design and should be investigated:

Strength I Limit State will control for high live to dead load ratios.

Strength III or V will control for structures subjected to high wind loads

All loads at the top of the footing are the same as those at the bottom of the column except for the dynamic load allowance which does not apply to design of foundations.

A spread footing foundation will be evaluated for the following failure conditions:

[10.5.2]
[10.5.3]

1. Bearing Resistance – Strength Limit States

2. Settlement – Service I Limit State

3. Sliding Resistance – Strength Limit States

4. Limiting Eccentricity (Overturning) – Strength Limit States

5. Overall Stability – Service I Limit State

6. Structural Resistance – Service I and Strength Limit States

1. Bearing Resistance

[11.6.3]

Bearing resistance check is a strength limit state. The appropriate Strength Limit States are I, III and V. The maximum bearing load will be found by applying the maximum load factors to each applicable load. The factored bearing resistance will be provided in the Geotechnical Foundation Report. The factored bearing resistance of the rock is 60 ksf. The load factors and load combinations are shown below:

Strength I (max) Limit State:

Strength I = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH) + 1.75(LL + BR) + 1.00WA + 0.50TU + 1.00SE

[3.4.1]

$$P_{max} = 1.25(1263.6) + 1.50(66.2) + 1.35(130.7) + 1.75(297.4 + 0) + 1.00(0) + 0.50(0) + 1.00(13.3) = 2389 \text{ k}$$

<u>Longitudinal</u>

For second order effects, the axial load should include the applied load at the top plus half the column weight. For a hammerhead pier the entire cap should be added with half the remaining column weight. Since the column load in this example is small, conservatively use the reaction at the base of the column.

$$P_u = 1.25(1074.6) + 1.50(66.2) + 1.75(297.4) + 1.00(13.3) = 1976 \text{ k}$$

$$\delta_b = \delta_s = \frac{C_m}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1 - \frac{1976}{(0.75)\cdot(12{,}898)}} = 1.257$$

For a footing founded on rock the column will be fixed at the top of the footing. To determine the moment at the base of the footing, the moment magnifier should be applied to the moment at the base of the column and added to the moment caused by the shear times the footing thickness plus the loads directly applied to the footing. A conservative alternate method would be to apply the moment magnifier to the moment at the base of the footing. Both methods will be shown for this case but subsequent calculations will only use the later method.

Method 1 (primary moment at top of footing)

$$M_{long} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(935) + 1.75(0 + 594) + 1.00(101) + 0.50(785) + 1.00(0) = 2001 \text{ ft-k}$$

At base of footing:

$$M_{long} = (1.257)(2001) + (67.7)(3.00) + (1.365)(3.00)^2 \div 2 = 2724 \text{ ft-k}$$

Method 2 (At base of footing):

M_{long} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(1020) + 1.75(0 + 648) + 1.00(137) + 0.50(856) + 1.00(0) = 2209 ft-k

M_{long} = (1.257)(2209) = 2777 ft-k

The two values are close. Method 2 is higher because the loads below the bottom of the column were also amplified. A footing supported on soil can rotate and some amplification can occur over the depth of the footing but for a footing supported on solid rock one would not expect much rotation. However, for a conservative simplification Method 2 will be used in all subsequent calculations.

Transverse

$$\delta_b = \delta_s = \frac{C_m}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1 - \frac{1976}{(0.75)\cdot(29{,}020)}} = 1.100$$

M_{trans} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(0) + 1.75(1606 + 0) + 1.00(189) + 0.5(0) + 1.00(0) = 3000 ft-k

M_{trans} = (1.100)(3000) = 3300 ft-k

Factored Bearing Pressure

[11.6.1.4]

Based on an iterative design process, the footing will be 20 feet in the longitudinal direction, 21 feet in the transverse direction and 3 feet thick as shown in Figures 4 and 21. When proportioning footing dimensions to meet settlement and bearing resistance requirements, the distribution of bearing stress on the effective area shall be assumed to be linearly varying (triangular or trapezoidal as applicable) for footings on rock.

$$q_{\max} = \frac{P}{A} + \frac{M_{long}}{S_{long}} + \frac{M_{trans}}{S_{trans}}$$

A = (21.00)(20.00) = 420 ft^2
S_{long} = $(21.00)(20.00)^2 \div 6$ = 1400 ft^3
S_{trans} = $(20.00)(21.00)^2 \div 6$ = 1470 ft^3

$$q_{\max} = \frac{2389}{420} + \frac{2777}{1400} + \frac{3300}{1470} = 5.69 + 1.98 + 2.24 = 9.91 \text{ ksf}$$

q_{min} = 5.69 – 1.98 – 2.24 = 1.47 ksf > 0

Since there is no uplift for this load case, the factored bearing pressures are as shown above.

Strength III (max) Limit State:

[3.4.1]

Strength III = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH) + 1.00WA + 1.40WS + 0.50TU + 1.00SE

P_{max} = 1.25(1263.6) + 1.50(66.2) + 1.35(130.7) + 0.50(0) + 1.00(0) + 1.40(0) + 0.50(0) + 1.00(13.3) = 1869 k

Longitudinal

P_u = 1.25(1074.6) + 1.50(66.2) + 1.00(13.3) = 1456 k

$$\delta_b = \delta_s = \frac{C_m}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1 - \frac{1456}{(0.75)\cdot(12{,}898)}} = 1.177$$

M_{long} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(1020) + 1.00(137) + 1.40(946) + 0.50(856) + 1.00(0) = 2399 ft-k

M_{long} = (1.177)(2399) = 2824 ft-k

Transverse

$$\delta_b = \delta_s = \frac{C_m}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1 - \frac{1456}{(0.75)\cdot(29{,}020)}} = 1.072$$

M_{trans} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(0) + 1.00(189) + 1.40(1246 + 408) + 0.5(0) + 1.00(0) = 2505 ft-k

M_{trans} = (1.072)(2505) = 2685 ft-k

Factored Bearing Pressure

$$q_{\max} = \frac{1869}{420} + \frac{2824}{1400} + \frac{2685}{1470} = 4.45 + 2.02 + 1.83 = 8.30 \text{ ksf}$$

q_{min} = 4.45 – 2.02 – 1.83 = 0.60 ksf > 0

[3.4.1]

Strength V (max) Limit State:

Strength V = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH)
+ 1.35(LL+BR) + 1.00WA + 0.40WS + 1.00WL + 0.50TU
+ 1.00SE

P_{max} = 1.25(1263.6) + 1.50(66.2) + 1.35(130.7) + 0.50(0)
+ 1.35(297.4 + 0) + 1.00(0) + 0.40(0) + 1.00(0) + 0.50(0)
+ 1.00(13.3) = 2270 k

Longitudinal

P_u = 1.25(1074.6) + 1.50(66.2) + 1.35(297.4) + 1.00(13.3) = 1857 k

$$\delta_b = \delta_s = \frac{C_m}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1 - \frac{1857}{(0.75)\cdot(12{,}898)}} = 1.238$$

M_{long} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(1020) + 1.35(0 + 648)
+ 1.00(137) + 0.40(946) + 1.00(176) + 0.50(856) + 1.00(0)
= 2504 ft-k

M_{long} = (1.238)(2504) = 3100 ft-k

Transverse

$$\delta_b = \delta_s = \frac{C_m}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1 - \frac{1857}{(0.75)\cdot(29{,}020)}} = 1.093$$

M_{trans} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(0) + 1.35(1606 + 0)
+ 1.00(189) + 0.40(1246) + 1.00(388) + 0.50(0) + 1.00(0)
= 3244 ft-k

M_{trans} = (1.093)(3244) = 3546 ft-k

Factored Bearing Pressure

$$q_{max} = \frac{2270}{420} + \frac{3100}{1400} + \frac{3546}{1470} = 5.40 + 2.21 + 2.41 = \underline{10.02} \text{ ksf} \Leftarrow \text{Critical}$$

q_{min} = 5.40 – 2.21 – 2.41 = 0.78 ksf > 0

Since the maximum factored soil stress of 10.02 ksf (Strength V max Limit State) is less than the factored bearing resistance of 60 ksf, the bearing strength criterion is satisfied.

2. Settlement

[11.5.2]

Settlement is a service limit state. For a multi-span bridge differential settlement can cause unacceptable structural distress in the bridge. There are also limits to total settlement to ensure a smooth ride and for aesthetics. The Geotechnical Foundation Report will provide information on settlement typically in the form of settlement versus factored bearing stress. The bridge engineer will determine the amount of deformation the structure can tolerate and ensure that the service limit bearing stress does not exceed the factored resistance for the given deformation.

[BDG]

For a footing on spread footings supported on rock the maximum elastic settlement will generally be less than ½ inch. For the multi-span precast box beam bridge, where continuity is considered for only the composite dead loads, the maximum settlement should be less than ½ inch. Based on this tolerable total settlement, the factored bearing stress for the given settlement is 10.0 ksf. The maximum allowable rotation is 0.000625 radians to avoid in-depth analysis. For this problem the allowable settlement is 0.000625(86.50)(12) = 0.65 inches. An in-depth analysis is not required but will be performed to demonstrate how the analysis is performed

[3.4.1]

Service I Limit State:

Service I = 1.00(DC + DW + EV + CR + SH) + 1.00(LL + BR) + 1.00WA + 0.30WS + 1.00WL + 1.00TU + 1.00SE

P_{max} = 1.00(1263.6 + 66.2 + 130.7) + 1.00(297.4) + 1.00(13.3) = 1771 k

Longitudinal

For second order effects, the axial load should include the applied load at the top plus half the column weight. For a hammerhead pier the entire cap should be added with half the remaining column weight. Since the column load in this example is small, conservatively use the reaction at the base of the column.

P_{col} = 1.00(1074.6 + 66.2 + 0) + 1.00(297.4) + 1.00(13.3) = 1452 kips

$$\delta_b = \delta_s = \frac{C_m}{1-\frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1-\frac{1452}{(0.75)\cdot(12{,}898)}} = 1.177$$

M_{long} = 1.00(0 + 0 + 0 + 1020) + 1.00(0 + 648) + 1.00(137) + 0.30(946) + 1.00(176) + 1.00(856) + 1.00(0) = 3121 ft-k

M_{long} = (1.177)(3121) = 3673 ft-k

Transverse

$$\delta_b = \delta_s = \frac{C_m}{1 - \frac{P_u}{\varphi_K P_e}} = \frac{1.0}{1 - \frac{1452}{(0.75)\cdot(29{,}020)}} = 1.071$$

M_{trans} = 1.00(0 + 0 + 0 + 0) + 1.00(1606 + 0) + 1.00(189) + 0.30(1246) + 1.00(388) + 1.00(0) + 1.00(0) = 2557 ft-k

M_{trans} = (1.071)(2557) = 2739 ft-k

[11.6.1.4]

When proportioning footing dimensions to meet settlement and bearing resistance requirements, the distribution of bearing stress on the effective area shall be assumed to be linearly varying (triangular or trapezoidal as applicable) for footings on rock.

$$q_{\max} = \frac{P}{A} + \frac{M_{long}}{S_{long}} + \frac{M_{trans}}{S_{trans}}$$

$$q_{\max} = \frac{1771}{420} + \frac{3673}{1400} + \frac{2739}{1470} = 4.22 + 2.62 + 1.86 = 8.70\,\text{ksf}$$

q_{min} = 4.22 – 2.62 – 1.86 = -0.26 ksf < 0

Since uplift occurs use of the above formula for q_{max} underestimates the actual maximum stress. It is not desirable to have any uplift in a footing but it is not always avoidable for pier footings with moments in both directions. When uplift occurs, the determination of the maximum soil stress is very difficult and involves trial and error methods. Curves such as Figure 4.4.7.1.1.1C in the AASHTO Standard Specifications 17th Edition 2002 may be used to obtain an approximate solution. The result of using this curve is that the maximum soil stress is computed to be 8.86 ksf which is slightly higher than that determined above.

$q_r = \varphi q_n$ = (1.0)(8.86) = 8.86 ksf

Since the service limit bearing resistance of 10.0 ksf is greater than the applied service bearing pressure of 8.86 ksf, the settlement criterion is satisfied.

3. Sliding Resistance

[11.6.3.6]
[10.6.3.3]

Spread footings must be designed to resist lateral loads without sliding failure of the foundation. The sliding resistance of a footing on rock is based on the normal stress and the interface friction between the foundation and the rock. The Geotechnical Foundation Report should provide the interface friction value and the resistance factor for sliding for use in design.

[11.5.3]

The Strength Limit States are used for this analysis. However, since the resistance is based on the reaction, minimum factors are used for all vertical loads. The most critical case is for the pier footing overburden soil to be removed, so EV will be taken as zero assuming scour has occurred.

[10.5.5-1]

From given geotechnical data, the factored interface coefficient of friction is 0.45 and the resistance factor is 0.80.

[3.4.1]

Strength I (min) Limit State:

Max Strength I = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH)
+ 1.75(LL+BR) + 1.00WA + 0.50TU + 1.00SE

Min Strength I = 0.90DC + 0.65DW + 1.00EV + 0.50(CR + SH)
+ 1.75(LL+BR) + 1.00WA + 0.50TU + 1.00SE

$$P_{min} = 0.90(1263.6) + 0.65(0) + 1.00(0) + 0.50(0) + 1.75(-30.4 + 0) + 1.00(0) + 0.50(0) + 1.00(-13.3) = 1071 \text{ k}$$

$$V_{long} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(28.32) + 1.75(0 + 18.00) + 1.00(14.24) + 0.50(23.79) + 1.00(0) = 71.8 \text{ k}$$

$$V_{trans} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(0) + 1.75(0 + 0) + 1.00(20.34) + 0.50(0) + 1.00(0) = 20.3 \text{ k}$$

$$V_u = \sqrt{(71.8)^2 + (20.3)^2} = 74.6\ k$$

$$V_n = (0.45)(1071) = 482 \text{ k}$$

$$V_r = \varphi_\tau V_n = (0.80)(482) = 386 \text{ k}$$

[3.4.1]

Strength III (min) Limit State:

Max Strength III = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH)
+ 1.00WA + 1.40WS + 0.50TU + 1.00SE

Min Strength III = 0.90DC + 0.65DW + 1.00EV + 0.50(CR + SH)
+ 1.00WA + 1.40WS + 0.50TU + 1.00SE

P_{min} = 0.90(1263.6) + 0.65(0) + 1.35(0) + 0.50(0) + 1.00(0) + 1.40(-53.0)
+ 0.50(0) + 1.00(-13.3) = 1050 k

V_{long} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(28.32) + 1.00(14.24)
+ 1.40(29.84) + 0.50(23.79) + 1.00(0) = 82.1 k

V_{trans} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(0) + 1.00(20.34)
+ 1.40(34.09) + 0.50(0) + 1.00(0) = 68.1 k

$$V_u = \sqrt{(82.1)^2 + (68.1)^2} = 106.7\ k$$

V_n = (0.45)(1050) = 473 k

$V_r = \varphi_\tau V_n$ = (0.80)(473) = 378 k

[3.4.1]

Strength V (min) Limit State:

Max Strength V = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH)
+ 1.35(LL+BR) + 1.00WA + 0.40WS + 1.00WL
+ 0.50TU + 1.00SE

Min Strength V = 0.90DC + 0.65DW + 1.00EV + 0.50(CR + SH)
+ 1.35(LL+BR) + 1.00WA + 0.40WS + 1.00WL
+ 0.50TU + 1.00SE

P_{min} = 0.90(1263.6) + 0.65(0) + 1.00(0) + 0.50(0) + 1.35(-30.4 + 0)
+ 1.00(0) + 0.40(0) + 1.00(0) + 0.50(0) + 1.00(-13.3) = 1083 k

V_{long} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(28.32) + 1.35(0 + 18.00)
+ 1.00(14.24) + 0.40(29.84) + 1.00(4.88) + 0.50(23.79)
+ 1.00(0) = 81.4 k

V_{trans} = 1.25(0) + 1.50(0) + 1.35(0) + 0.50(0) + 1.35(0 + 0) + 1.00(20.34)
+ 0.40(34.09) + 1.00(8.59) + 0.50(0) + 1.00(0) = 42.6 k

$$V_u = \sqrt{(81.4)^2 + (42.6)^2} = 91.9\ k$$

V_n = (0.45)(1083) = 487 k

$V_r = \varphi_\tau V_n$ = (0.80)(487) = 390 k

Since the factored sliding resistance for all limit states exceeds the factored sliding force, the sliding resistance criterion is satisfied.

[11.6.3.3]

4. Limiting Eccentricity (Overturning)

Spread footing foundations must be designed to resist overturning which results from lateral and eccentric vertical loads. For LRFD, the criteria were revised to reflect the factoring of loads. As a result, the eccentricity of footings for factored loads must be less than 3/8B and 3/8L for footings on rock. These new limits were developed by direct calibration with ASD. The effect of factoring the loads is to increase the eccentricity of the load resultant such that the permissible eccentricity is increased.

The appropriate Strength Limit States are I, III and V. The maximum eccentricity will be found by applying the maximum load factors to each lateral or eccentrically applied load but to apply the minimum load factors to the resisting loads. The load combinations are the same as for sliding except moments are grouped instead of lateral loads. The maximum moments were previously calculated to determine the bearing pressure.

Strength I (min) Limit State

Max Strength I = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH)
+ 1.75(LL+BR) + 1.00WA + 0.50TU + 1.00SE

Min Strength I = 0.90DC + 0.65DW + 1.00EV + 0.50(CR + SH)
+ 1.75(LL+BR) + 1.00WA + 0.50TU + 1.00SE

P_{min} = 0.90(1263.6) + 0.65(0) + 1.00(0) + 0.50(0) + 1.75(-30.4 + 0)
+ 0.50(0) + 1.00(-13.3) = 1071 k

M_{long} = 2777 ft-k

M_{trans} = 3300 ft-k

e_{long} = 2777 / 1071 = 2.59 ft < 3/8B = (3/8)(20.00) = 7.50 ft
e_{trans} = 3300 / 1071 = 3.08 ft < 3/8L = (3/8)(21.00) = 7.88 ft

Strength III (min) Limit State

Max Strength III = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH)
+ 1.00WA + 1.40WS + 0.50TU + 1.00SE

Min Strength III = 0.90DC + 0.65DW + 1.00EV + 0.50(CR + SH)
+ 1.00WA + 1.40WS + 0.50TU + 1.00SE

P_{min} = 0.90(1263.6) + 0.65(0) + 1.00(0) + 0.50(0) + 1.00(0) + 1.40(-53.0)
+ 0.50(0) + 1.00(-13.3) = 1050 k

M_{long} = 2824 ft-k

M_{trans} = 2685 ft-k

e_{long} = 2824 / 1050 = 2.69 ft < 7.50 ft
e_{trans} = 2685 / 1050 = 2.56 ft < 7.88 ft

Strength V (min) Limit State

Max Strength V = 1.25DC + 1.50DW + 1.35EV + 0.50(CR + SH)
+ 1.35(LL+BR) + 1.00WA + 0.40WS +1.00WL
+ 0.50TU + 1.00SE

Min Strength V = 0.90DC + 0.65DW + 1.00EV + 0.50(CR + SH)
+ 1.35(LL+BR) + 1.00WA + 0.40WS +1.00WL
+ 0.50TU + 1.00SE

P_{min} = 0.90(1263.6) + 0.65(0) + 1.00(0) + 0.50(0) + 1.35(-30.4 + 0)
+ 1.00(0) + 0.40(0) + 1.00(0) + 0.50(0) + 1.00(-13.3) = 1083 k

M_{long} = 3100 ft-k

M_{trans} = 3546 ft-k

e_{long} = 3100 / 1083 = 2.86 ft < 7.50 ft
e_{trans} = 3546 / 1083 = 3.27 ft < 7.88 ft

Since the eccentricity for each strength limit state is within the allowable limit, the pier is stable.

5. Overall Stability

Overall stability is a service limit state that depends upon the properties and orientation of the joints of the supporting rock mass. Satisfying the design criteria is the responsibility of the geotechnical engineer. The results of this analysis should be included in the Geotechnical Foundation Report.

6. Structural Resistance

[11.6.4]

All parts of the footing must satisfy the appropriate strength and serviceability requirements. In this example service limit state is investigated first since it will often control the amount of reinforcing in a footing.

Transverse Design

The transverse direction will be investigated first. To determine the critical moment and shear the distribution of the soil stress must be known. A one foot wide strip will be used for the analysis and design of the footing. A linear variation in the soil stress is assumed for the structural design of the footing.

Service I Limit State

P_{max} = 1771 k

M_{trans} = 2739 ft-k

Area = (20.00)(21.00) = 420 ft^2

$S_{trans} = (20.00)(21.00)^2 \div 6 = 1470\ ft^3$

f_{max} = 1771 / 420 + 2739 / 1470 = 6.080 ksf

f_{min} = 1771 / 420 - 2739 / 1470 = 2.353 ksf

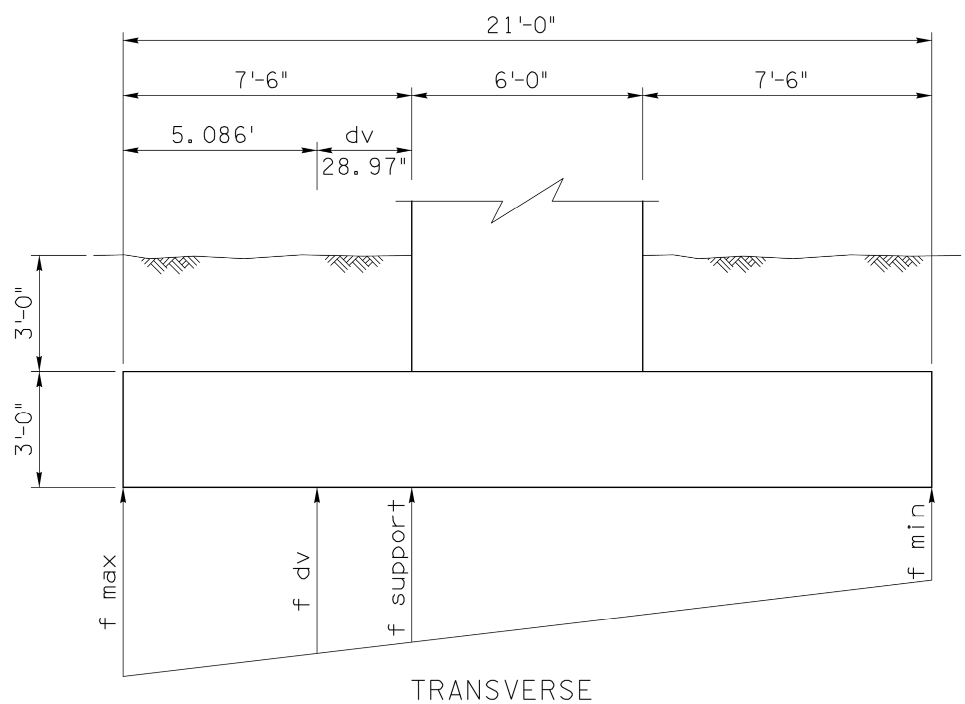


Figure 18

[5.13.3.4]

At the face of support:

$$f_{support} = 6.080 - (6.080 - 2.353)(7.50) / (21.00) = 4.749 \text{ ksf}$$

$$M_{trans} = 4.749(7.50)^2 \div 2 + (6.080 - 4.749)(7.50)^2 \div 3 - 1.00(0.15)(3.00)(7.50)^2 \div 2 - 1.00(0.11)(3.00)(7.50)^2 \div 2 = 136.58 \text{ ft-k/ft}$$

Try #10 @ 6 inches

$$A_s = (1.27)(12 / 6) = 2.540 \text{ in}^2\text{/ft}$$

Control of Cracking
[5.7.3.4]

The footing must be checked for cracking loads. This section applies to all members in which tension in the cross-section exceeds 80 percent of the modulus of rupture at service limit state.

$$0.80 f_r = (0.80) \cdot \left(0.24\sqrt{3.5}\right) = 0.359 \; ksi$$

The stress in the footing under service loads is:

$$S_c = \frac{bh^2}{6} = \frac{(12.0) \cdot (36.0)^2}{6} = 2592 \; in^3\text{/ft}$$

$$f_s = \frac{M_s}{S_c} = \frac{(136.58) \cdot (12.0)}{2592} = 0.632 \; ksi$$

Since the service limit stress exceeds 80 percent of the cracking load stress, the crack control criteria must be satisfied.

[5.7.3.4-1]

$$s \le \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c$$

in which:

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

$$d_c = 3.00 \text{ clr} + 1.27 + 1.27 / 2 = 4.91 \text{ in}$$

$$\beta_s = 1 + \frac{4.91}{0.7 \cdot (36.00 - 4.91)} = 1.226$$

γ_e = 0.75 for class 2 exposure since the footing is exposed to soil and water.

Determine the service limit state stress in the reinforcement.

d_s = 36.00 – 3.0 clr – 1.27 – 1.27 / 2 = 31.10 in

$$p = \frac{A_s}{bd_s} = \frac{2.540}{(12)\cdot(31.10)} = 0.00681$$

np = 9(0.00681) = 0.0613

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.0613) + (0.0613)^2} - 0.0613 = 0.294$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.294}{3} = 0.902$$

$$f_{ss} = \frac{M_s}{A_s jd_s} = \frac{(136.58)\cdot(12)}{(2.540)\cdot(0.902)\cdot(31.10)} = 23.00 \text{ ksi} <= 0.60f_y = 36.0 \text{ ksi}$$

$$s \le \frac{700\cdot(0.75)}{(1.226)\cdot(23.00)} - 2\cdot(4.91) = 8.8\ in$$

Since the 6 inch spacing of the reinforcement is less than the maximum allowable spacing, the service limit state criteria for crack control is satisfied.

An alternate method to satisfy this requirement is to solve the equation for the maximum spacing for the maximum stress. For a given reinforcing spacing the more traditional comparison of stresses can be performed.

$$f_s = \frac{700\gamma_e}{\beta_s(s + 2d_c)} = \frac{700\cdot(0.75)}{(1.226)\cdot(6 + 2\cdot(4.91))} = 27.07ksi$$

Since the stress in the reinforcing of 23.00 ksi is less than 27.07 ksi the criteria for crack control is satisfied.

[3.4.1] **Strength I Limit State**

P_{max} = 2389 k
M_{trans} = 3300 ft-k

f_{max} = 2389 / 420 + 3300 / 1470 = 7.933 ksf

f_{min} = 2389 / 420 - 3300 / 1470 = 3.443 ksf

[5.13.3.4] At the face of support:

$f_{support}$ = 7.933 – (7.933 – 3.443)(7.50) / (21.00) = 6.329 ksf

$M_{Utrans} = 6.329(7.50)^2 \div 2 + (7.933 - 6.329)(7.50)^2 \div 3$
$- 1.25(0.15)(3.00)(7.50)^2 \div 2 - 1.35(0.11)(3.00)(7.50)^2 \div 2$
= <u>179.73</u> ft-k/ft <= Critical

[5.8.2.9] The shear depth, d_v, is determined as the <u>greatest</u> of the following criteria:

d_e = 36.00 – 3.00 clr – 1.27(1.5) = 31.10 in
a = (2.540)(60) / [0.85(3.5)(12)] = 4.27 in

1) d_v = 31.10 – 4.27 / 2 = <u>28.97 in</u> <= Critical

but not less than:
2) 0.9 d_e = (0.9)(31.10) = 27.99 in
3) 0.72 h = (0.72)(36.00) = 25.92 in

Use d_v = 28.97 in

[5.8.3.2] At the distance "d_v" from the face of the support:

Edge distance = 7.50 – 28.97 ÷ 12 = 5.086 ft

f_{dv} = 7.933 – (7.933 – 3.443)(5.086) / (21.00) = 6.846 ksf

V_{dv} = (7.933 + 6.846)(5.086) ÷2 – 1.25(0.15)(3.00)(5.086)
– 1.35(0.11)(3.00)(5.086) = <u>32.46</u> k/ft <= Critical

[3.4.1]

Strength III Limit State

P_{max} = 1869 k
M_{trans} = 2685 ft-k

f_{max} = 1869 / 420 + 2685 / 1470 = 6.277 ksf

f_{min} = 1869 / 420 - 2685 / 1470 = 2.623 ksf

[5.13.3.4]

At the face of support:
$f_{support}$ = 6.277 – (6.277 – 2.623)(7.50) / (21.00) = 4.972 ksf

$M_{Utrans} = 4.972(7.50)^2 \div 2 + (6.277 - 4.972)(7.50)^2 \div 3$
$- 1.25(0.15)(3.00)(7.50)^2 \div 2 - 1.35(0.11)(3.00)(7.50)^2 \div 2$
= 135.96 ft-k/ft

[5.8.3.2]

At the distance "d_v" equal to 28.97 inches from the face of the support:

f_{dv} = 6.277 – (6.277 – 2.623)(5.086) / (21.00) = 5.392 ksf

V_{dv} = (6.277 + 5.392)(5.086) ÷2 – 1.25(0.15)(3.00)(5.086)
– 1.35(0.11)(3.00)(5.086) = 24.55 k/ft

[3.4.1]

Strength V Limit State

P_{max} = 2270 k
M_{trans} = 3546 ft-k

f_{max} = 2270 / 420 + 3546 / 1470 = 7.817 ksf

f_{min} = 2270 / 420 - 3546 / 1470 = 2.993 ksf

[5.13.3.4]

At the face of support:
$f_{support}$ = 7.817 – (7.817 – 2.993)(7.50) / (21.00) = 6.094 ksf

$M_{Utrans} = 6.094(7.50)^2 \div 2 + (7.817 - 6.094)(7.50)^2 \div 3$
$- 1.25(0.15)(3.00)(7.50)^2 \div 2 - 1.35(0.11)(3.00)(7.50)^2 \div 2$
= 175.35 ft-k/ft

[5.8.3.2]

At the distance "d_v" equal to 28.97 inches from the face of the support:

f_{dv} = 7.817 – (7.817 – 2.993)(5.086) / (21.00) = 6.649 ksf

V_{dv} = (7.817 + 6.649)(5.086) ÷2 – 1.25(0.15)(3.00)(5.086)
– 1.35(0.11)(3.00)(5.086) = 31.66 k/ft

Flexural Resistance [5.7.3]

The flexural resistance must be greater than the largest factored moment. For this problem, Strength I Limit State produced the largest factored moment M_u equal to 179.73 ft-k/ft.

Try #10 @ 6 inches

A_s = 2.540 in^2/ft

[5.7.3.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(2.540)\cdot(60)}{(0.85)\cdot(3.5)\cdot(0.85)\cdot(12.00)} = 5.022 \text{ in}$$

[5.7.2.1]

$$\frac{c}{d_s} = \frac{5.022}{31.10} = 0.161 < 0.6$$ Therefore f_y may be used in above equation.

[5.7.3.2.3]

$$a = \beta_1 c = (0.85)\cdot(5.022) = 4.27\ in$$

The net tensile strain in the reinforcing is:

[C5.7.2.1-1]

$$\varepsilon_t = 0.003\left(\frac{d_t}{c} - 1\right) = 0.003\cdot\left(\frac{31.10}{5.022} - 1\right) = 0.016$$

[5.5.4.2.1]

Since the net tensile strain, ε_t = 0.016 > 0.005, the section is tension-controlled and the reduction factor φ = 0.90.

[5.7.3.2.2-1]

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = (2.540)\cdot(60)\cdot\left(31.10 - \frac{4.27}{2}\right) \div 12 = 367.86 \text{ ft-k/ft}$$

φM_n = (0.90)(367.86) = 331.07 ft-k/ft

Since the flexural resistance φM_n = 331.07 ft-k/ft is greater than the factored load M_u = 179.73 ft-k/ft, the strength criterion is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The provision that limited the amount of reinforcing in a section was deleted in 2005.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 times the factored moment.

S_c = (12.0)(36.00)^2 ÷ 6 = 2592 in^3/ft

$$f_r = 0.37\sqrt{3.5} = 0.692\ ksi$$

The amount of reinforcing shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

$1.2M_{cr} = 1.2 f_r S_c = (1.2) \cdot (0.692) \cdot (2592) \div 12 = 179.37$ ft-k/ft <= Critical
$1.33M_u$ = 1.33(179.73) = 239.04 ft-k/ft

Since the flexural resistance, φM_n = 331.07 ft-k/ft > 179.37 ft-k/ft, the minimum reinforcing criteria is satisfied.

Longitudinal Design

The longitudinal direction must also be investigated for the footing design. To determine the critical moment and shear the distribution of the soil stress must be determined. A linear variation is assumed for the structural design of the footing. A one foot side strip will be used for the analysis and design of the footing.

Service I Limit State

P_{max} = 1771 k
M_{long} = 3673 ft-k

Area = (20.00)(21.00) = 420 ft^2

S_{long} = $(21.00)(20.00)^2 \div 6$ = 1400 ft^3

f_{max} = 1771 / 420 + 3673 / 1400 = 6.840 ksf

f_{min} = 1771 / 420 - 3673 / 1400 = 1.593 ksf

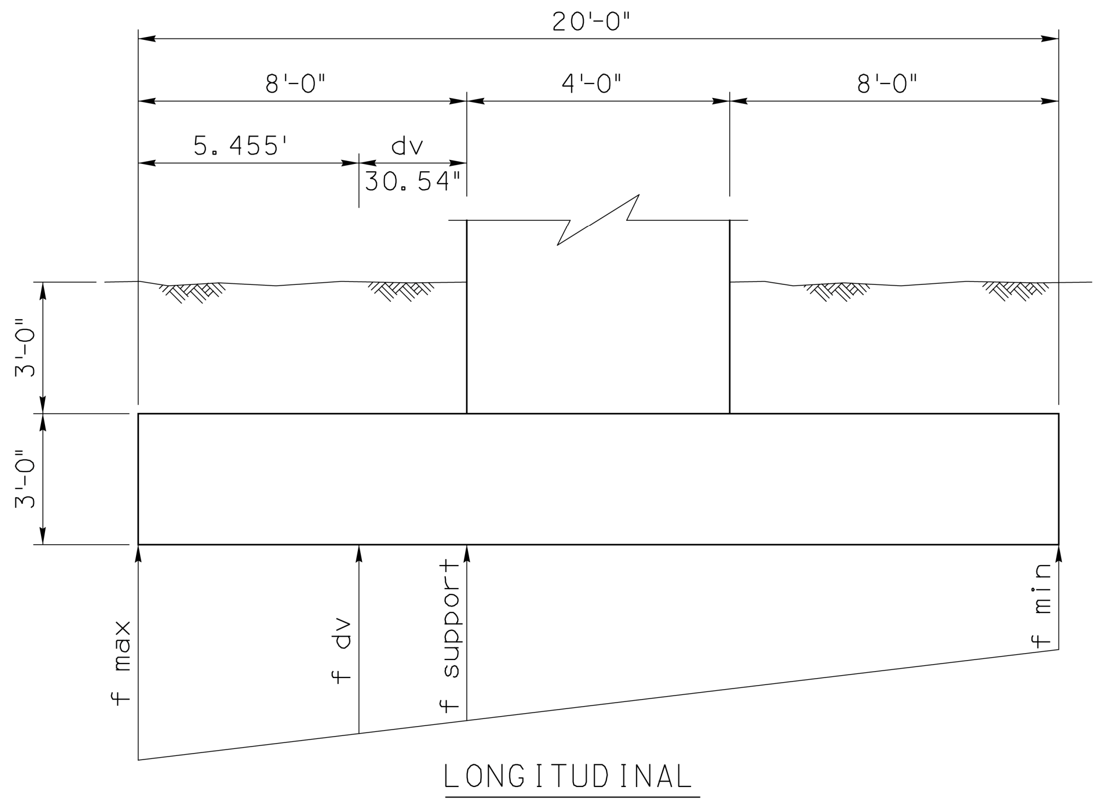


Figure 19

[5.13.3.4]

At the face of support:

$f_{support}$ = 6.840 – (6.840 – 1.593)(8.00) / (20.00) = 4.741 ksf

M_{long} = 4.741(8.00)^2 ÷ 2 + (6.840 – 4.741)(8.00)^2 ÷ 3
– 1.00(0.15)(3.00)(8.00)^2 ÷ 2 – 1.00(0.11)(3.00)(8.00)^2 ÷ 2
= 171.53 ft-k/ft

Try #10 @ 7 inches

A_s = (1.27)(12 / 7) = 2.177 in^2/ft

Control of Cracking
[5.7.3.4]

The footing must be checked for cracking loads. This section applies to all members in which tension in the cross-section exceeds 80 percent of the modulus of rupture at service limit state.

$$0.80\, f_r = (0.80)\cdot\left(0.24\sqrt{3.5}\right) = 0.359\ ksi$$

The stress in the footing under service loads is:

$$S_c = \frac{bh^2}{6} = \frac{(12.0)\cdot(36.0)^2}{6} = 2592\ in^3/\text{ft}$$

$$f_s = \frac{M_s}{S_c} = \frac{(171.53)\cdot(12.0)}{2592} = 0.794\ ksi$$

Since the service limit stress exceeds 80 percent of the cracking load stress, the crack control criteria must be satisfied.

[5.7.3.4-1]

$$s \le \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c$$

in which:

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

d_c = 3.00 clr + 1.27 / 2 = 3.64 in

$$\beta_s = 1 + \frac{3.64}{0.7\cdot(36.00 - 3.64)} = 1.161$$

γ_e = 0.75 for class 2 exposure since the footing is exposed to soil and water.

Determine the service limit state stress in the reinforcement.

d_s = 36.00 – 3.0 clr – 1.27 / 2 = 32.37 in

$$p = \frac{A_s}{bd_s} = \frac{2.177}{(12)\cdot(32.37)} = 0.00560$$

np = 9(0.00560) = 0.0504

$$k = \sqrt{2np + np^2} - np = \sqrt{2\cdot(0.0504) + (0.0504)^2} - 0.0504 = 0.271$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.271}{3} = 0.910$$

$$f_{ss} = \frac{M_s}{A_s jd_s} = \frac{(171.53)\cdot(12)}{(2.177)\cdot(0.910)\cdot(32.37)} = 32.10\,\text{ksi} <= 0.60f_y = 36.0\text{ ksi}$$

Solving for the allowable stress yields:

$$f_s = \frac{700\gamma_e}{\beta_s(s + 2d_c)} = \frac{700\cdot(0.75)}{(1.161)\cdot(7 + 2\cdot(3.64))} = 31.67ksi$$

The service limit stress in the reinforcing of 32.10 slightly exceeds the service limit allowable stress of 31.67 ksi by 1.4%. Since this overstress is within the 2% normal allowable overstress, the service limit state criteria for crack control is satisfied.

[3.4.1] **Strength I Limit State**

P_{max} = 2389 k
M_{long} = 2777 ft-k

f_{max} = 2389 / 420 + 2777 / 1400 = 7.672 ksf

f_{min} = 2389 / 420 - 2777 / 1400 = 3.705 ksf

[5.13.3.4] At the face of support

$f_{support}$ = 7.672 – (7.672 – 3.705)(8.00) / (20.00) = 6.085 ksf

$M_{long} = 6.085(8.00)^2 \div 2 + (7.672 - 6.085)(8.00)^2 \div 3$
$- 1.25(0.15)(3.00)(8.00)^2 \div 2 - 1.35(0.11)(3.00)(8.00)^2 \div 2$
= <u>196.32</u> ft-k/ft <= Critical

[5.8.2.9] The shear depth, d_v, is determined as the <u>greatest</u> of the following criteria:

d_e = 36.00 – 3.00 clr – 1.27 / 2 = 32.37 in
a = (2.177)(60) / [0.85(3.5)(12)] = 3.66 in

1) d_v = 32.37 – 3.66 / 2 = 30.54 in <= Critical

but not less than:
2) 0.9 d_e = (0.9)(32.37) = 29.13 in
3) 0.72 h = (0.72)(36.00) = 25.92 in

Use d_v = 30.54 in

[5.8.3.2] At the distance "d_v" from the face of the support:

Edge distance = 8.00 – 30.54 ÷ 12 = 5.455 ft

f_{dv} = 7.672 – (7.672 – 3.705)(5.455) / (20.00) = 6.590 ksf

V_{dv} = (7.672 + 6.590)(5.455) ÷2 – 1.25(0.15)(3.00)(5.455)
– 1.35(0.11)(3.00)(5.455) = <u>33.40</u> k/ft <= Critical

Strength III Limit State

P_{max} = 1869 k
M_{long} = 2824 ft-k

f_{max} = 1869 / 420 + 2824 / 1400 = 6.467 ksf

f_{min} = 1869 / 420 - 2824 / 1400 = 2.433 ksf

At the face of support:

[5.13.3.4] $f_{support}$ = 6.467 – (6.467 – 2.433)(8.00) / (20.00) = 4.853 ksf

$M_{long} = 4.853(8.00)^2 \div 2 + (6.467 - 4.853)(8.00)^2 \div 3 - 1.25(0.15)(3.00)(8.00)^2 \div 2 - 1.35(0.11)(3.00)(8.00)^2 \div 2$
= 157.47 ft-k/ft

[5.8.3.2] At the distance "d_v" equal to 30.54 inches from the face of the support:

f_{dv} = 6.467 – (6.467 – 2.433)(5.455) / (20.00) = 5.367 ksf

$V_{dv} = (6.467 + 5.367)(5.455) \div 2 - 1.25(0.15)(3.00)(5.455) - 1.35(0.11)(3.00)(5.455)$ = 26.78 k/ft

Strength V Limit State

P_{max} = 2270 k
M_{long} = 3100 ft-k

f_{max} = 2270 / 420 + 3100 / 1400 = 7.619 ksf

f_{min} = 2270 / 420 - 3100 / 1400 = 3.190 ksf

At the face of support:

[5.13.3.4] $f_{support}$ = 7.619 – (7.619 – 3.190)(8.00) / (20.00) = 5.847 ksf

$M_{long} = 5.847(8.00)^2 \div 2 + (7.619 - 5.847)(8.00)^2 \div 3 - 1.25(0.15)(3.00)(8.00)^2 \div 2 - 1.35(0.11)(3.00)(8.00)^2 \div 2$
= 192.65 ft-k/ft

[5.8.3.2] At the distance "d_v" equal to 30.54 inches from the face of the support:

f_{dv} = 7.619 – (7.619 – 3.190)(5.455) / (20.00) = 6.411 ksf

$V_{dv} = (7.619 + 6.411)(5.455) \div 2 - 1.25(0.15)(3.00)(5.455) - 1.35(0.11)(3.00)(5.455)$ = 32.77 k/ft

Flexural Resistance [5.7.3]

The flexural resistance must be greater than the largest factored moment. For this problem, Strength I Limit State produced the largest factored moment equal to 196.32 ft-k/ft.

Try #10 @ 7 inches

A_s = 2.177 in^2/ft
d_s = 36.00 – 3.00 clr – 1.27 / 2 = 32.37 in

[5.7.33.1.1-4]

$$c = \frac{A_s f_y}{0.85 f'_c \beta_1 b} = \frac{(2.177)\cdot(60)}{(0.85)\cdot(3.5)\cdot(0.85)\cdot(12.00)} = 4.304 \text{ in}$$

[5.7.2.1]

$$\frac{c}{d_s} = \frac{4.304}{32.37} = 0.133 < 0.6$$ Therefore f_y may be used in above equation.

[5.7.3.2.3]

$$a = \beta_1 c = (0.85)\cdot(4.304) = 3.66\ in$$

The net tensile strain in the reinforcing is:

[C5.7.2.1-1]

$$\varepsilon_t = 0.003\left(\frac{d_t}{c} - 1\right) = 0.003\cdot\left(\frac{32.37}{4.304} - 1\right) = 0.020$$

[5.5.4.2.1]

Since the net tensile strain, ε_t = 0.020 > 0.005, the section is tension-controlled and the reduction factor φ = 0.90.

[5.7.3.2.2-1]

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = (2.177)\cdot(60)\cdot\left(32.37 - \frac{3.66}{2}\right) \div 12 = 332.43 \text{ ft-k/ft}$$

φM_n = (0.90)(332.43) = 299.19 ft-k/ft

Since the flexural resistance φM_n = 299.19 ft-k/ft is greater than the factored load M_u = 196.32 ft-k/ft, the strength criterion is satisfied.

Maximum Reinforcing [5.7.3.3.1]

The provision that limited the amount of reinforcing in a section was deleted in 2005.

Minimum Reinforcing [5.7.3.3.2]

The LRFD Specification specifies that all sections requiring reinforcing must have sufficient strength to resist a moment equal to at least 1.2 times the moment that causes a concrete section to crack or 1.33 times the factored moment.

S_c = (12.0)(36.00)^2 ÷ 6 = 2592 in^3/ft
$f_r = 0.37\sqrt{3.5} = 0.692\ ksi$

The amount of reinforcing shall be adequate to develop a factored flexural resistance at least equal to the lesser of:

$1.2M_{cr} = 1.2 f_r S_c = (1.2) \cdot (0.692) \cdot (2592) \div 12 = 179.37$ ft-k/ft <= Critical
$1.33M_u = 1.33(196.32) = 261.11$ ft-k/ft

Since the flexural resistance, $\varphi M_n = 299.19$ ft-k/ft > 179.37 ft-k/ft, the minimum reinforcing criteria is satisfied.

Distribution [5.13.3.5]

Reinforcing in a footing must be properly distributed. For a rectangular shaped footing, the reinforcing in the long direction (transverse moment) may be equally distributed across the width of the footing.

In the short direction (longitudinal moment), a portion of the total reinforcing as specified below shall be distributed uniformly across the width equal to the length of the short side.

$$\beta = \frac{21.00}{20.00} = 1.05$$

$$A_{s \cdot BW} = A_{s \cdot SD}\left(\frac{2}{\beta + 1}\right) = A_{s \cdot SD}\left(\frac{2}{1.05 + 1}\right) = 0.978 A_{s \cdot SD}$$

Since the area is close to 1.0, ignore the requirement.

Development Length [5.11.2.1.1]

The reinforcing must be properly developed on both sides of the critical section. The minimum available development length is (7.50)(12) – 2 clear = 88 inches. The required development length of a #10 reinforcing bar is:

$$l_d = \frac{1.25 A_b f_y}{\sqrt{f'_c}} = \frac{(1.25)\cdot(1.27)\cdot(60)}{\sqrt{3.5}} = 50.9 \text{ inches}$$

but not less than: $0.4 d_b f_y = (0.4)\cdot(1.27)\cdot(60) = 30.5 \text{ inches}$

[5.11.2.1.3]

Modification factors that reduce l_d:

Not less than 6 inch spacing	0.8
Excess Reinforcing: 196.32 / 299.19 =	0.656

Required development:

l_d = (0.8)(0.656)(50.9) = 26.7 inch

Since the available development length is greater than the required, the development length criterion is satisfied.

Transverse Footing Shear [5.8]

Determine Shear

The maximum factored one-way shear in the footing is the Strength I Limit State value of 32.46 k/ft.

Determine Analysis Model

[5.8.1.1] The sectional model of analysis is appropriate for the design of pier footings since the assumptions of traditional beam theory are valid. Since concentrated loads are not applied directly to the footings the sectional model may be used.

Shear Depth, d_v

[5.8.3.2]
[5.8.2.9]

The shear depth has previously been calculated as d_v equal to 28.97 inches.

Estimate Crack Angle θ

[5.8.3.4.1] For concrete footings the simplified procedure may be used.

$\beta = 2.0$
$\theta = 45°$

Calculate Concrete Shear Strength, V_c

The nominal shear resistance from concrete, V_c, is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v$$

]

$$V_c = 0.0316 \cdot (2.0) \cdot \sqrt{3.5} \cdot (12.00) \cdot (28.97) = 41.10 \text{ k/ft}$$

The nominal shear strength is the lesser of:

[5.8.3.3-1] $V_n = V_c + V_s + V_p = \underline{41.10}$ k/ft <= Critical

[5.8.3.3-2] $V_n = 0.25f'_c b_v d_v + V_p = 0.25(3.5)(12.00)(28.97) + 0 = 304.19$ k/ft

$\varphi V_n = (0.90)(41.10) = 36.99 \text{ k/ft} > V_u = 32.46 \text{ k/ft}$

Since the factored applied shear in the transverse direction is less than the strength, the one-way shear criterion is satisfied.

Longitudinal Footing Shear [5.8]

Determine Shear

The maximum factored one-way shear in the footing is the Strength I Limit State value of 33.40 k/ft.

Determine Analysis Model

[5.8.1.1]

The sectional model of analysis is appropriate for the design of pier footings since the assumptions of traditional beam theory are valid. Since concentrated loads are not applied directly to the footings the sectional model may be used.

Shear Depth, d_v

[5.8.3.2] [5.8.2.9]

The shear depth has previously been calculated as d_v equal to 30.54 inches.

Estimate Crack Angle θ

[5.8.3.4.1]

For concrete footings the simplified procedure may be used.

$\beta = 2.0$
$\theta = 45°$

Calculate Concrete Shear Strength, V_c

The nominal shear resistance from concrete, V_c, is calculated as follows:

[5.8.3.3-3]

$$V_c = 0.0316\beta\sqrt{f'_c}b_v d_v$$

$$V_c = 0.0316 \cdot (2.0) \cdot \sqrt{3.5} \cdot (12.00) \cdot (30.54) = 43.33 \text{ k/ft}$$

The nominal shear strength is the lesser of:

[5.8.3.3-1]

$V_n = V_c + V_s + V_p = \underline{43.33}$ k/ft <= Critical

[5.8.3.3-2]

$V_n = 0.25f'_c b_v d_v + V_p = 0.25(3.5)(12.00)(30.54) + 0 = 320.67$ k/ft

$\varphi V_n = (0.90)(43.33) = 39.00 \text{ k/ft} > V_u = 33.40$ k/ft

Since the factored applied shear in the transverse direction is less than the strength, the shear criterion is satisfied.

Two-Way Shear

Two-way shear shall also be checked for the footing. For investigation of two-way shear the maximum vertical load will be the controlling limit state. Strength I Limit State controls with a factored axial load at the top of the footing of:

$$P_{max} = 1.25(1074.6) + 1.50(66.2) + 1.75(297.4) + 1.00(13.3) = 1976.3 \text{ kips}$$

For the effective shear depth use the average of the shear depth in the two different directions.

$$d_v = (28.97 + 30.54) \div 2 = 29.76 \text{ inches}$$

[5.13.3.6.1]

The perimeter is taken at a distance of $d_v / 2$ from the face of the column.

Trans = 72.00 + 28.97 = 100.97 inches
Long = 48.00 + 30.54 = 78.54 inches

$$b_o = (100.97 + 78.54)(2) = 359.02 \text{ inches}$$

In addition to the reaction from the bottom of the column, the weight of the footing and cover soil inside the shear perimeter will also add to the shear.

Footing: 1.25(0.150)(3.00)(78.54)(100.97) / 144 = 31.0 k
Soil: 1.35(0.110)(3.00)[(78.54)(100.97) / 144 - (4)(6)] = 13.8 k

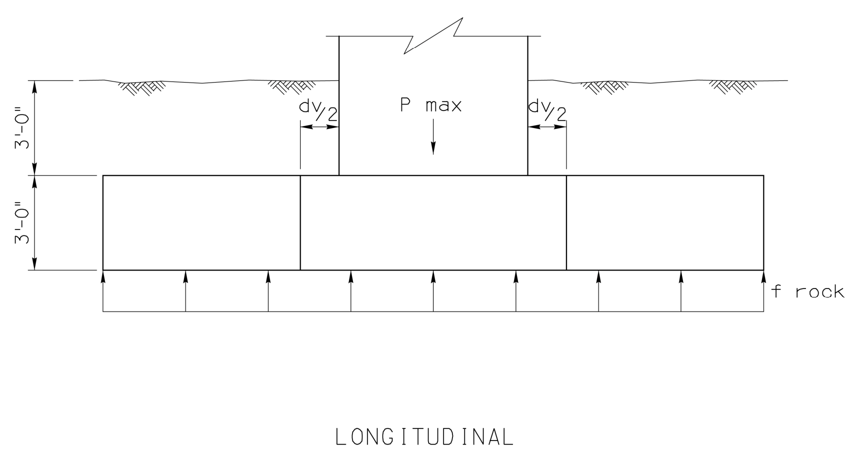


Figure 20

The soil reaction at the base of the footing must be subtracted from this applied load to determine the net shear acting on the footing. Since all four sides of the perimeter are considered, the effect of the bending moments is canceled and can be ignored.

P_{max} = 1.25(1263.6) + 1.50(66.2) + 1.35(130.7) + 1.75(297.4) + 1.00(13.3)
= 2389 kips

f_{soil} = 2389 / 420 = 5.688 ksf

R_{soil} = 5.688(100.97)(78.54) / 144 = 313.2 kips

V_{2way} = 1976.3 + 31.0 + 13.8 – 313.2 = 1707.9 kips

The equation for the nominal shear resistance of the concrete is:

[5.13.3.6.3-1]

$$V_n = \left(0.063 + \frac{0.126}{\beta_c}\right)\sqrt{f'_c}\,b_o d_v \le 0.126\sqrt{f'_c}\,b_o d_v$$

$$\beta_c = \frac{21.00}{20.00} = 1.05$$

$$V_n = \left(0.063 + \frac{0.126}{1.05}\right)\sqrt{3.5}\cdot(359.02)\cdot(29.76) = 3658 \text{ kips}$$

$$V_n \le 0.126\sqrt{3.5}\cdot(359.02)\cdot(29.76) = 2519 \text{ kips}$$

The shear strength of the footing for two-way shear from the concrete is:

φV_n = (0.90)(2519) = 2267 kips ≥ V_u = 1708 kips

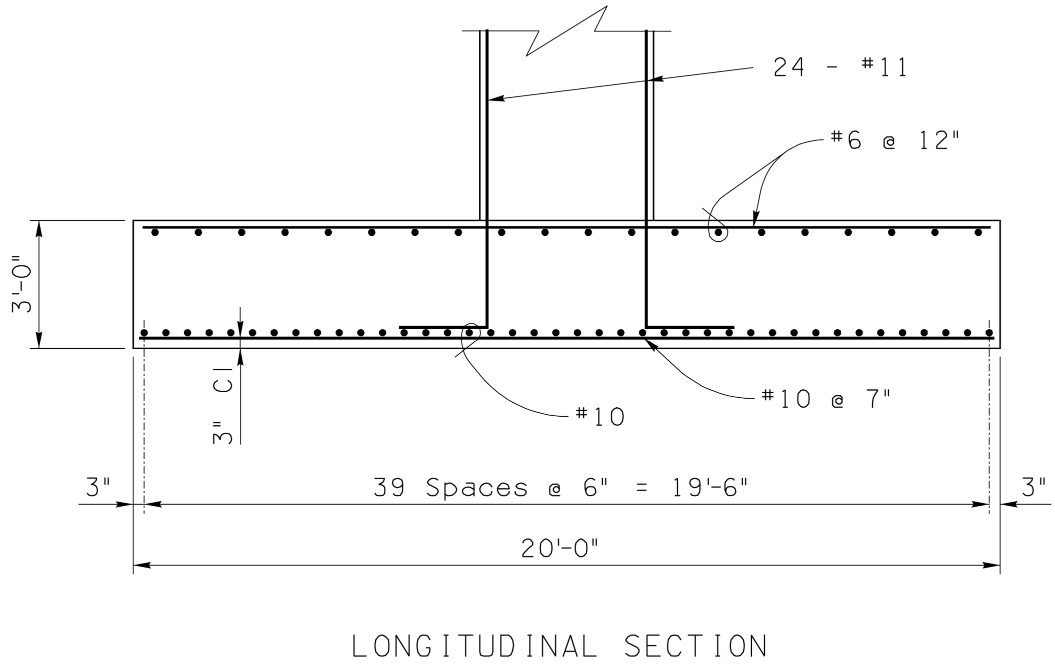
∴The two-way shear criterion is satisfied.

Development Length [5.11.2.4]

The vertical column reinforcing must be adequately developed into the footing. The #11 reinforcement will be hooked with an available embedment length of 36.00 – 3.00 clr – 1.27(2) = 30.5 inches. The required development length of a hooked reinforcing bar is:

$$l_{hb} = \frac{38.0 d_b}{\sqrt{f'_c}} = \frac{(38.0)\cdot(1.41)}{\sqrt{3.5}} = 28.6 \text{ in}$$

Since the available length is greater than the required, the development length of the vertical column reinforcing into the footing is satisfied.


24 - #11
#6 @ 12"
3'-0"
3" Cl
#10
#10 @ 7"
3"
39 Spaces @ 6" = 19'-6"
3"
20'-0"
LONGITUDINAL SECTION

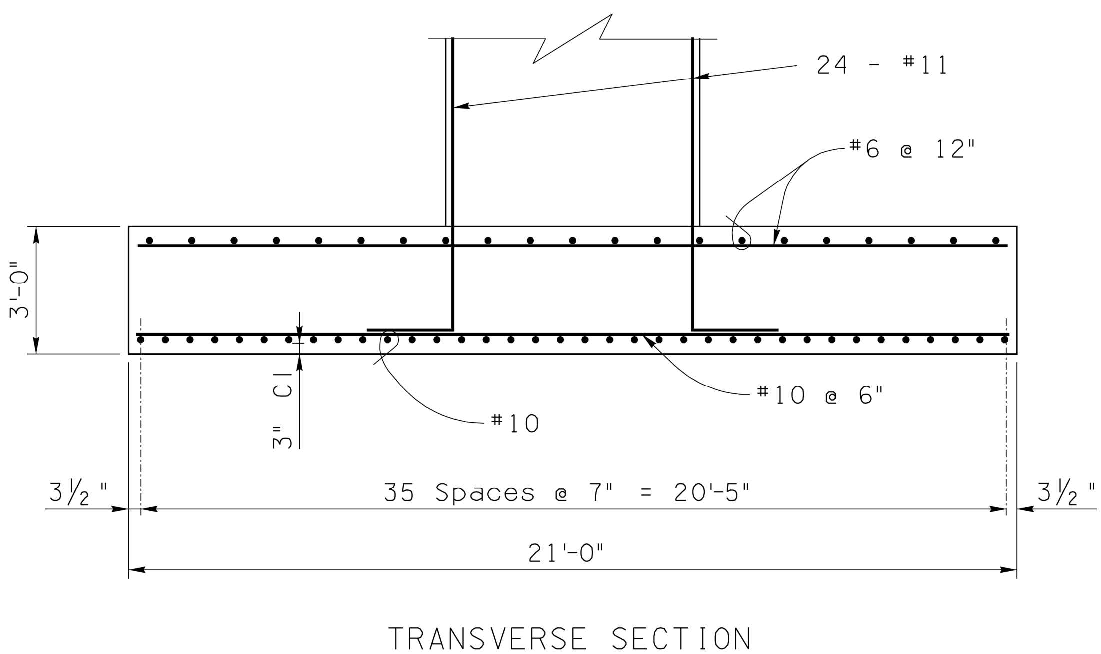

24 - #11
#6 @ 12"
3'-0"
3" Cl
#10
#10 @ 6"
3½"
35 Spaces @ 7" = 20'-5"
3½"
21'-0"
TRANSVERSE SECTION

Figure 21